

Pregunta 1

$$TF(f(t)) = \int_{-\infty}^{\infty} e^{-2\pi j\nu t} e^{-at} \sin(bt) dt$$

Como

$$\sin(bt) = \frac{e^{jbt} - e^{-jbt}}{2j}$$

Entonces:

$$\begin{aligned} TF(f(t)) &= \int_{-\infty}^{\infty} e^{-2\pi j\nu t} e^{-at} \frac{e^{jbt} - e^{-jbt}}{2j} dt \\ &= \frac{1}{2j} \left[\int_{-\infty}^{\infty} e^{(-2\pi j\nu + jb - a)t} dt - \int_{-\infty}^{\infty} e^{-(2\pi j\nu - jb + a)t} dt \right] \\ &= \frac{1}{2j} \left[\frac{e^{(-2\pi j\nu + jb - a)t}}{(-2\pi j\nu + jb - a)} \Big|_0^\infty - \frac{e^{-(2\pi j\nu - jb + a)t}}{(2\pi j\nu - jb + a)} \Big|_0^\infty \right] \\ &= \frac{1}{2j} \left[\frac{1}{(2\pi j\nu + a - jb)} - \frac{1}{(2\pi j\nu + a + jb)} \right] \end{aligned}$$

Pregunta 2 Se pueden aplicar las propiedades de escalamiento y desplazamiento que queda:

$$TF(g(at) \cdot e^{ibt}) = \frac{1}{a} G\left(\frac{\nu}{a} - \frac{b}{2\pi a}\right)$$

Se puede demostrar por definición.

Pregunta 3

$$TF(f(t)\cos(2\pi\nu_0 t)) = TF(f(t)) * \frac{e^{2\pi j\nu_0 jt} + e^{-2\pi j\nu_0 t}}{2}$$

$$= \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{-2\pi j\nu t} f(t) e^{2\pi j\nu_0 jt} dt + \int_{-\infty}^{\infty} e^{-2\pi j\nu t} f(t) e^{-2\pi j\nu_0 jt} dt \right]$$

$$= \frac{1}{2} \left[\int_{-\infty}^{\infty} e^{-2\pi j(\nu - \nu_0)t} f(t) dt + \int_{-\infty}^{\infty} e^{-2\pi j(\nu + \nu_0)t} f(t) dt \right]$$

$$= \frac{1}{2}[F(\nu - \nu_0) + F(\nu + \nu_0)]$$

Pregunta 4 Dem. que: $\delta(t - t_0) = \int_{-\infty}^{\infty} e^{2\pi j\nu(t-t_0)} d\nu$

$$TF(\delta(t - t_0)) = \int_{-\infty}^{\infty} e^{-2\pi j\nu t} \delta(t - t_0) dt = e^{-2\pi j\nu t_0}$$

Aplicando Transformada de Fourier Inversa:

$$\delta(t - t_0) = \int_{-\infty}^{\infty} e^{2\pi j\nu t} e^{-2\pi j\nu t_0} d\nu = \int_{-\infty}^{\infty} e^{2\pi j\nu(t-t_0)} d\nu$$

Pregunta 5

$$TF(\delta(t - t_0) * \cos(2\pi\nu_0 t)) = TF(\delta(t - t_0)) \cdot TF(\cos(2\pi\nu_0 t))$$

$$TF(\delta(t - t_0) * \cos(2\pi\nu_0 t)) = \int_{-\infty}^{\infty} e^{-2\pi j\nu t} \delta(t - t_0) dt \cdot TF(\cos(2\pi\nu_0 t))$$

$$TF(\delta(t - t_0) * \cos(2\pi\nu_0 t)) = e^{-2\pi j\nu t_0} \cdot TF(\cos(2\pi\nu_0 t))$$

Expresando el cos en forma exponencial se tiene:

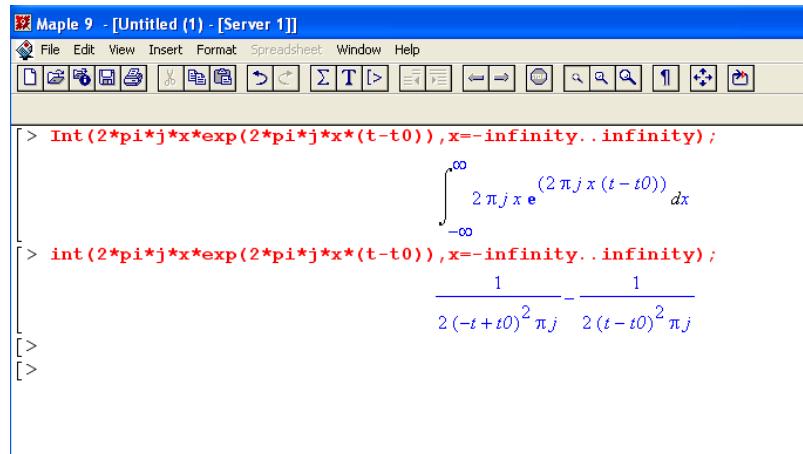
$$TF(\delta(t - t_0) * \cos(2\pi\nu_0 t)) = e^{-2\pi j\nu t_0} \cdot [\delta(\nu - \nu_0) + \delta(\nu + \nu_0)]$$

Pregunta 5

$$TF\left(\frac{d\delta(t - t_0)}{dt}\right) = 2\pi j\nu TF(\delta(t - t_0))$$

$$TF\left(\frac{d\delta(t - t_0)}{dt}\right) = 2\pi j\nu e^{2\pi j\nu t_0}$$

Aplicando la transformada inversa se tiene que la derivada puede expresarse como en la figura donde $x = \nu$:



The screenshot shows a Maple 9 interface with a menu bar (File, Edit, View, Insert, Format, Spreadsheet, Window, Help) and a toolbar with various mathematical symbols. The main workspace displays the following code and its resulting output:

```
> Int(2*pi*j*x*exp(2*pi*j*x*(t-t0)),x=-infinity..infinity);
          ∫∞
          2 π j x e^(2 π j x (t - t0)) dx
-> int(2*pi*j*x*exp(2*pi*j*x*(t-t0)),x=-infinity..infinity);
          1           1
          2 (-t + t0)² π j - 2 (t - t0)² π j
[>
[>
```