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Letter to the Editor

Reply to Lajaunie's Comments

Thank you to Ch. Lajaunie for nicely rephrasing the essence of our paper "Indicator Principal Component Kriging" (Suro-Pérez and Journel, 1991, referred hereafter as SPJ), and for bringing up some interesting points for discussion.

- With regard to Lajaunie's footnote, we maintain our remark at the top of p. 763 in SPJ about the major difference between Disjunctive Kriging (DK) and Indicator Cokriging (CoIK). There is no mix-up between estimation method and bivariate distribution models: any conditional cumulative distribution function (ccdf) can be seen as the projection of the proper indicator transform onto the space of functions of all the data. The CoIK or Indicator Principal Component Kriging (IPCK) ccdf model is simply the projection of that projection onto a specific subspace.
- Can the factors be orthogonal?

Principal Component Analysis (PCA) and Correspondence Analysis (CA) can provide only approximate orthogonal factors ($Y_k(x)$), whether the orthogonalization is achieved for one (PCA) or two lags (CA). In the case of PCA, we have documented in SPJ and elsewhere that use of cumulative indicators achieve almost orthogonal factors (Suro-Pérez, 1988; Suro-Pérez and Journel, 1990). In all the cases we have considered, the level of crosscovariance is negligible for any h. However, application of IPCK does require verification of the approximate orthogonality between factors, as recommended in SPJ.

- Does the model produce valid indicator covariances (after back transform through matrix A)?

Not necessarily, as Lajaunie rightfully points out. However, the point is academic since we are not after building bivariate distribution models but modeling ccdf's. Such indicator covariances after back transform are never called for in the IPCK system. Order relations corrections which are a must for all IK approaches (and also DK) ensures a licit ccdf.

Correspondence analysis used as orthogonalization method does not guar-

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antee either that the bivariate distribution obtained by back transform is a licit bivariate distribution. Isofactorial models do not provide either licit bivariate cdf's when not expanded to order ∞.

- We disagree on the point that CA is more appropriate than PCA when looking for the conditional expectation of an indicator variable. Indeed, the correct scalar product is that leading to the conditional expectation which happens to be exactly the (non-centered) indicator covariance as defined in SPJ. Our comparison between CoIK and IPCK (Tables 4 to 7 and Fig. 8 in SPJ) indicates a strong similarity between both estimators, with IPCK requiring much less modeling effort.
- Regarding of the choice of class indicators or cumulative indicators, we advocate the former for modeling conditional probabilities of categorical variables (Suro-Pérez and Journel, 1990) and the latter for modeling ccdf's of continuous variables. Indeed, in the case of continuous variables, cumulative indicator data carry information from one cutoff to another, thus reducing the need for CoIK and supporting the use of Indicator Kriging (IK) as a further approximation to model ccdf's (see Tables 4–7 and Fig. 9 in SPJ). Furthermore, inference of class indicator autocovariances is generally more difficult.
- Change of support models.

The DK approach provides change of support models only through severe additional hypotheses that go well beyond the initial point–point bivariate model; there is no way to check *a priori* such hypotheses. We prefer the approach of short scale simulations, computer intensive indeed, but which allows some control on the underlying hypotheses, essentially, the short scale bivariate distribution (see Isaaks, 1990).

Again it bears repeating that the major difference between CoIK-IPCK-IK on the one side and DK on the other is the decision in the former case to extract from data more than a single covariance function. Besides order relation problems, we have not met "unfortunate consequences" or "unacceptable inconsistencies" in the IPCK application. On the contrary, it has been our experience that IPCK models are consistent approximations to CoIK models, requiring less modeling and fewer systems solving.

V. Suro-Pérez and A. G. Journel
Department of Applied Earth Science
Stanford University
Stanford, California 94305

REFERENCES

Isaaks, E. H., 1990, The application of Monte Carlo methods to the analysis of spatially correlated data: Ph.D. thesis, Stanford University.

Suro-Pérez, V., 1988, Indicator kriging based on principal component analysis: MS thesis, Stanford University.

Suro-Pérez, V., and Journel, A. G., 1990, Stochastic Simulation of Lithofacies: An Improved Sequential Indicator Approach, *in* Guerillot and Guillon (Ed.), Proc. of 2nd Europ. Conf. on Math of Oil Recov: Technip, Paris, p. 3–10.

Suro-Pérez, V., and Journel, A. G., 1991, Indicator Principal Component Kriging: Math. Geol., v. 23, n. 5 p. 759–788.