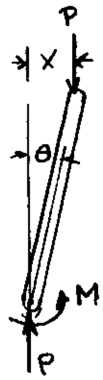
CHAPTER 10

PROBLEM 10.1

10.1 Knowing that the torsional spring at B is of constant K and that the bar AB is rigid, determine the critical load P_{cr}.



SOLUTION

Let θ be the angle change of bar AB.

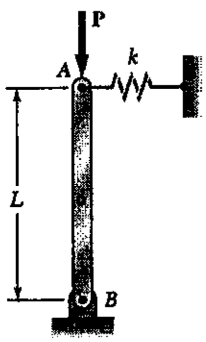
$M = K\theta \quad , \quad x = L\sin\theta \approx L\theta$

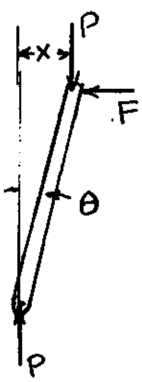
$\circlearrowleft M_B = 0 \quad M - Px = 0 \quad K\theta - PL\theta = 0$

$(K - PL)\theta = 0 \quad\quad P_{cr} = K/L$ ◄

PROBLEM 10.2

10.2 Knowing that the spring at A is of constant k and that the bar AB is rigid, determine the critical load P_{cr}.





SOLUTION

Let θ be the angle change of bar AB.

$F = kx = kL\sin\theta$

$\circlearrowleft \Sigma M_B = 0 \quad FL\cos\theta - Px = 0$

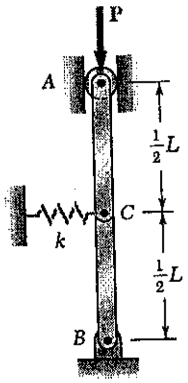
$kL^2\sin\theta\cos\theta - PL\sin\theta = 0$

Using $\sin\theta \approx \theta$ and $\cos\theta \approx 1 \quad\quad kL^2\theta - PL\theta = 0$

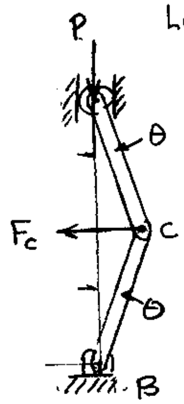
$(kL^2 - PL)\theta = 0 \quad\quad P_{cr} = kL$ ◄

PROBLEM 10.3

10.3 Two rigid bars *AC* and *BC* are connected as shown to a spring of constant *k*. Knowing that the spring can act in either tension or compression, determine the critical load P_{cr} for the system.



SOLUTION



Let x be the lateral deflection of point C

$x = \frac{1}{2}L \sin\theta \qquad F_c = kx = \frac{1}{2}kL\sin\theta$

Joint C: $+\uparrow \Sigma F_y = 0 \quad F_{AC}\cos\theta - F_{CB}\cos\theta = 0$

$F_{AC} = F_{CB}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad F_{AB}\sin\theta + F_{CB}\sin\theta - F_c = 0$

$-2F_{AB}\sin\theta - \frac{1}{2}kL\sin\theta = 0$

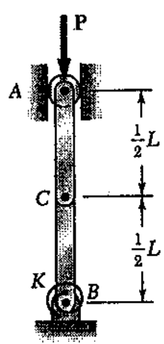
$-(F_{AB} + \frac{1}{4}kL)\sin\theta = 0 \qquad F_{AB} = -\frac{1}{4}kL$

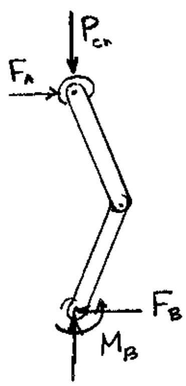
Joint A: $\Sigma F_y = 0 \qquad -P - F_{AB}\cos\theta = 0 \qquad P = -F_{AB}\cos\theta = \frac{1}{4}kL\cos\theta$

With $\theta \to 0 \qquad P_{cr} = \frac{1}{4}kL$

PROBLEM 10.4

10.4 Two rigid bars *AC* and *BC* are connected by a pin at *C* as shown. Knowing that the torsional spring at *B* is of constant *K*, determine the critical load P_{cr} for the system.





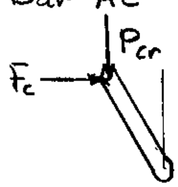
SOLUTION

Let θ be the angle change of each bar.

$M_B = K\theta$

$\circlearrowleft M_B = 0 \qquad K\theta - F_A L = 0 \qquad F_A = \frac{K\theta}{L}$

Bar AC $\quad \circlearrowleft \Sigma M_c = 0$

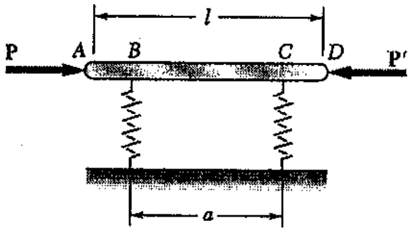


$P_{cr}\frac{1}{2}L\theta - \frac{1}{2}L F_A = 0$

$P_{cr} = \frac{F_A}{\theta} = \frac{K}{L}$

PROBLEM 10.5

10.5 The rigid bar AD is attached to two springs of constant k and is in equilibrium in the position shown. Knowing that the equal and opposite loads **P** and **P′** *remain horizontal*, determine the magnitude P_{cr} of the critical load for the system.



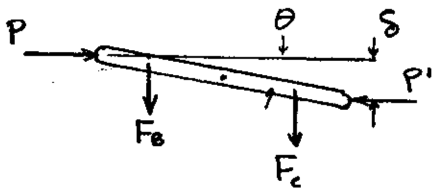
SOLUTION

Let y_B and y_C be the deflections of points B and C, positive upward.

Then $\quad F_B = -k y_B \qquad F_C = -k y_C$

$+\uparrow \Sigma F_y = 0 \qquad F_B + F_C = 0 \qquad F_C = -F_B$

$y_C = -y_B \qquad F_B$ and F_C form a couple ⟲



Let θ be the angle change: $\quad y_B = -y_C = \frac{1}{2} a \sin\theta, \qquad \delta = l \sin\theta$

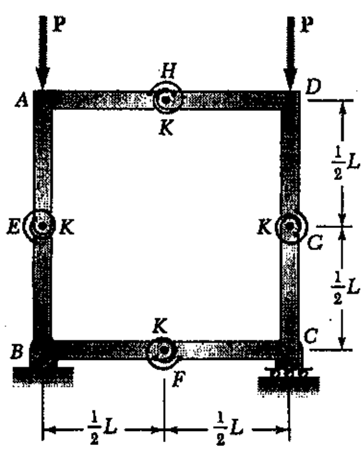
P and P′ form a couple ⟳ of amount $P\delta$

$+⟲ \Sigma M = 0; \quad k\left(\frac{1}{2} a \sin\theta\right) a \cos\theta - P l \sin\theta = 0 \qquad P = \frac{ka^2}{2l}\cos\theta$

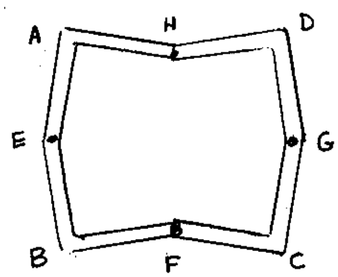
Let $\theta \to 0 \qquad P_{cr} = \dfrac{ka^2}{2l}$ ◄

PROBLEM 10.6

10.6 A frame consists of four L-shaped members connected by four torsional springs, each of constant K. Knowing that equal loads **P** are applied at points A and D as shown, determine the critical value P_{cr} of the loads applied to the frame.

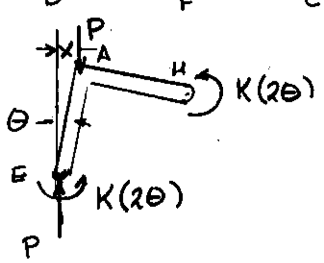


SOLUTION



Let θ be the rotation of each L-shaped member.

Angle change across each torsional spring is 2θ



$x = \frac{1}{2} L \sin\theta \approx \frac{1}{2} L\theta$

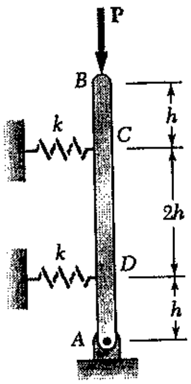
$\Sigma M_E = 0$

$K(2\theta) + K(2\theta) - Px = 0$

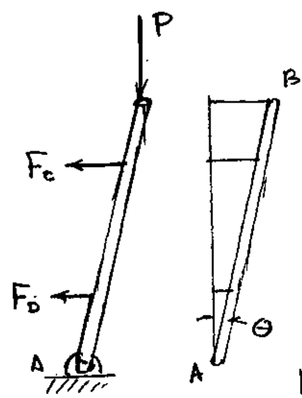
$P_{cr} = \dfrac{4K\theta}{x} = \dfrac{8K}{L}$ ◄

PROBLEM 10.7

10.7 The rigid rod AB is attached to a hinge at A and to two springs, each of constant $k = 2.0$ kip/in., that can act in either tension or compression. Knowing that $h = 2.0$ ft, determine the critical load.



SOLUTION



Let θ be the small rotation angle

$x_D \approx h\theta$, $x_C \approx 3h\theta$, $x_B \approx 4h\theta$

$F_C = kx_C \approx 3kh\theta$

$F_D = kx_D \approx kh\theta$

$\circlearrowleft \Sigma M_A = 0 \qquad hF_D + 3hF_C - Px_B = 0$

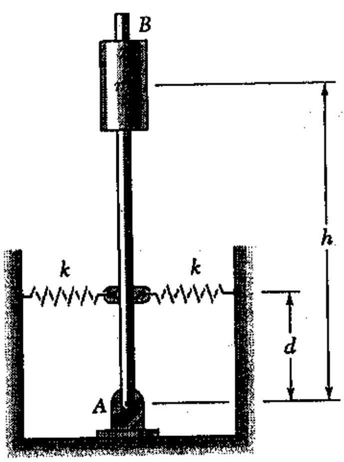
$kh^2\theta + 9kh^2\theta - 4hP = 0, \quad P = \frac{5}{2}kh$

Data: $k = 2.0$ kip/in , $h = 2$ ft $= 24$ in

$P = \frac{5}{2}(2.0)(24) = 120$ kips.

PROBLEM 10.8

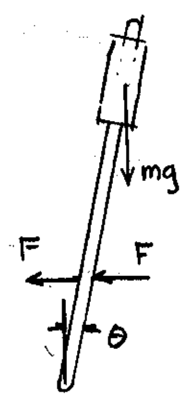
10.8 If $m = 125$ kg, $h = 700$, and the constant of each spring is $k = 2.8$ kN/m, determine the range of values of the distance d for which the equilibrium of the rigid rod AB is stable in the position shown. Each spring can act in either tension or compression.



SOLUTION

$h = 700$ mm. $= 700 \times 10^{-3}$ m

Let θ be the small rotation of AB



$x = d\theta \qquad F = kx = kd\theta$

$\circlearrowleft \Sigma M_A = 0 \qquad 2Fd - mgh\theta = 0$

$2kd^2\theta - mgh = 0$

$d_{cr}^2 = \frac{mgh}{2k}$

$d_{cr} = \sqrt{\frac{mgh}{k}} = \sqrt{\frac{(125)(9.81)(700 \times 10^{-3})}{(2)(2.8 \times 10^3)}}$

$= 0.392$ m $= 392$ mm

$d > 392$ mm for stability

PROBLEM 10.9

10.9 Determine the critical load of a round wooden dowel that is 48-in. long and has a diameter of (*a*) 0.375 in., (*b*) 0.5 in. Use $E = 1.6 \times 10^6$ psi.

SOLUTION

(a) $c = \frac{1}{2}d = 0.1875$ in $\qquad I = \frac{\pi}{4}c^4 = 970.7 \times 10^{-6}$ in^4

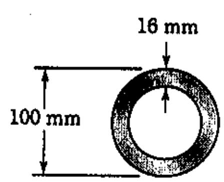
$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(1.6\times10^6)(970.7\times10^{-6})}{(48)^2} = 6.65 \text{ lb.}$$

(b) $c = \frac{1}{2}d = 0.25$ in. $\qquad I = \frac{\pi}{4}c^4 = 3.068 \times 10^{-3}$ in^4

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(1.6\times10^6)(3.068\times10^{-3})}{(48)^2} = 21.0 \text{ lb.}$$

PROBLEM 10.10

10.10 Determine the critical load of a steel tube that is 5.0 m long and has a 100-mm outer diameter and a 16 mm wall thickness. Use $E = 200$ GPa.



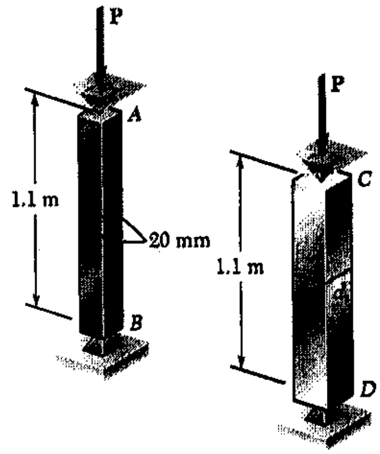
SOLUTION

$c_o = \frac{1}{2}d_o = 50$ mm $\qquad c_i = c_o - t = 50 - 16 = 34$ mm

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 3.859 \times 10^6$ mm^4 $= 3.859 \times 10^{-6}$ m^4

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(200\times10^9)(3.859\times10^{-6})}{(5.0)^2} = 305\times10^3 \text{ N} = 305 \text{ kN}$$

PROBLEM 10.11



Brass
$E = 120$ GPa
$\rho = 8740$ kg/m^3

Aluminum
$E = 70$ GPa
$\rho = 2710$ kg/m^3

10.11 Determine (*a*) the critical load for the brass strut, (*b*) the dimension *d* for which the aluminum strut will have the same critical load, (*c*) the weight of the aluminum strut as a percent of the weight of the brass strut.

SOLUTION

(a) Brass strut $\quad I = \frac{1}{12}(20)(20)^3 = 13.333 \times 10^3 \text{ mm}^4 = 13.333 \times 10^{-9} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 E_b I_b}{L^2} = \frac{\pi^2 (120 \times 10^9)(13.333 \times 10^{-9})}{(1.1)^2}$$

$$= 13.06 \times 10^3 \text{ N} = 13.06 \text{ kN}$$

(b) Aluminum strut

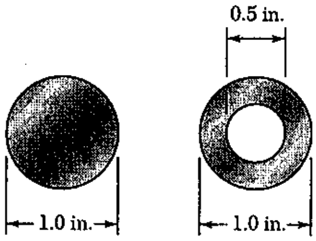
$$P_{cr} = \frac{\pi^2 E_a I_a}{L^2} = \frac{\pi^2 E_a (d^4/12)}{L^2}$$

$$d^4 = \frac{12 P_{cr} L^2}{\pi^2 E_a} = \frac{(12)(13.06 \times 10^3)(1.1)^2}{\pi^2 (70 \times 10^9)} = 274.3 \times 10^{-9} \text{ m}^4$$

$$d = 22.9 \times 10^{-3} \text{ m} = 22.9 \text{ mm}$$

(c) $$\frac{m_a}{m_b} = \frac{\gamma_a L d^2}{\gamma_b L d_b^2} = \left(\frac{\gamma_a}{\gamma_b}\right)\left(\frac{d}{d_b}\right)^2 = \left(\frac{2710}{8740}\right)\left(\frac{22.9}{20}\right)^2 = 0.406 = 40.6\%$$

PROBLEM 10.12



10.12 A compression member of 20 in. effective length consists of a solid 1.0-in.-diameter aluminum rod. In order to reduce the weight of the member by 25%, the solid rod is replaced by a hollow rod of the cross section shown. Determine (*a*) the percent reduction in the critical load, (*b*) the value of the critical load for the hollow rod. Use $E = 10.6 \times 10^6$ psi.

SOLUTION

Solid $\quad A_s = \frac{\pi}{4} d_o^2 \qquad I_s = \frac{\pi}{4}\left(\frac{d_o}{2}\right)^4 = \frac{\pi}{64} d_o^4$

Hollow: $\quad A_H = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{3}{4} A_s = \frac{3}{4}\frac{\pi}{4} d_o^2$

$$d_i^2 = \frac{1}{4} d_o^2 \qquad d_i = \frac{1}{2} d_o = 0.5 \text{ in.}$$

Solid rod: $\quad I_s = \frac{\pi}{64}(1.0)^4 = 0.049087 \text{ in}^4$

$$P_{cr} = \frac{\pi^2 E I_s}{L^2} = \frac{\pi^2 (10.6 \times 10^6)(0.049087)}{(20)^2} = 12.839 \times 10^3 \text{ lb.}$$

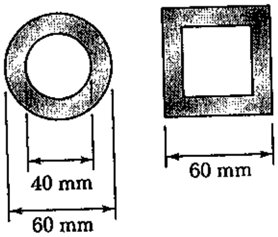
Hollow rod: $\quad I_H = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}\left[(1)^4 - \left(\frac{1}{2}\right)^4\right] = 0.046019 \text{ in}^4$

(b) $\quad P_{cr} = \frac{\pi^2 E I_H}{L^2} = \frac{\pi^2 (10.6 \times 10^6)(0.046019)}{(20)^2} = 12.036 \times 10^3 \text{ lb.} = 12.04 \text{ kips}$ ◄

(a) $\quad \frac{P_s - P_H}{P_s} = \frac{12.839 \times 10^3 - 12.036 \times 10^3}{12.839 \times 10^3} = 0.0625 = 6.25\%$ ◄

PROBLEM 10.13

10.13 Two brass rods used as compression members, each of 3-m effective length, have the cross sections shown. (*a*) Determine the wall thickness of the hollow square rod for which the rods have the same cross-sectional area. (*b*) Using $E = 105$ GPa, determine the critical load of each rod.



SOLUTION

(a) Same area $\quad \frac{\pi}{4}(d_o^2 - d_i^2) = b_o^2 - b_i^2$

$$b_i^2 = b_o^2 - \frac{\pi}{4}(d_o^2 - d_i^2)$$

$$= 60^2 - \frac{\pi}{4}(60^2 - 40^2) = 2.0292 \text{ mm}^2$$

$$b_i = 45.047 \text{ mm} \qquad t = \frac{1}{2}(b_o - b_i) = 7.48 \text{ mm}$$

(b) Circular: $\quad I = \frac{\pi}{64}(d_o^4 - d_i^4) = 510.51 \times 10^3 \text{ mm}^4 = 510.51 \times 10^{-9} \text{ m}^4$

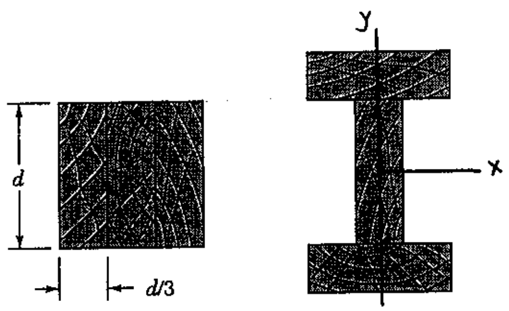
$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(105 \times 10^9)(510.51 \times 10^{-9})}{(3.0)^2} = 58.8 \times 10^3 \text{ N} = 58.8 \text{ kN}$$

Square: $\quad I = \frac{1}{12}(b_o^4 - b_i^4) = 736.85 \times 10^3 \text{ mm}^4 = 736.85 \times 10^{-9} \text{ m}^4$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(105 \times 10^9)(736.85 \times 10^{-9})}{(3.0)^2} = 84.8 \times 10^3 \text{ N} = 84.8 \text{ kN}$$

PROBLEM 10.14

10.14 A column of effective length L can be made by gluing together identical planks in each of the arrangements shown. Determine the ratio of the critical load using the arrangement *a* to the critical load using the arrangement *b*.



(*a*) (*b*)

SOLUTION

Arrangement (a)

$$I_a = \frac{1}{12} d^4$$

$$P_{cr,a} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E d^4}{12 L_e^2}$$

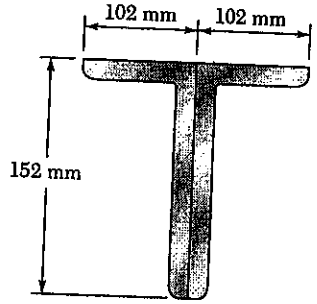
Arrangement (b) $\quad I_{min} = I_y = \frac{1}{12}\left(\frac{d}{3}d^3\right) + \frac{1}{12}(d)\left(\frac{d}{3}\right)^3 + \frac{1}{12}\left(\frac{d}{3}\right)(d)^3 = \frac{19}{324} d^4$

$$P_{cr,b} = \frac{\pi^2 EI}{L_e^2} = \frac{19\pi^2 E d^4}{324 L_e^2}$$

$$\frac{P_{cr,a}}{P_{cr,b}} = \frac{1}{12} \cdot \frac{324}{19} = \frac{27}{19} = 1.421$$

PROBLEM 10.15

10.15 A compression member of 7-m effective length is made by welding together two L152 × 102 × 12.7 angles as shown. Using E = 200 GPa, determine the allowable centric load for the member if a factor of safety of 2.2 is required.



SOLUTION

Angle L 152 × 102 × 12.7 $\quad A = 3060\ \text{mm}^2$

$I_x = 7.20 \times 10^6\ \text{mm}^4 \qquad I_y = 2.64 \times 10^6\ \text{mm}^4$

$y = 50.3\ \text{mm} \qquad x = 25.3\ \text{mm}$

Two angles: $I_x = (2)(7.20 \times 10^6) = 14.00 \times 10^6\ \text{mm}^4$

$I_y = 2[2.64 \times 10^6 + (3060)(25.3)^2] = 9.197 \times 10^6\ \text{mm}^4$

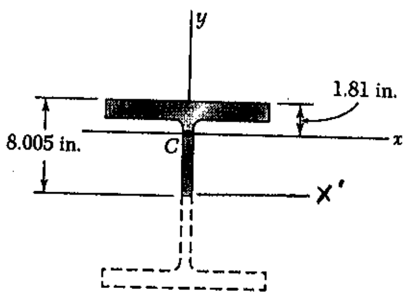
$I_{min} = I_y = 9.197 \times 10^6\ \text{mm}^4 = 9.197 \times 10^{-6}\ \text{m}^4$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(9.197 \times 10^{-6})}{(7.0)^2} = 370.5 \times 10^3\ \text{N} = 370.5\ \text{kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{370.5}{2.2} = 168.4\ \text{kN}$$

PROBLEM 10.16

10.16 A column of 26-ft effective length is made from half a W16 × 40 rolled-steel shape. Knowing that the centroid of the cross section is located as shown, determine the factor of safety if the allowable centric load is 20 kips. Use $E = 29 \times 10^6$ psi.



SOLUTION

Full W 16 × 40 $\quad A = 11.8\ \text{in}^2$

$I_x = 518\ \text{in}^4, \quad I_y = 28.9\ \text{in}^4$

Half W 16 × 40

$A = (\tfrac{1}{2})(11.8) = 5.90\ \text{in}^2$

$I_x = \tfrac{1}{2}(518) - (5.90)(8.005 - 1.81)^2 = 32.57\ \text{in}^4$

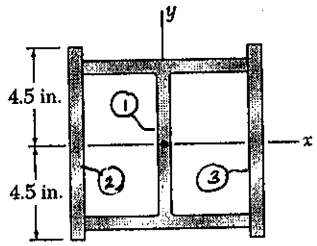
$I_y = \tfrac{1}{2}(28.9) = 14.45\ \text{in}^4 = I_{min}$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 (29 \times 10^6)(14.45)}{(26 \times 12)^2} = 42.5 \times 10^3\ \text{lb} = 42.5\ \text{kips}$$

$$P_{all} = \frac{P_{cr}}{F.S.} \qquad F.S. = \frac{P_{cr}}{P_{all}} = \frac{42.5}{20} = 2.125$$

PROBLEM 10.17

10.17 A column of 22-ft effective length is to be made by welding two 9 × 0.5 in. plates to a W8 × 35 as shown. Determine the allowable centric load if a factor of safety of 2.3 is required. Use $E = 29 \times 10^6$ psi.



SOLUTION

(1) W 8 × 35 $\quad I_x = 127 \text{ in}^4 \quad I_y = 42.6 \text{ in}^4$

$b_f = 8.02$ in

(2) and (3) For each plate $\quad A = (0.5)(9.0) = 4.5 \text{ in}^2$

$$I_x = \frac{1}{12}(0.5)(9)^3 = 30.375 \text{ in}^4$$

$$I_y = \frac{1}{12}(9)(0.5)^3 + (4.5)\left(\frac{8.02}{2} + \frac{0.5}{2}\right)^2\Big] = 81.758 \text{ in}^4$$

Total: $\quad I_x = 127 + (2)(30.375) = 187.75 \text{ in}^4 = I_{min}$

$\quad I_y = 42.6 + (2)(81.758) = 206.12$ in

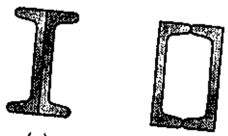
$L = 22$ ft $= 264$ in.

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2(29 \times 10^6)(187.75)}{264^2} = 771.0 \times 10^3 \text{ lb} = 771 \text{ kips}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{771}{2.3} = 335 \text{ kips}$$

PROBLEM 10.18

10.18 A column of 3-m effective length is to be made by welding together two C130 × 13 rolled-steel channels. Using $E = 200$ GPa, determine for each arrangement shown the allowable centric load if a factor of safety of 2.4 is required.



(a) (b)

SOLUTION

For channel C 130 × 13 $\quad A = 1710 \text{ mm}^2 \quad b_f = 48 \text{ mm}$

$I_x = 3.70 \times 10^6 \text{ mm}^4 \quad I_y = 0.264 \times 10^6 \text{ mm}^4 \quad \bar{x} = 12.2 \text{ mm}$

Arrangement (a) $\quad I_x = (2)(3.70 \times 10^6) = 7.40 \times 10^6 \text{ mm}^4$

$$I_y = 2[0.264 \times 10^6 + (1710)(12.2)^2] = 1.0370 \times 10^6 \text{ mm}^2$$

$$I_{min} = I_y = 1.0370 \times 10^6 \text{ mm} = 1.0370 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(1.0370 \times 10^{-6})}{(3.0)^2} = 227 \times 10^3 \text{ N} = 227 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{227}{2.4} = 94.8 \text{ kN}$$

Arrangement (b) $\quad I_x = (2)(3.70 \times 10^6) \text{ mm}^4$

$$I_y = 2[0.264 \times 10^6 + (1710)(48 - 12.2)^2] = 4.911 \times 10^6 \text{ mm}^4$$

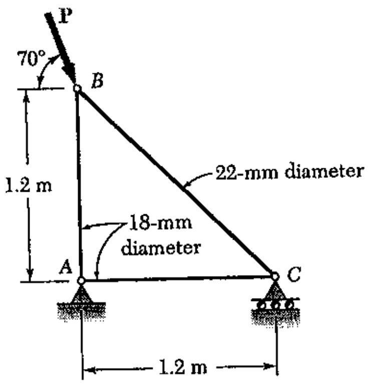
$$I_{min} = I_y = 4.911 \times 10^6 \text{ mm}^4 = 4.911 \times 10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(4.911 \times 10^{-6})}{(3.0)^2} = 1077 \times 10^3 \text{ N} = 1077 \text{ kN}$$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{1077}{2.4} = 449 \text{ kN}$$

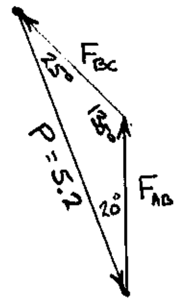
PROBLEM 10.19

10.19 Knowing that $P = 5.2$ kN, determine the factor of safety for the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



SOLUTION

Joint B: From force triangle



$$\frac{F_{AB}}{\sin 25°} = \frac{F_{BC}}{\sin 20°} = \frac{5.2}{\sin 135°}$$

$F_{AB} = 3.1079$ kN (comp)

$F_{BC} = 2.5152$ kN (comp)

Member AB: $I_{AB} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{18}{2}\right)^4 = 5.153 \times 10^3 \text{ mm}^4 = 5.153 \times 10^{-9} \text{ m}^4$

$$F_{AB,cr} = \frac{\pi^2 E I_{AB}}{L_{AB}^2} = \frac{\pi^2(200\times10^9)(5.153\times10^{-9})}{(1.2)^2} = 7.0636\times10^3 \text{ N} = 7.0636 \text{ kN}$$

$$\text{F.S.} = \frac{F_{AB,cr}}{F_{AB}} = \frac{7.0636}{3.1079} = 2.27$$

Member BC: $I_{BC} = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{22}{2}\right)^4 = 11.499\times10^3 \text{ mm}^4 = 11.499\times10^{-9} \text{ m}^4$

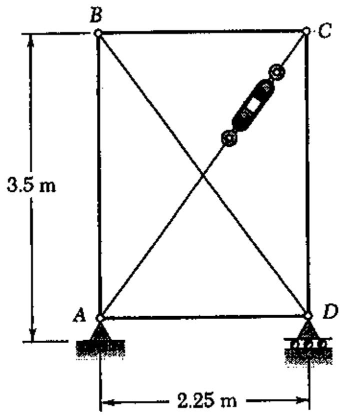
$L_{BC}^2 = 1.2^2 + 1.2^2 = 2.88 \text{ m}^2$

$$F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2(200\times10^9)(11.499\times10^{-9})}{2.88} = 7.8813\times10^3 \text{ N} = 7.8813 \text{ kN}$$

$$\text{F.S.} = \frac{F_{BC,cr}}{F_{BC}} = \frac{7.8813}{2.5152} = 3.13$$

Smallest F.S. governs. F.S. = 2.27

PROBLEM 10.20

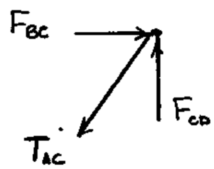


10.20 Members *AB* and *CD* are 30-mm-diameter steel rods, and members *BC* and *AD* are 22-mm-diameter steel rods. When the turnbuckle is tightened, the diagonal member *AC* is put in tension. Knowing that a factor of safety with respect to buckling of 2.75 is required, determine the largest allowable tension in *AC*. Use *E* = 200 GPa and consider only buckling in the plane of the structure.

SOLUTION

$$L_{AC} = \sqrt{(3.5)^2 + (2.25)^2} = 4.1608 \text{ m}$$

Joint C



$$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad F_{BC} - \frac{2.25}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.84926\, F_{BC}$$

$$+\uparrow \Sigma F_y = 0 \qquad F_{CD} - \frac{3.5}{4.1608} T_{AC} = 0$$

$$T_{AC} = 1.1888\, F_{CD}$$

Members BC and AD: $I_{BC} = \frac{\pi}{4}\left(\frac{d_{BC}}{2}\right)^4 = \frac{\pi}{4}\left(\frac{22}{2}\right)^4 = 11.499 \times 10^3 \text{ mm}^4 = 11.499 \times 10^{-9} \text{ m}^4$

$L_{BC} = 2.25 \text{m} \qquad F_{BC,cr} = \frac{\pi^2 E I_{BC}}{L_{BC}^2} = \frac{\pi^2 (200 \times 10^9)(11.499 \times 10^{-9})}{(2.25)^2} = 4.4836 \times 10^3 \text{ N}$

$F_{BC,al} = \frac{F_{BC,cr}}{F.S.} = 1.6304 \times 10^3 \text{ N} \qquad T_{AC,al} = 3.02 \times 10^3 \text{ N}$

Members AB and CD: $I_{CD} = \frac{\pi}{4}\left(\frac{d_{CD}}{2}\right)^4 = \frac{\pi}{4}\left(\frac{30}{2}\right)^4 = 39.761 \times 10^3 \text{ mm}^4 = 39.761 \times 10^{-9} \text{ m}^4$

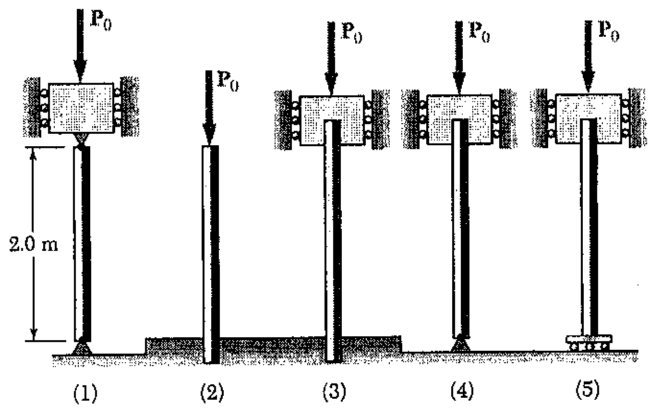
$L_{CD} = 3.5 \text{m} \qquad F_{CD,cr} = \frac{\pi^2 E I_{CD}}{L_{CD}^2} = \frac{\pi^2 (200 \times 10^9)(39.761 \times 10^{-9})}{(3.5)^2} = 6.4069 \times 10^3 \text{ N}$

$F_{CD,al} = \frac{F_{CD,cr}}{F.S.} = 2.3298 \times 10^3 \text{ N} \qquad T_{AC,al} = 2.77 \times 10^3 \text{ N}$

Smaller value for $T_{AC,al}$ governs $\qquad T_{AC,al} = 2.77 \times 10^3 \text{ N} = 2.77 \text{ kN}$ ◄

PROBLEM 10.21

10.21 Each of the five struts consists of an aluminum tube that has a 32-mm outer diameter and a 4-mm wall thickness. Using $E = 70$ GPa and a factor of safety of 2.3, determine the allowable load $\mathbf{P}_0$ for each support condition shown.



SOLUTION

$c_o = \frac{1}{2} d_o = \frac{1}{2}(32) = 16 \text{ mm}$

$c_i = c_o - t = 16 - 4 = 12 \text{ mm}$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 35.1858 \times 10^3 \text{ mm}^4$
$= 35.1858 \times 10^{-9} \text{ m}^4$

$\pi^2 EI = \pi^2 (70 \times 10^9)(35.1858 \times 10^{-9})$
$= 24309 \text{ N}\cdot\text{m}^2$

$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{24309}{L_e^2}$

$P_{all} = \frac{P_{cr}}{F.S.} = \frac{10569}{L_e^2}$

(1) $L_e = (1)(2.0) = 2.0 \text{ m}$, $P_{all} = 2642 \text{ N} = 2.64 \text{ kN}$ ◄

(2) $L_e = (2)(2.0) = 4.0 \text{ m}$, $P_{all} = 661 \text{ N} = 0.661 \text{ kN}$ ◄

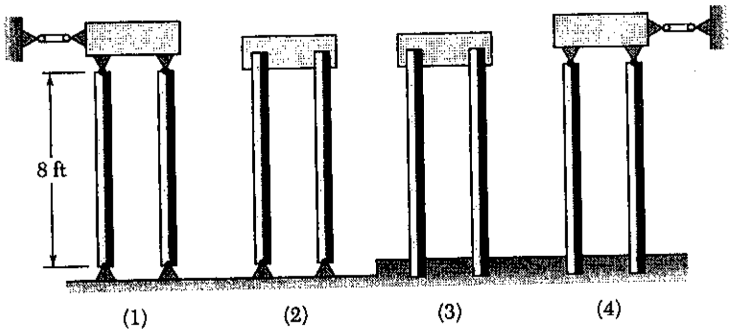
(3) $L_e = (\frac{1}{2})(2.0) = 1.0 \text{ m}$, $P_{all} = 10569 \text{ N} = 10.57 \text{ kN}$ ◄

(4) $L_e = (0.7)(2.0) = 1.4 \text{ m}$, $P_{all} = 5392 \text{ N} = 5.39 \text{ kN}$ ◄

(5) $L_e = (1.0)(2.0) = 2.0 \text{ m}$, $P_{all} = 2642 \text{ N} = 2.64 \text{ kN}$ ◄

PROBLEM 10.22

10.22 Two columns are used to support a block weighing 3.25 kips in each of the four ways shown. (*a*) Knowing that the column of Fig. (1) is made of steel with a 1.25-in.-diameter, determine the factor of safety with respect to buckling for the loading shown. (*b*) Determine the diameter of each of the other columns for which the factor of safety is the same as the factor of safety obtained in part *a*. Use $E = 29 \times 10^6$ psi.



SOLUTION

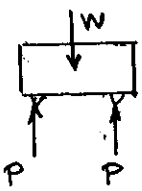
(a) $I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{1.25}{2}\right)^4$

$= 0.119842 \text{ in}^4$

$L = 8 \text{ ft} = 96 \text{ in}$

$P_{cr} = \frac{\pi^2 EI}{L^2}$

$P_{cr} = \frac{\pi^2 (29 \times 10^6)(0.119842)}{(96)^2} = 3722 \text{ lb} = 3.722 \text{ kip.}$ for one column.



$P = \frac{1}{2} W = \frac{3.25}{2} = 1.625 \text{ kip.}$

$F.S. = \frac{P_{cr}}{P} = \frac{3.722}{1.625} = 2.29$

$P_{cr(1)} = \frac{\pi^2 EI}{L^2} \qquad P_{cr(\alpha)} = \frac{\pi^2 E I_\alpha}{(L_{e,\alpha})^2}$

$\frac{P_{cr(\alpha)}}{P_{cr(1)}} = 1 \qquad \frac{I_\alpha}{I_{(1)}} \cdot \frac{L^2}{L_e^2} = 1 \qquad \left(\frac{d_\alpha}{d_{(1)}}\right)^4 \left(\frac{L_e}{L}\right)^2 = 1$

$d_\alpha = d_{(1)} \sqrt{\frac{L_{e(\alpha)}}{L}}$

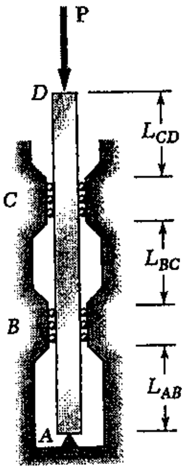
(2) $L_{e(2)} / L = 2.0 \qquad d_{(2)} = 1.25 \sqrt{2.0} = 1.768 \text{ in.}$

(3) $L_{e(3)} / L = 1.0 \qquad d_{(3)} = 1.25 \text{ in.}$

(4) $L_{e(4)} / L = 0.7 \qquad d_{(4)} = 1.25 \sqrt{0.7} = 1.046 \text{ in.}$

PROBLEM 10.23

10.23 A 25-mm-square aluminum strut is maintained in the position shown by a pin support at A and by sets of rollers at B and C that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 1.0$ m, $L_{BC} = 1.25$ m, and $L_{CD} = 0.5$ m, determine the allowable load **P** using a factor of safety with respect to buckling of 2.8. Consider only buckling in the plane of the figure and use $E = 75$ GPa.



SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(25)^3 = 32.552\times10^3 \text{ mm}^3 = 32.552\times10^{-9} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} \qquad P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{(F.S.)L_e^2} = \frac{\pi^2 EI}{(F.S.)(L_{e,max})^2}$$

Portion AB: $L_e = 0.7\,L_{AB} = (0.7)(1.0) = 0.7$ m

Portion BC: $L_e = 0.5\,L_{BC} = (0.5)(1.25) = 0.625$ m

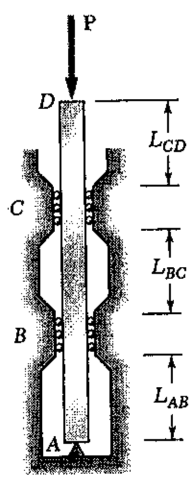
Portion CD: $L_e = 2\,L_{CD} = (2.0)(0.5) = 1.0$ m

$L_{e,max} = 1.0$ m

$$P_{all} = \frac{\pi^2(75\times10^9)(35.552\times10^{-9})}{(2.8)(1.0)^2} = 8.61\times10^3 \text{ N} = 8.61 \text{ kN}$$

PROBLEM 10.24

10.24 A 32-mm-square aluminum strut is maintained in the position shown by a pin support at A and by sets of rollers at B and C that prevent rotation of the strut in the plane of the figure. Knowing that $L_{AB} = 1.4$ m, determine (*a*) the largest values of L_{BC} and L_{CD} that may be used if the allowable load **P** is to be as large as possible, (*b*) the magnitude of the corresponding allowable load if the factor of safety is 2.8. Consider only buckling in the plane of the figure and use $E = 72$ GPa.



SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(32)(32)^3 = 87.381\times10^3 \text{ mm}^3 = 87.381\times10^{-9} \text{ m}^4$$

Equivalent lengths: AB $\quad L_e = 0.7\,L_{AB} = 0.98$ m

BC $\quad L_e = 0.5\,L_{BC}$

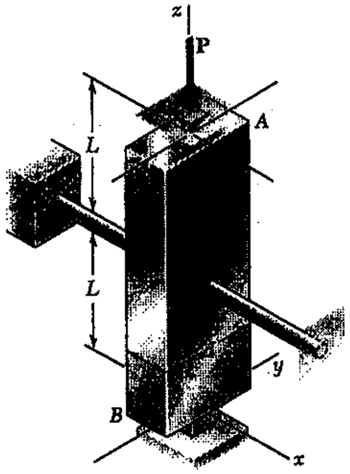
CD $\quad L_e = 2\,L_{CD}$

Equating $\quad L_{BC} = \frac{0.7}{0.5}L_{AB} = (1.4)(1.4) = 1.96$ m

$L_{CD} = \frac{0.7}{2}L_{AB} = (0.35)(1.4) = 0.49$ m

(b) $$P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 EI}{(F.S.)L_e^2} = \frac{\pi^2(72\times10^9)(87.381\times10^{-9})}{(2.8)(0.98)^2} = 23.1\times10^3 \text{ N} = 23.1 \text{ kN}$$

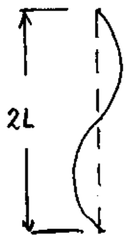
PROBLEM 10.25



10.25 Column *ABC* has a uniform rectangular cross section and is braced in the *xz* plane at its midpoint *C*. (*a*) Determine the ratio *b*/*d* for which the factor of safety is the same with respect to buckling in the *xz* and *yz* planes. (*b*) Using the ratio found in part *a*, design the cross section of the column so that the factor of safety will be 2.7 when *P* = 1.2 kips, *L* = 24 in., and *E* = 10.6 × 10⁶ psi.

SOLUTION

Buckling in xz-plane: $L_e = L = 24$ in.



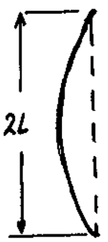
$$I = \frac{1}{12} d b^3$$

$$P = \frac{P_{cr}}{F.S} = \frac{\pi^2 E I}{2.8\, L_e^2} = \frac{\pi^2 E\, d b^3}{12 (F.S)\, L_e^2}$$

$$d b^3 = \frac{12\, P\, (F.S.)\, L_e^2}{\pi^2 E} = \frac{(12)(1.2 \times 10^3)(2.7)(24)^2}{\pi^2\, (10.6 \times 10^6)}$$

$$= 0.21406 \text{ in}^4$$

Buckling in yz-plane: $L_e = 2L = (2)(24) = 48$ in $\qquad I = \frac{1}{12} b d^3$



$$P = \frac{P_{cr}}{F.S} = \frac{\pi^2 E I}{2.8\, L_e^2} = \frac{\pi^2 E\, b d^3}{12 (F.S)\, L_e^2}$$

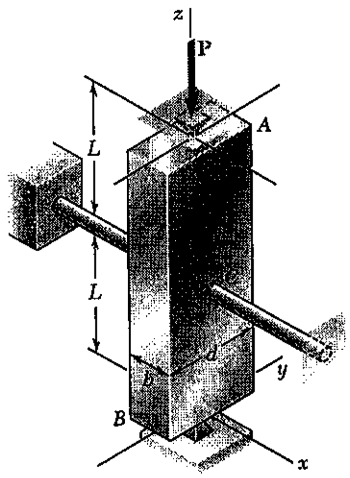
$$b d^3 = \frac{12\, P\, (F.S.)\, L_e^2}{\pi^2 E} = \frac{(12)(1.2 \times 10^3)(2.7)(48)^2}{\pi^2\, (10.6 \times 10^6)} = 0.85625 \text{ in}^4$$

(a) $$\frac{d b^3}{b d^3} = \frac{b^2}{d^2} = \frac{0.21406}{0.85625} = \frac{1}{4} \qquad \frac{b}{d} = \frac{1}{2}$$

$$d b^3 = d\left(\tfrac{1}{8} d^3\right) = \tfrac{1}{8} d^4 = 0.21406 \text{ in}^4, \quad d = 1.144 \text{ in.}$$

$$b = \tfrac{1}{2} d = 0.572 \text{ in.}$$

PROBLEM 10.26



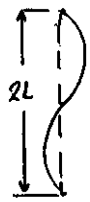
10.26 The aluminum column *ABC* has a uniform rectangular cross section with $b = \frac{1}{2}$ in. and $d = \frac{7}{8}$ in. The column is braced in the *xz* plane at its midpoint *C* and carries a centric load **P** of magnitude 1.1 kips. Knowing that a factor of safety of 2.5 is required, determine the largest allowable length *L*. Use $E = 10.6 \times 10^6$ psi.

SOLUTION

$P_{cr} = (F.S.)P = (2.5)(1.1\times10^3) = 2.75\times10^3$ lb.

$P_{cr} = \dfrac{\pi^2 EI}{L_e^2}$ $\qquad L_e = \pi\sqrt{\dfrac{EI}{P_{cr}}}$

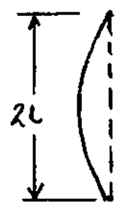
Buckling in xz-plane: $I = \frac{1}{12}db^3 = \frac{1}{12}\left(\frac{7}{8}\right)\left(\frac{1}{2}\right)^3 = 9.1146\times10^{-3}$ in^4



$L = L_e = \pi\sqrt{\dfrac{EI}{P_{cr}}} = \pi\sqrt{\dfrac{(10.6\times10^6)(9.1146\times10^{-3})}{2.75\times10^3}}$

$= 18.62$ in.

Buckling in yz-plane

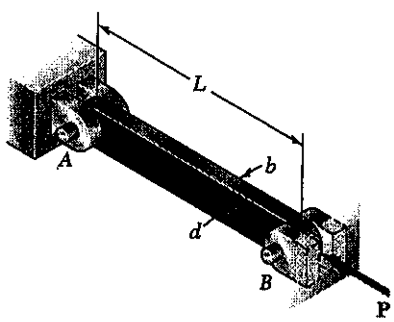


$I = \frac{1}{12}bd^3 = \frac{1}{12}\left(\frac{1}{2}\right)\left(\frac{7}{8}\right)^3 = 27.913\times10^{-3}$ in^4 $\qquad L_e = 2L$

$L = \frac{1}{2}L_e = \dfrac{\pi}{2}\sqrt{\dfrac{EI}{P_{cr}}} = \dfrac{\pi}{2}\sqrt{\dfrac{(10.6\times10^6)(27.913\times10^{-3})}{2.75\times10^3}} = 16.29$ in.

Smaller length governs $\qquad L = 16.29$ in. ◀

PROBLEM 10.27



10.27 The uniform brass bar *AB* has a rectangular cross section and is supported by pins and brackets as shown. Each end of the bar can rotate freely about a horizontal axis through the pin, but rotation about a vertical axis is prevented by the brackets. (*a*) Determine the ratio *b*/*d* for which the factor of safety is the same about the horizontal and vertical axes. (*b*) Determine the factor of safety if *P* = 1.8 kips, *L* = 7 ft, *d* = 1.5 in., and $E = 15 \times 10^6$ psi.

SOLUTION

Buckling in horizontal plane: $L_e = \frac{1}{2}L$, $I = \frac{1}{12}db^3$

$P_{cr1} = \dfrac{\pi^2 EI}{L_e^2} = \dfrac{4\pi^2 Edb^3}{12L^2}$ $\qquad$ (1)

Buckling in vertical plane: $L_e = L$, $I = \frac{1}{12}bd^3$

$P_{cr2} = \dfrac{\pi^2 EI}{L_e^2} = \dfrac{\pi^2 Ebd^3}{12L^2}$ $\qquad$ (2)

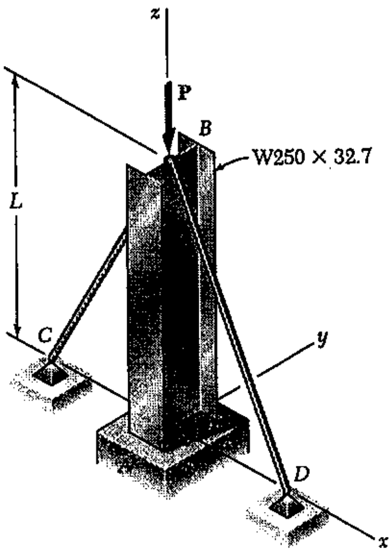
(a) Equating $P_{cr1} = P_{cr2}$ $\qquad \dfrac{4\pi^2 Edb^3}{12L^2} = \dfrac{\pi^2 Ebd^3}{12L^2}$ $\qquad 4b^2 = d^2$ $\qquad b = \frac{1}{2}d$ ◀

(b) $b = \frac{1}{2}d = 0.75$ in $\qquad L = 7$ ft $= 84$ in

Using (2) $\qquad P_{cr} = \dfrac{\pi^2(15\times10^6)(0.75)(1.5)^3}{(12)(84)^2} = 4.426\times10^3$ lb $= 4.426$ kips.

$F.S. = \dfrac{P_{cr}}{P} = \dfrac{4.426}{1.8} = 2.46$ ◀

PROBLEM 10.28



10.28 Column *AB* carries a centric load **P** of magnitude 72 kN. Cables *BC* and *BD* are taut and prevent motion of point *B* in the *xz* plane. Using Euler's formula and a factor of safety of 2.3, and neglecting the tension in the cables, determine the maximum allowable length *L*. Use *E* = 200 GPa.

SOLUTION

W 250 × 32.7 $\quad I_x = 48.9 \times 10^6 \text{ mm}^4 = 48.9 \times 10^{-6} \text{ m}^4$

$I_y = 4.73 \times 10^6 \text{ mm}^4 = 4.73 \times 10^{-6} \text{ m}^4$

$P = 72 \times 10^3$ N $\qquad P_{cr,\,min} = (F.S.)(P) = 165.3 \times 10^3$ N

Buckling in xz-plane: $L_e = 0.7L$

$$P_{cr} = \frac{\pi^2 E I_y}{(0.7L^2)}$$

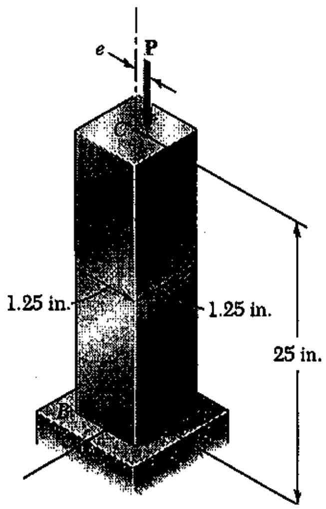
$$L = \frac{\pi}{0.7}\sqrt{\frac{EI_y}{P_{cr}}} = \frac{\pi}{0.7}\sqrt{\frac{(200\times10^9)(4.73\times10^{-6})}{165.3\times10^3}}$$

$$= 10.74 \text{ m}$$

Buckling in yz-plane: $\quad L_e = 2L \qquad P_{cr} = \dfrac{\pi^2 E I_x}{(2L)^2}$

$$L = \frac{\pi}{2}\sqrt{\frac{EI_x}{P_{cr}}} = \frac{\pi}{2}\sqrt{\frac{(200\times10^9)(48.9\times10^{-6})}{165.3\times10^3}} = 12.08 \text{ m}$$

PROBLEM 10.29



10.29 An axial load **P** is applied to the 1.25-in.-square aluminum bar *ABC* as shown. When *P* = 3.8 kips, the horizontal deflection at end *C* is 0.16 in. Using *E* = 10.1 × 10⁶ psi, determine (*a*) the eccentricity *e* of the load, (*b*) the maximum stress in the rod.

SOLUTION

$I = \frac{1}{12}(1.25)^4 = 0.20345 \text{ in}^4 \qquad A = 1.25^2 = 1.5625 \text{ in}^2$

$L_e = 2L = 50$ in $\qquad L_e = 2L = 50$ in.

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(10.1\times10^6)(0.20845)}{(50)^2} = 8.1122\times10^3 \text{ lb.}$$

$$\frac{P}{P_{cr}} = \frac{3.8\times10^3}{8.1122\times10^3} = 0.46842$$

(a) $y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = e\left[\sec\left(\frac{\pi}{2}\sqrt{0.46842}\right) - 1\right]$

$= e\left[\sec(1.07508) - 1\right] = 1.1023\, e$

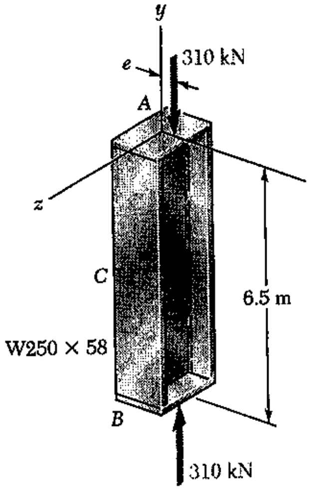
$$e = \frac{y_{max}}{1.1023} = \frac{0.16}{1.1023} = 0.1451 \text{ in.}$$

(b) $M_{max} = P(e + y_{max}) = (3.8\times10^3)(0.1451 + 0.16) = 1.15957$ lb·in

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{3.8\times10^3}{1.5625} + \frac{(1.15957)(0.625)}{0.20345} = 5.99\times10^3 \text{ psi} = 5.99 \text{ ksi}$$

PROBLEM 10.30

10.30 The line of action of the 310-kN axial load is parallel to the geometric axis of the column AB and intersects the x axis at $x = e$. Using $E = 200$ GPa, determine (*a*) the eccentricity e when the deflection of the midpoint C of the column is 9 mm, (*b*) the corresponding maximum stress in the column.



SOLUTION

For W250 × 58 $\quad A = 7420\ mm^2 = 7420 \times 10^{-6}\ m^2$

$I_y = 18.8 \times 10^6\ mm^4 = 18.8 \times 10^{-6}\ m^4$

$S_y = 185 \times 10^3\ mm^3 = 185 \times 10^{-6}\ m^3$

$L = 6.5\ m \qquad L_e = 6.5\ m$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200 \times 10^9)(18.8 \times 10^{-6})}{(6.5)^2} = 878.3 \times 10^3\ N$$

$$\frac{P}{P_{cr}} = \frac{310 \times 10^3}{878.3 \times 10^3} = 0.35294$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = 0.67990\ e$$

(a) $$e = \frac{y_{max}}{0.67990} = \frac{9 \times 10^{-3}}{0.67990} = 13.24 \times 10^{-3}\ m = 13.24\ mm$$

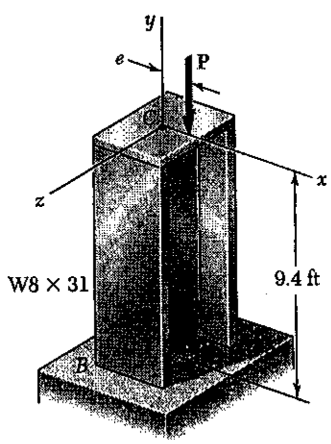
(b) $$M_{max} = P(e + y_{max}) = (310 \times 10^3)(9 + 13.24)(10^{-3}) = 6893.5\ N\cdot m$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{310 \times 10^3}{7420 \times 10^{-6}} + \frac{6893.5}{185 \times 10^{-6}}$$

$$= 41.78 \times 10^6 + 37.26 \times 10^6 = 79.04 \times 10^6\ Pa = 79.0\ MPa$$

PROBLEM 10.31

10.31 The axial load **P** is applied at a point located on the x axis at a distance e from the geometric axis of the rolled-steel column BC. When $P = 82$ kips, the horizontal deflection of the top of the column is 0.20 in. Using $E = 29 \times 10^6$ psi, determine (*a*) the eccentricity e of the load, (*b*) the maximum stress in the column.



SOLUTION

W8×31: $A = 9.13\ in^2, \quad I_y = 37.1\ in^4, \quad S_y = 9.27\ in^3$

$L = 9.4\ ft = 112.8\ in \qquad L_e = 2L = 225.6\ in$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29 \times 10^6)(37.1)}{(225.6)^2} = 208.63 \times 10^3$$

$$\frac{P}{P_{cr}} = \frac{82 \times 10^3}{208.63 \times 10^3} = 0.39304$$

(a) $$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = 0.80816\ e$$

$$e = \frac{y_{max}}{0.80816} = \frac{0.20}{0.80816} = 0.247\ in.$$

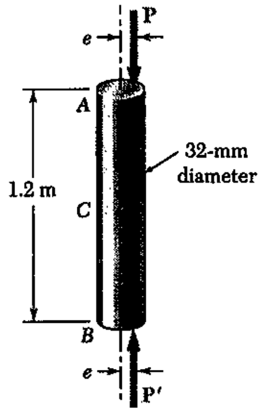
(b) $$M_{max} = P(e + y_{max}) = (82 \times 10^3)(0.247 + 0.20) = 36.693 \times 10^3\ lb\cdot in$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{82 \times 10^3}{9.13} + \frac{36.693 \times 10^3}{9.27} = 12.94 \times 10^3\ psi$$

$$= 12.94\ ksi$$

PROBLEM 10.32

10.32 An axial load **P** is applied to the 32-mm-diameter steel rod *AB* as shown. For $P = 37$ kN and $e = 1.2$ mm, determine (*a*) the deflection at the midpoint *C* of the rod, (*b*) the maximum stress in the rod. Use $E = 200$ GPa.



SOLUTION

$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi}{4}\left(\frac{32}{2}\right)^4 = 51.47\times10^3\ \text{mm}^4 = 51.47\times10^{-9}\ \text{m}^4$$

$$L_e = L = 1.2\ \text{m}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200\times10^9)(51.47\times10^{-9})}{(1.2)^2} = 70.556\times10^3\ \text{N}$$

$$\frac{P}{P_{cr}} = \frac{37\times10^3}{70.556\times10^3} = 0.52440$$

(a) $$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = 1.3817e = (1.3817)(1.2)$$
$$= 1.658\ \text{mm}$$

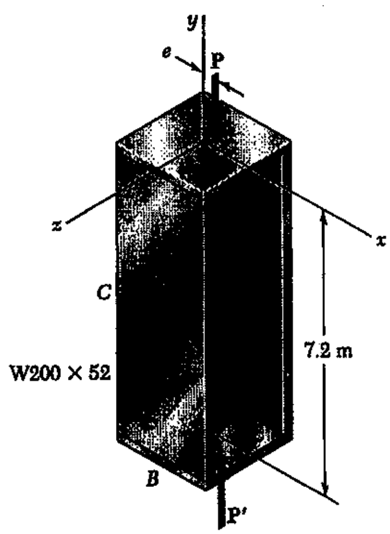
(b) $$M_{max} = P(e + y_{max}) = (37\times10^3)(1.2 + 1.658)(10^{-3}) = 105.75\ \text{N}\cdot\text{m}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(32)^2 = 804.25\ \text{mm}^2 = 804.25\times10^{-6}\ \text{m}^2, \quad c = 16\times10^{-3}\ \text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{37\times10^3}{804.25\times10^{-6}} + \frac{(105.75)(16\times10^{-3})}{51.47\times10^{-9}} = 78.9\times10^6\ \text{Pa}$$
$$= 78.9\ \text{MPa}$$

PROBLEM 10.33

10.33 The line of action of the axial load **P** of magnitude 270 kN is parallel to the geometric axis of the column *AB* and intersects the *x* axis at $e = 14$ mm. Using $E = 200$ GPa, determine (*a*) the deflection of the midpoint *C* of the column, (*b*) the maximum stress in the column.



SOLUTION

W 200 × 52 $\quad A = 6660\ \text{mm}^2 = 6660\times10^{-6}\ \text{m}^2$

$I_y = 17.8\times10^6\ \text{mm}^4 = 17.8\times10^{-6}\ \text{m}^4$

$S_y = 175\times10^3\ \text{mm}^3 = 175\times10^{-6}\ \text{m}^3$

$L = 7.2\ \text{m} \qquad L_e = 7.2\ \text{m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200\times10^9)(17.8\times10^{-6})}{(7.2)^2}$$
$$= 677.77\times10^3\ \text{N}$$

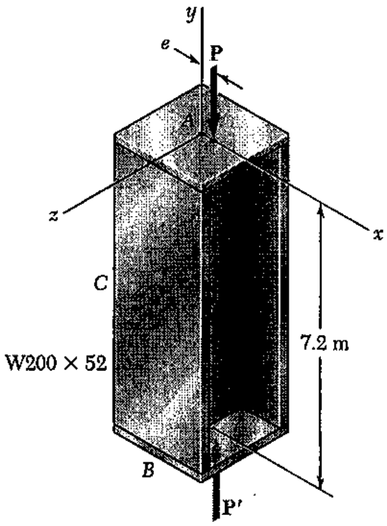
$$\frac{P}{P_{cr}} = \frac{270\times10^3}{677.77\times10^3} = 0.39836$$

(a) $$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = 0.82648e = (0.82648)(14) = 11.57\ \text{mm}$$

(b) $$M_{max} = P(e + y_{max}) = (270\times10^3)(14 + 11.57)(10^{-3}) = 6904\ \text{N}\cdot\text{m}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{270\times10^3}{6660\times10^{-6}} + \frac{6904}{175\times10^{-6}} = 80.0\times10^6\ \text{Pa} = 80.0\ \text{MPa}$$

PROBLEM 10.34



10.33 The line of action of the axial load **P** of magnitude 270 kN is parallel to the geometric axis of the column *AB* and intersects the *x* axis at *e* = 14 mm. Using *E* = 200 GPa, determine (*a*) the deflection of the midpoint *C* of the column, (*b*) the maximum stress in the column.

10.34 Solve Prob. 10.33 if the load **P** is applied parallel to the geometric axis of the column *AB* so that it intersects the *x* axis at *e* = 21 mm.

SOLUTION

W 200 × 52 $\quad A = 6660\ mm^2 = 6660 \times 10^{-6}\ m^2$

$I_y = 17.8 \times 10^6\ mm^4 = 17.8 \times 10^{-6}\ m^4$

$S_y = 175 \times 10^3\ mm^3 = 175 \times 10^{-6}\ m^3$

$L = 7.2\ m \qquad L_e = 7.2\ m$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200 \times 10^9)(17.8 \times 10^{-6})}{(7.2)^2}$$

$$= 677.77 \times 10^3\ N$$

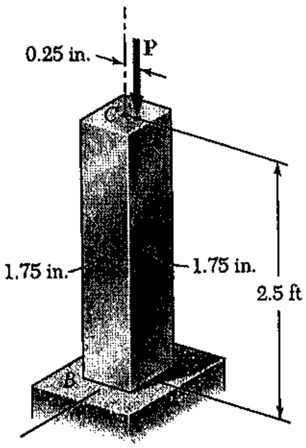
$$\frac{P}{P_{cr}} = \frac{270 \times 10^3}{677.77 \times 10^3} = 0.39836$$

(a) $y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] = 0.82648\ e = (0.82648)(21) = 17.36\ mm$

(b) $M_{max} = P(e + y_{max}) = (270 \times 10^3)(21 + 17.36)(10^{-3}) = 10356\ N \cdot m$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{270 \times 10^3}{6660 \times 10^{-6}} + \frac{10356}{175 \times 10^{-6}} = 99.7 \times 10^6\ Pa = 99.7\ MPa$$

PROBLEM 10.35



10.35 An axial load **P** is applied at a point D that is 0.25 in. from the geometric axis of the square aluminum bar BC. Determine (*a*) the load **P** for which the horizontal deflection of end C is 0.50 in., (*b*) the corresponding maximum stress in the column. Use $E = 10.1 \times 10^3$ ksi.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(1.75)(1.75)^3 = 0.78157 \text{ in}^4$$

$$A = (1.75)^2 = 3.0625 \text{ in}^2 \qquad c = \frac{1}{2}(1.75) = 0.875 \text{ in.}$$

$$L = 2.5 \text{ ft} = 30 \text{ in} \qquad L_e = 2L = 60 \text{ in.}$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(10.1\times10^3)(0.78157)}{(60)^2} = 21.641 \text{ kips.}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_c}}\right) - 1\right]$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e}, \qquad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max}+e}$$

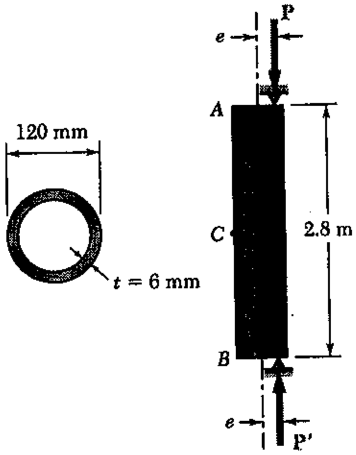
(a) $$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{e+y_{max}}\right]^2 = \left[\frac{2}{\pi}\arccos\frac{0.25}{0.25+0.50}\right]^2$$

$$= 0.61411 \qquad P = 0.61411\, P_{cr} = 13.29 \text{ kips}$$

(b) $$M_{max} = P(e + y_{max}) = (13.29)(0.25+0.50) = 9.9675 \text{ kip·in.}$$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{13.29}{3.0625} + \frac{(9.9675)(0.875)}{0.78157} = 15.50 \text{ ksi}$$

PROBLEM 10.36



10.36 A brass pipe having the cross section shown has an axial load **P** applied 5 mm from its geometric axis. Using $E = 120$ GPa, determine (*a*) the load **P** for which the horizontal deflection at the midpoint *C* is 5 mm, (*b*) the corresponding maximum stress in the column.

SOLUTION

$c_o = \frac{1}{2} d_o = 60 \text{ mm} \qquad c_i = c_o - t = 54 \text{ mm}$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 3.5005 \times 10^6 \text{ mm}^4 = 3.5005 \times 10^{-6} \text{ m}^4$

$L = 2.8 \text{ m} \qquad L_e = 2.8 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (120 \times 10^9)(3.5005 \times 10^{-6})}{(2.8)^2}$$

$$= 528.8 \times 10^3 \text{ N} = 528.8 \text{ kN}$$

(a) $y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e} \qquad \frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e}\right]^2$

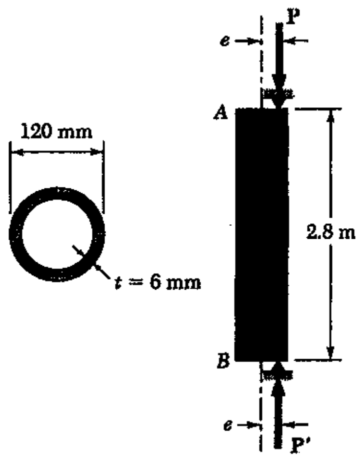
$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{5}{5+5}\right]^2 = 0.44444 \qquad P = 0.44444\, P_{cr} = 235 \text{ kN}$ ◄

(b) $M_{max} = P(e + y_{max}) = (235 \times 10^3)(5 + 5)(10^{-3}) = 2350 \text{ N}\cdot\text{m}$

$A = \pi(c_o^2 - c_i^2) = \pi(60^2 - 54^2) = 2.1488 \times 10^3 \text{ mm}^2 = 2.1488 \times 10^{-3} \text{ m}^2$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{235 \times 10^3}{2.1488 \times 10^{-3}} + \frac{(2350)(60 \times 10^{-3})}{3.5005 \times 10^{-6}} = 149.6 \times 10^6 \text{ Pa} = 149.6 \text{ MPa}$$ ◄

PROBLEM 10.37



10.36 A brass pipe having the cross section shown has an axial load **P** applied 5 mm from its geometric axis. Using $E = 120$ GPa, determine (*a*) the load **P** for which the horizontal deflection at the midpoint C is 5 mm, (*b*) the corresponding maximum stress in the column.

10.37 Solve Prob. 10.36, assuming that the axial load **P** is applied 10 mm from the geometric axis of the column.

SOLUTION

$c_o = \frac{1}{2} d_o = 60 \text{ mm} \qquad c_i = c_o - t = 54 \text{ mm}$

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 3.5005 \times 10^6 \text{ mm}^4 = 3.5005 \times 10^{-6} \text{ m}^4$

$L = 2.8 \text{ m} \qquad L_e = 2.8 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (120 \times 10^9)(3.5005 \times 10^{-6})}{(2.8)^2}$$

$$= 528.8 \times 10^3 \text{ N} = 528.8 \text{ kN}$$

(a) $y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e}$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e} \qquad \frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{e}{y_{max} + e}\right]^2$

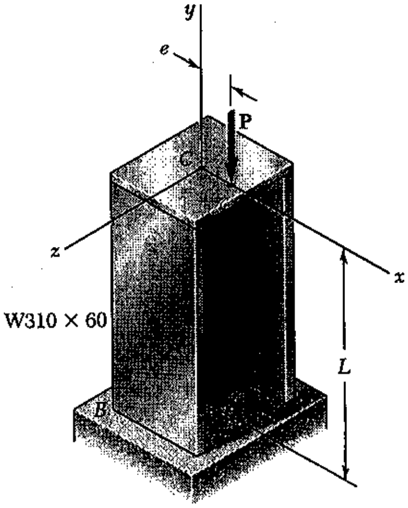
$\frac{P}{P_{cr}} = \left[\frac{2}{\pi} \arccos \frac{10}{5 + 10}\right]^2 = 0.28670 \qquad P = 0.28670\, P_{cr} = 151.6 \text{ kN}$ ◄

(b) $M_{max} = P(e + y_{max}) = (151.6 \times 10^3)(10 + 5)(10^{-3}) = 2274 \text{ N}\cdot\text{m}$

$A = \pi(c_o^2 - c_i^2) = \pi(60^2 - 54^2) = 2.1488 \times 10^3 \text{ mm}^2 = 2.1488 \times 10^{-3} \text{ m}^2$

$$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{151.6 \times 10^3}{2.1488 \times 10^{-3}} + \frac{(2274)(60 \times 10^{-3})}{3.5005 \times 10^{-6}} = 109.5 \times 10^6 \text{ Pa} = 109.5 \text{ MPa}$$ ◄

PROBLEM 10.38



10.38 An axial load **P** is applied at a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W310 × 60 rolled-steel column BC. Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (*a*) the load **P** for which the horizontal deflection at end C is 15 mm, (*b*) the corresponding maximum stress in the column.

SOLUTION

W310×60 $\quad A = 7590\ \text{mm}^2 = 7590 \times 10^{-6}\ \text{m}^2$

$I_y = 18.3 \times 10^6\ \text{mm}^4 = 18.3 \times 10^{-6}\ \text{m}^4$

$S_y = 180 \times 10^3\ \text{mm}^3 = 180 \times 10^{-6}\ \text{m}^3$

$L = 3.5\ \text{m} \qquad L_e = 2L = 7.0\ \text{m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.3 \times 10^{-6})}{(7.0)^2}$$

$$= 737.2 \times 10^3\ \text{N} = 737.2\ \text{kN}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e} \qquad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max}+e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{max}+e}\right]^2 = \left[\frac{2}{\pi}\arccos\frac{12}{15+12}\right]^2 = 0.49957$$

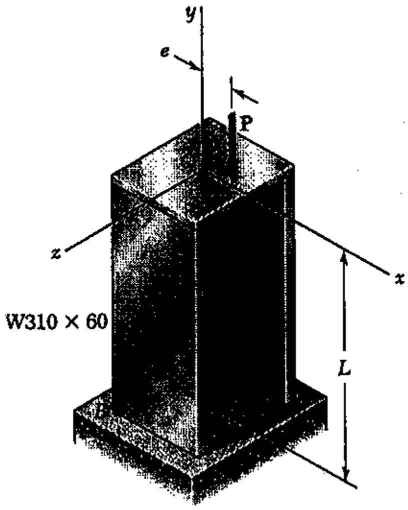
(a) $\qquad P = 0.49957\, P_{cr} = 368.28\ \text{kN}$ ◀

$$M_{max} = P(e + y_{max}) = (368.28 \times 10^3)(12 + 15)(10^{-3}) = 9944\ \text{N}\cdot\text{m}$$

(b) $$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{368.28 \times 10^3}{7590 \times 10^{-6}} + \frac{9944}{180 \times 10^{-6}} = 103.8 \times 10^6\ \text{Pa}$$

$$= 103.8\ \text{MPa}$$ ◀

PROBLEM 10.39



10.38 An axial load **P** is applied at a point located on the *x* axis at a distance $e = 12$ mm from the geometric axis of the W310 × 60 rolled-steel column *BC*. Assuming that $L = 3.5$ m and using $E = 200$ GPa, determine (*a*) the load **P** for which the horizontal deflection at end *C* is 15 mm, (*b*) the corresponding maximum stress in the column.

10.39 Solve Prob. 10.38, assuming that *L* is 4.5 m.

SOLUTION

W 310 × 60 $\quad A = 7590 \text{ mm}^2 = 7590 \times 10^{-6} \text{ m}^2$

$I_y = 18.3 \times 10^6 \text{ mm}^4 = 18.3 \times 10^{-6} \text{ m}^4$

$S_y = 180 \times 10^3 \text{ mm}^3 = 180 \times 10^{-6} \text{ m}^3$

$L = 4.5 \text{ m} \qquad L_e = 2L = 9.0 \text{ m}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (200 \times 10^9)(18.3 \times 10^{-6})}{(9.0)^2}$$

$$= 445.96 \times 10^3 \text{ N} = 445.96 \text{ kN}$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max} + e}{e} \qquad \cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max} + e}$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{max} + e}\right]^2 = \left[\frac{2}{\pi}\arccos\frac{12}{15 + 12}\right]^2 = 0.49957$$

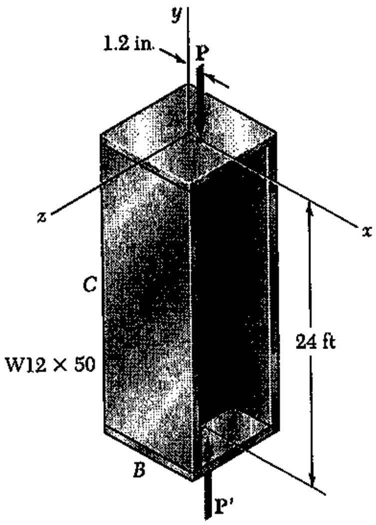
(a) $\qquad P = 0.49957\, P_{cr} = 222.79 \text{ kN}$

$$M_{max} = P(e + y_{max}) = (222.79 \times 10^3)(12 + 15)(10^{-3}) = 6015 \text{ N}\cdot\text{m}$$

(b) $$\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{222.79 \times 10^3}{7590 \times 10^{-6}} + \frac{6015}{180 \times 10^{-6}} = 62.8 \times 10^6 \text{ Pa}$$

$$= 62.8 \text{ MPa}$$

PROBLEM 10.40



10.40 The line of action of an axial load **P** is parallel to the geometric axis of the column *AB* and intersects the *x* axis at $x = 1.2$ in. Using $E = 29 \times 10^6$ psi., determine (*a*) the load **P** for which the horizontal deflection of the midpoint *C* of the column is 0.8 in., (*b*) the corresponding maximum stress in the column.

SOLUTION

W 12 × 50 $\quad A = 14.7\ in^2,\ I_y = 56.3\ in^4,\ S_y = 13.9\ in^3$

$L = 24\ ft = 288\ in \qquad L_e = 288\ in^2$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29\times 10^6)(56.3)}{(288)^2} = 194.28 \times 10^3\ lb.$$

$$= 194.28\ kips$$

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{y_{max}+e}{e}$$

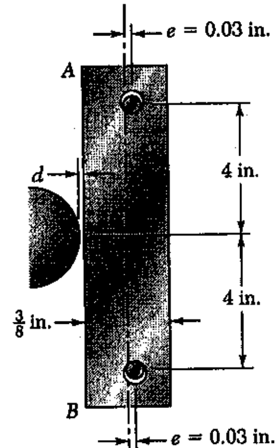
$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{e}{y_{max}+e} \qquad \frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{e}{y_{max}+e}\right]^2$$

(a) $\frac{P}{P_{cr}} = \left[\frac{2}{\pi}\arccos\frac{1.2}{0.8+1.2}\right]^2 = 0.34849 \qquad P = 0.34849\ P_{cr} = 67.7\ kips$ ◄

$$M_{max} = P(e + y_{max}) = (67.7)(1.2 + 0.8) = 135.4\ kip\cdot in$$

(b) $\sigma_{max} = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{M}{S_y} = \frac{67.7}{14.7} + \frac{135.4}{13.9} = 14.3\ ksi$ ◄

PROBLEM 10.41



10.41 The steel bar AB has a $\frac{3}{8} \times \frac{3}{8}$-in. square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.03$ in. from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point C if $d = 0.01$ in. Use $E = 29 \times 10^6$ psi. and the coefficient of thermal expansion $\alpha = 6.5 \times 10^{-6}/°F$.

SOLUTION

$$A = (\tfrac{3}{8})(\tfrac{3}{8}) = 0.140625 \text{ in}^2$$

$$I = \tfrac{1}{12}(\tfrac{3}{8})^4 = 1.64795 \times 10^{-3} \text{ in}^4$$

$$EI = (29 \times 10^6)(1.64795 \times 10^{-3}) = 47791 \text{ lb}\cdot\text{in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (47791)}{(8)^2} = 7370 \text{ lb.}$$

Calculate P using the secant formula

$$y_{max} = d = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right] \qquad \sec\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1 + \frac{d}{e}$$

$$\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = \cos^{-1}\left(1 + \frac{d}{e}\right)^{-1} = \cos^{-1}\left(1 + \frac{0.01}{0.03}\right)^{-1} = \cos^{-1}(0.75) = 0.72273$$

$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.72273)\right]^2 = 0.21170 \qquad P = 0.21170\, P_{cr} = 1560.2 \text{ lb.}$$

Thermal analysis.

(1) Simple approximation by ignoring eccentricity.

$$\text{Total elongation} = \alpha L(\Delta T) - \frac{PL}{EA} = 0$$

$$\Delta T = \frac{PL}{EA}\frac{1}{\alpha L} = \frac{P}{EA\alpha} = \frac{1560.2}{(29 \times 10^6)(0.140625)(6.5 \times 10^{-6})} = 58.9\ °F$$

(2) Analysis with inclusion of eccentricity.

$$\text{Total elongation of centroidal axis} = \alpha L(\Delta T) - \frac{PL}{EA} = 2e\left.\frac{dy}{dx}\right|_{x=0}$$

To calculate $\frac{dy}{dx}$, differentiate eq. (10.26)

$$\frac{dy}{dx} = e\left(p \tan\frac{pL}{2}\cos px - p\sin px\right)$$

At $x = 0$

$$\left.\frac{dy}{dx}\right|_{x=0} = ep\tan\frac{pL}{2} = e\sqrt{\frac{P}{EI}}\tan\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}$$

The elongation of the centroidal axis is $2e^2\sqrt{\frac{P}{EI}}\tan\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$

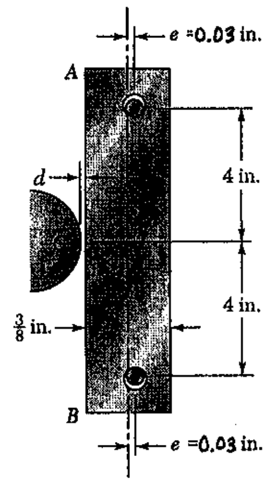
$$= (2)(0.03)^2\sqrt{\frac{1560.2}{47791}}\tan(0.72273) = 286.8 \times 10^{-6} \text{ in.}$$

$$\alpha L(\Delta T) = \frac{PL}{EA} + 2e\left.\frac{dy}{dx}\right|_{x=0}$$

$$\Delta T = \frac{P}{EA\alpha} + \frac{286.8 \times 10^{-6}}{\alpha L} = 58.9 + \frac{286.8 \times 10^{-6}}{(6.5 \times 10^{-6})(8)} = 58.9 + 5.5°$$

$$= 64.4°\ F$$

PROBLEM 10.42



10.41 The steel bar *AB* has a $\frac{3}{8} \times \frac{3}{8}$ -in. square cross section and is held by pins that are a fixed distance apart and are located at a distance $e = 0.03$ in. from the geometric axis of the bar. Knowing that at temperature T_0 the pins are in contact with the bar and that the force in the bar is zero, determine the increase in temperature for which the bar will just make contact with point *C* if $d = 0.01$ in. Use $E = 29 \times 10^6$ psi. and the coefficient of thermal expansion $\alpha = 6.5 \times 10^{-6}/°F$.

10.42 For the bar of Prob. 10.41, determine the required distance *d* for which the bar will just make contact with point *C* when the temperature increases by 120 °F.

SOLUTION

$$A = \left(\tfrac{3}{8}\right)\left(\tfrac{3}{8}\right) = 0.140625 \text{ in}^2$$

$$I = \tfrac{1}{12}\left(\tfrac{3}{8}\right)^4 = 1.64795 \times 10^{-3} \text{ in}^4$$

$$EI = (29 \times 10^6)(1.64795 \times 10^{-3}) = 47791 \text{ lb}\cdot\text{in}^2$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 (47791)}{(8)^2} = 7370 \text{ lb.}$$

Calculate P from thermal analysis. To obtain an approximate value, neglect the effect of eccentricity in the thermal analysis.

Total elongation $= \alpha L (\Delta T) - \frac{PL}{EA} = 0$

$$P = EA\alpha(\Delta T) = (29 \times 10^6)(0.140625)(6.5 \times 10^{-6})(120) = 3181 \text{ lb.}$$

Calculate the deflection using the secant formula

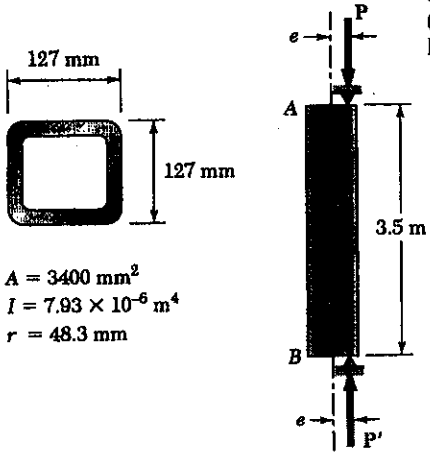
$$d = y_{max} = e\left[\sec\left(\tfrac{\pi}{2}\sqrt{\tfrac{P}{P_{cr}}}\right) - 1\right] = (0.03)\left[\sec\left(\tfrac{\pi}{2}\sqrt{\tfrac{3181}{7370}}\right) - 1\right]$$

$$= (0.03)\left[\sec(1.03197) - 1\right] = (0.03)(0.94883) = 0.0285 \text{ in.} \blacktriangleleft$$

For an improved thermal analysis including eccentricity, see solution of Prob. 10.41.

PROBLEM 10.43

10.43 A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load **P** when the eccentricity e is (*a*) 15 mm, (*b*) 7.5 mm. (*Hint:* Since the factor of safety must be applied to the load **P**, not to the stress, use Fig. 10.24 to determine P_Y).



$A = 3400$ mm²
$I = 7.93 \times 10^{-6}$ m⁴
$r = 48.3$ mm

SOLUTION

$A = 3400 \times 10^{-6}\ m^2 \qquad r = 48.3 \times 10^{-3}\ m$

$L_e = 3.5\ m \qquad \frac{L_e}{r} = \frac{3.5}{48.3 \times 10^{-3}} = 72.46$

$c = \frac{127}{2} = 63.5\ mm$

(a) $e = 15\ mm \qquad \frac{ec}{r^2} = \frac{(15)(63.5)}{(48.3)^2} = 0.40829$

Using Fig. 10.24 with $L_e/r = 72.46$ and $ec/r^2 = 0.40829$

$P/A = 144.75\ MPa = 144.75 \times 10^6\ Pa$

$P = (144.75 \times 10^6)(3400 \times 10^{-6}) = 492 \times 10^3\ N$

Using factor of safety $\quad P_{all} = \frac{492 \times 10^3}{2.6} = 189 \times 10^3\ N = 189\ kN$ ◄

(b) $e = 7.5\ mm \qquad \frac{ec}{r^2} = \frac{(7.5)(63.5)}{(48.3)^2} = 0.20415$

Using Fig. 10.24 with $L_e/r = 72.46$ and $ec/r^2 = 0.20415$

$P/A = 175.2\ MPa = 175.2 \times 10^6\ Pa$

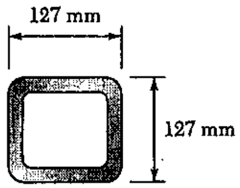
$P = (175.2 \times 10^6)(3400 \times 10^{-6}) = 596 \times 10^3\ N$

Using factor of safety $\quad P_{all} = \frac{596 \times 10^3}{2.6} = 229 \times 10^3\ N = 229\ kN$ ◄

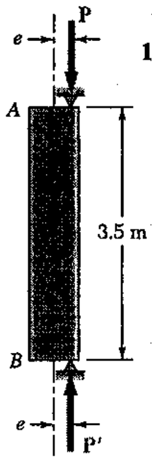
PROBLEM 10.44

10.43 A 3.5-m-long steel tube having the cross section and properties shown is used as a column. For the grade of steel used $\sigma_Y = 250$ MPa and $E = 200$ GPa. Knowing that a factor of safety of 2.6 with respect to permanent deformation is required, determine the allowable load **P** when the eccentricity e is (*a*) 15 mm, (*b*) 7.5 mm. (*Hint:* Since the factor of safety must be applied to the load **P**, not to the stress, use Fig. 10.24 to determine P_Y).

10.44 Solve Prob. 10.43, assuming that the length of the steel tube is increased to 5 m.



$A = 3400 \text{ mm}^2$
$I = 7.93 \times 10^{-6} \text{ m}^4$
$r = 48.3 \text{ mm}$



SOLUTION

$A = 3400 \times 10^{-6} \text{ m}^2 \qquad r = 48.3 \times 10^{-3} \text{ m}$

$L_e = 5 \text{ m} \qquad \dfrac{L_e}{r} = \dfrac{5}{48.3 \times 10^{-3}} = 103.52$

$c = \dfrac{127}{2} = 63.5 \text{ mm}$

(a) $e = 15 \text{ mm} \qquad \dfrac{ec}{r^2} = \dfrac{(15)(63.5)}{(48.3)^2} = 0.40829$

Using Fig. 10.24 with $\dfrac{L_e}{r} = 103.52$

and $\dfrac{ec}{r^2} = 0.40829$ gives $\dfrac{P}{A} = 112.75 \text{ MPa} = 112.75 \times 10^{-6} \text{ Pa}$

$P = (112.75 \times 10^{6})(3400 \times 10^{6}) = 383 \times 10^3 \text{ N}$

Using factor of safety $\quad P_{all} = \dfrac{383 \times 10^3}{2.6} = 147 \times 10^3 \text{ N} = 147 \text{ kN}$ ◄

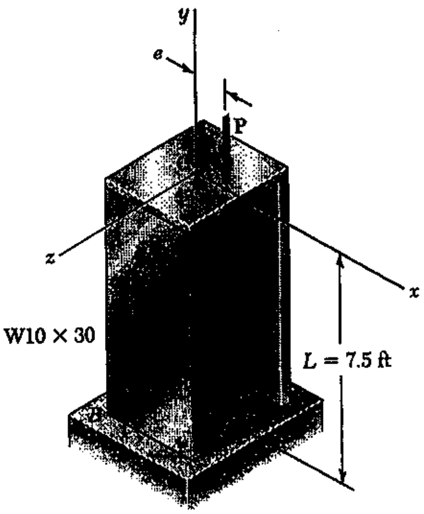
(b) $e = 7.5 \text{ mm} \qquad \dfrac{ec}{r^2} = \dfrac{(7.5)(63.5)}{(48.3)^2} = 0.20415$

Using Fig. 10.24 gives $\dfrac{P}{A} = 133.2 \text{ MPa} = 133.2 \times 10^6 \text{ Pa}$

$P = (133.2 \times 10^6)(3400 \times 10^{-6}) = 453 \times 10^3 \text{ N}$

Using factor of safety $\quad P_{all} = \dfrac{453 \times 10^3}{2.6} = 174 \times 10^3 \text{ N} = 174 \text{ kN}$ ◄

PROBLEM 10.45



10.45 An axial load **P** is applied to the W10 × 30 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is $e = 0.5$ in. and that for the grade of steel used $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi., determine (*a*) the magnitude of *P* of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (*b*) the ratio of the load found in part *a* to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43.)

SOLUTION

W 10 × 30 $\quad A = 8.84 \text{ in}^2 \quad r_y = 1.37$ in.

$c = \frac{b_f}{2} = \frac{5.810}{2} = 2.905$ in $\quad I_y = 16.7 \text{ in}^4$

$L = 7.5$ ft $= 90$ in. $\quad L_e = 2L = 180$ in.

$\frac{L_e}{r} = \frac{180}{1.37} = 131.39 \quad e = 0.5$ in

$\frac{ec}{r^2} = \frac{(0.5)(2.905)}{(1.37)^2} = 0.7739$

Using Fig 10.24 $\quad \frac{P}{A} = 10.47$ ksi

$P = (10.47)(8.84) = 92.6$ kips

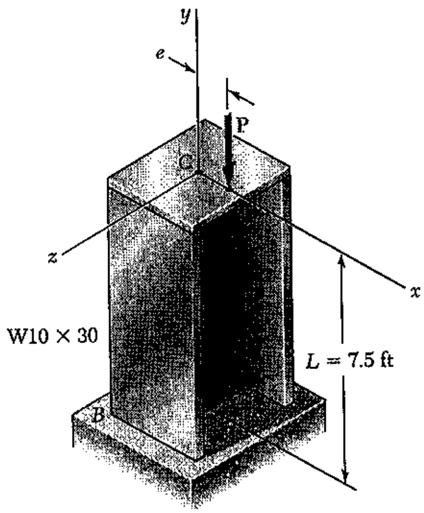
(a) Using factor of safety $\quad P_{all} = \frac{92.6}{2.4} = 38.6$ kips

$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 (29000)(16.7)}{(180)^2} = 147.5$ kips

Using factor of safety $\quad P_{all} = \frac{147.5}{2.4} = 61.5$ kips

(b) $\quad$ ratio $= \frac{38.6}{61.5} = 0.628$

PROBLEM 10.46



10.45 An axial load **P** is applied to the W10 × 30 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is e = 0.5 in. and that for the grade of steel used $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi., determine (*a*) the magnitude of *P* of the allowable load when a factor of safety of 2.4 with respect to permanent deformation is required, (*b*) the ratio of the load found in part *a* to the magnitude of the allowable centric load for the column. (See hint of Prob. 10.43.)

10.46 Solve Prob. 10.45, assuming that the length of the column is reduced to 5.0 ft.

SOLUTION

W10 × 30 $\quad A = 8.84\ in^2 \quad I_y = 16.7\ in^4$

$r_y = 1.37$ in $\quad c = \frac{b_f}{2} = \frac{5.810}{2} = 2.905$ in

$L = 5.0$ ft $= 60$ in $\quad L_e = 2L = 120$ in.

$$\frac{L_e}{r} = \frac{120}{1.37} = 87.6$$

$$\frac{ec}{r^2} = \frac{(0.5)(2.905)}{(1.37)^2} = 0.7739$$

Using Fig 10.24 $\quad \frac{P}{A} = 14.90$ ksi $\quad P = (14.90)(8.84) = 131.7$ kips

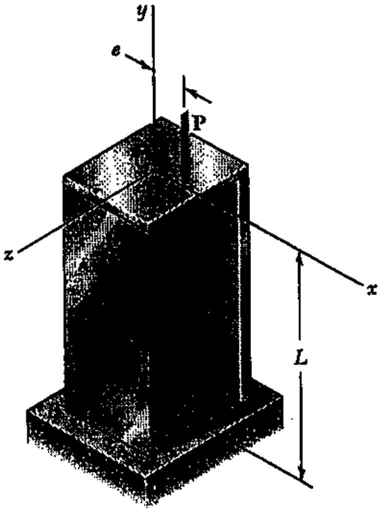
(a) Using factor of safety $\quad P_{all} = \frac{131.7}{2.4} = 54.9$ kips ◄

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(29000)(16.7)}{(120)^2} = 332 \text{ kips}$$

Using factor of safety $\quad P_{all} = \frac{332}{2.4} = 138.3$ kips

(b) $\quad$ ratio $= \frac{54.9}{138.3} = 0.397$ ◄

PROBLEM 10.47



10.47 A 55-kip axial load **P** is applied to a W8 × 24 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is $e = 0.25$ in., determine the largest permissible length *L* if the allowable stress in the column is 14 ksi. Use $E = 29 \times 10^6$ psi.

SOLUTION

Data: $P = 55$ kips, $e = 0.25$ in

$E = 29 \times 10^6$ psi $= 29000$ ksi

W 8 × 24 : $A = 7.08\ \text{in}^2$ $\quad b_f = 6.495$ in

$c = \frac{b_f}{2} = 3.25$ in, $\quad I_y = 18.3\ \text{in}^4$, $\quad r_y = 1.61$ in.

$\sigma_{max} = 14$ ksi

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec}\left(\frac{A\sigma_{max}}{P} - 1\right) = \frac{(1.61)^2}{(0.25)(3.25)}\left[\frac{(7.08)(14)}{55} - 1\right] = 2.5592$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.39075 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.16935$$

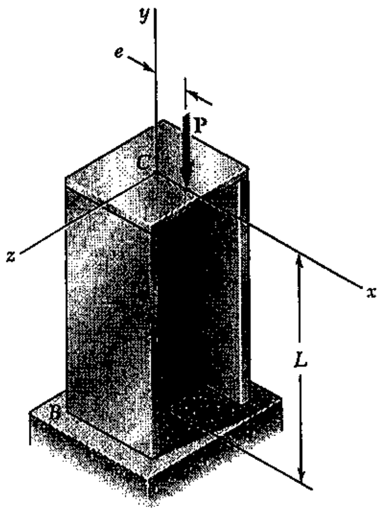
$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(1.16935)\right]^2 = 0.55418$$

$$P_{cr} = \frac{P}{0.55418} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.55418\,\pi^2 EI}{P} = \frac{0.55418\,\pi^2(29000)(18.3)}{55} = 52.78 \times 10^3\ \text{in}^2$$

$L_e = 229.7$ in $= 2L$ $\qquad L = 114.8$ in $= 9.57$ ft. ◄

PROBLEM 10.48



10.48 A 26-kip axial load **P** is applied to a W6 × 12 rolled-steel column *BC* that is free at its top *C* and fixed at its base *B*. Knowing that the eccentricity of the load is $e = 0.25$ in., determine the largest permissible length *L* if the allowable stress in the column is 14 ksi. Use $E = 29 \times 10^6$ psi.

SOLUTION

Data: $P = 26$ kips, $e = 0.25$ in.
$E = 29 \times 10^6$ psi $= 29000$ ksi

W 6 × 12: $A = 3.55$ in^2 $b_f = 4.000$ in.
$c = \frac{b_f}{2} = 2.000$ in, $I_y = 2.99$ in^4, $r_y = 0.918$ in.

$\sigma_{max} = 14$ ksi

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec}\left(\frac{A\sigma_{max}}{P} - 1\right) = \frac{(0.918)^2}{(0.25)(2.000)}\left[\frac{(3.55)(14)}{26} - 1\right] = 1.53635$$

$$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.65089 \qquad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.86204$$

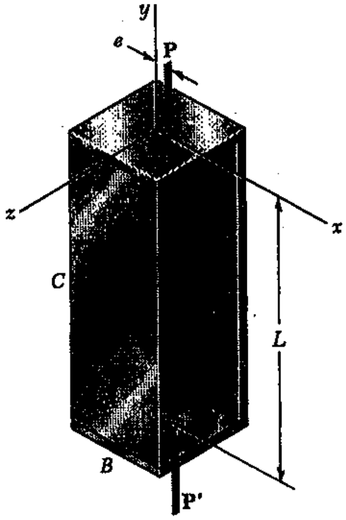
$$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.86204)\right]^2 = 0.30117$$

$$P_{cr} = \frac{P}{0.30117} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e^2 = \frac{0.30117\,\pi^2 EI}{P} = \frac{0.30117\,\pi^2(29000)(2.99)}{26} = 9.913 \times 10^3 \text{ in}^2$$

$L_e = 99.56$ in $= 2L$ $\qquad L = 49.78$ in. $= 4.15$ ft ◄

PROBLEM 10.49



10.49 Axial loads of magnitude $P = 84$ kN are applied parallel to the geometric axis of a W200 × 22.5 rolled-steel column AB and intersect the x axis at a distance e from its geometric axis. Knowing that allowable stress $\sigma_{all} = 75$ MPa and $E = 200$ GPa, determine the largest permissible length L when (*a*) $e = 5$ mm, (*b*) $e = 12$ mm.

SOLUTION

Data: $P = 84 \times 10^3$ N $\quad E = 200 \times 10^9$ Pa

W 200 × 22.5 $\quad A = 2860 \text{ mm}^2 = 2860 \times 10^{-6} \text{ m}^2$

$b_f = 102$ mm $\quad c = \frac{b_f}{2} = 51$ mm $\quad r_y = 22.3$ mm

$I_y = 1.42 \times 10^6 \text{ mm}^4 = 1.42 \times 10^{-6} \text{ m}^4$

$\sigma_{all} = \sigma_{max} = 75 \text{ MPa} = 75 \times 10^6 \text{ Pa}$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r^2}{ec}\left(\frac{A\sigma_{max}}{P} - 1\right)$$

(a) $e = 5$ mm $\quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(22.3)^2}{(5)(51)}\left[\frac{(2860\times10^{-6})(75\times10^6)}{84\times10^3} - 1\right] = 3.0297$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.33006 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.2344$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(1.2344)\right]^2 = 0.61757$

$P_{cr} = \frac{P}{0.61757} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.61757\,\pi^2 EI}{P} = \frac{0.61757\,\pi^2(200\times10^9)(1.42\times10^{-6})}{84\times10^3} = 20.61 \text{ m}^2$

$L_e = 4.54$ m $\quad L = L_e = 4.54$ m ◄

(b) $e = 12$ mm $\quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(22.3)^2}{(12)(51)}\left[\frac{(2860\times10^{-6})(75\times10^6)}{84\times10^3} - 1\right] = 1.26238$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.79216 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.6564635$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.65646)\right]^2 = 0.17466$

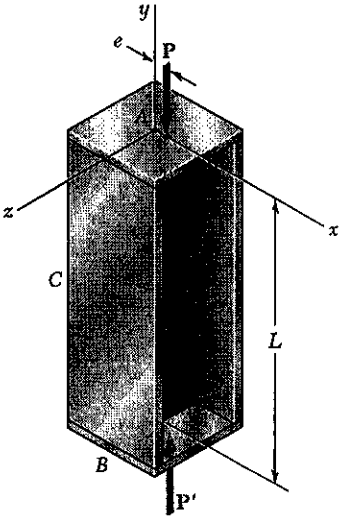
$P_{cr} = \frac{P}{0.17466} = \frac{\pi^2 EI}{L_e^2}$

$L_e^2 = \frac{0.17466\,\pi^2 EI}{P} = \frac{0.17466\,\pi^2(200\times10^9)(1.42\times10^{-6})}{84\times10^3} = 5.828 \text{ m}^2$

$L_e = 2.41$ m $\quad L = L_e = 2.41$ m ◄

PROBLEM 10.50

10.50 Axial loads of magnitude $P = 580$ kN are applied parallel to the geometric axis of a W250 × 80 rolled-steel column AB and intersect the x axis at a distance e from its geometric axis. Knowing that allowable stress $\sigma_{all} = 75$ MPa and $E = 200$ GPa, determine the largest permissible length L when (*a*) $e = 5$ mm, (*b*) $e = 10$ mm.



SOLUTION

Data: $P = 580 \times 10^3$ N $\quad E = 200 \times 10^9$ Pa

W 250 × 80 $\quad A = 10200 \text{ mm}^2 = 10200 \times 10^{-6} \text{ m}^2$

$b_f = 255$ mm $\quad c = \frac{b_f}{2} = 127.5$ mm $\quad r_y = 65.0$ mm

$I_y = 43.1 \times 10^6 \text{ mm}^4 = 43.1 \times 10^{-6} \text{ m}^4$

$\sigma_{all} = \sigma_{max} = 75 \text{ MPa} = 75 \times 10^6$ Pa

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r_y^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\frac{A\sigma_{max}}{P} - 1 = \frac{ec}{r_y^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

$$\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{r_y^2}{ec}\left(\frac{A\sigma_{max}}{P} - 1\right)$$

(a) $e = 5$ mm $\quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(65.0)^2}{(5)(127.5)}\left[\frac{(10200 \times 10^{-6})(75 \times 10^6)}{580 \times 10^3} - 1\right] = 2.1139$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.47305 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 1.07804$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(1.07804)\right]^2 = 0.47101$

$P_{cr} = \frac{P}{0.47101} = \frac{\pi^2 E I_y}{L_e^2}$

$L_e^2 = \frac{0.47101\,\pi^2 E I_y}{P} = \frac{0.47101\,\pi^2(200 \times 10^9)(43.1 \times 10^{-6})}{580 \times 10^3} = 69.09 \text{ m}^2$

$L_e = 8.31$ m $\quad L = L_e = 8.31$ m ◄

(b) $e = 10$ mm $\quad \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = \frac{(65)^2}{(10)(127.5)}\left[\frac{(10200 \times 10^{-6})(75 \times 10^6)}{580 \times 10^3} - 1\right] = 1.05696$

$\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) = 0.94611 \quad \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} = 0.32980$

$\frac{P}{P_{cr}} = \left[\frac{2}{\pi}(0.32980)\right]^2 = 0.044083$

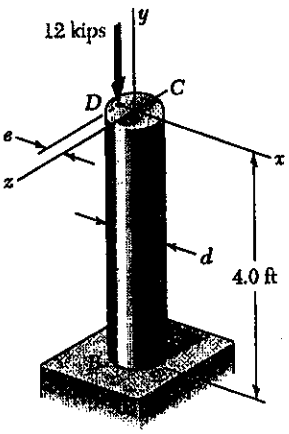
$P_{cr} = \frac{P}{0.044083} = \frac{\pi^2 E I}{L_e^2}$

$L_e^2 = \frac{0.044083\,\pi^2 E I}{P} = \frac{0.044083\,\pi^2(200 \times 10^9)(43.1 \times 10^{-6})}{580 \times 10^3} = 6.466 \text{ m}^2$

$L_e = 2.54$ m $\quad L = L_e = 2.54$ m ◄

PROBLEM 10.51

10.51 A 12-kip axial load is applied with an eccentricity $e = 0.375$ in. to the circular steel rod BC that is free at its top C and fixed at its base B. Knowing that the stock of rods available for use have diameters in increments of $\frac{1}{8}$ in. from 1.5 in. to 3.0 in., determine the lightest rod that may be used if $\sigma_{all} = 15$ ksi. Use $E = 29 \times 10^6$ psi.



SOLUTION

$E = 29 \times 10^6$ psi $= 29000$ ksi $\qquad d =$ diameter (in.)

$A = \frac{\pi}{4} d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{64} \qquad c = \frac{1}{2} d \qquad e = 0.375$ in

$L = 4.0$ ft $= 48$ in $\qquad L_e = 2L = 96$ in

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 (29000) \pi d^4}{(64)(96)^2} = 1.52449\, d^4 \text{ kips}$$

$$r^2 = \frac{I}{A} = \frac{\pi d^4}{64} \cdot \frac{4}{\pi d^2} = \frac{d^2}{16} \qquad P = 12 \text{ kips}$$

$$\frac{ec}{r^2} = \frac{(0.375)(\frac{1}{2} d)}{\frac{1}{16} d^2} = \frac{3}{d}$$

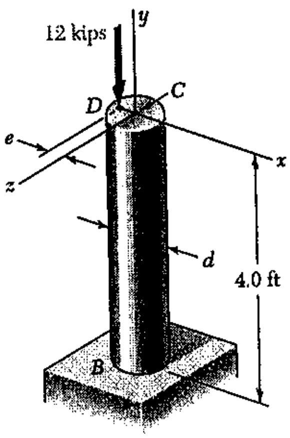
$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

d (in)	A (in²)	P_{cr} (kips)	$\frac{ec}{r^2}$	σ_{max} (ksi)
2.25	3.976	39.07	1.3333	9.26
2.0	3.1416	24.39	1.5	16.49
2.125	3.546	31.09	1.4118	11.90 ←

Use $d = 2.125$ in. ◄

$\sigma_{max} = 11.90$ ksi < 15 ksi

PROBLEM 10.52



10.51 A 12-kip axial load is applied with an eccentricity $e = 0.375$ in. to the circular steel rod BC that is free at its top C and fixed at its base B. Knowing that the stock of rods available for use have diameters in increments of $\frac{1}{8}$ in. from 1.5 in. to 3.0 in., determine the lightest rod that may be used if $\sigma_{all} = 15$ ksi. Use $E = 29 \times 10^6$ psi.

10.52 Solve Prob. 10.51, assuming that the 12-kip axial load will be applied to the rod with an eccentricity $e = \frac{1}{2} d$.

SOLUTION

$E = 29 \times 10^6$ psi $= 29000$ ksi $\qquad d =$ diameter (in)

$A = \frac{\pi}{4} d^2 \qquad I = \frac{\pi}{4}\left(\frac{d}{2}\right)^2 = \frac{\pi}{64} d^4 \qquad c = \frac{1}{2} d \qquad e = \frac{1}{2} d$

$L = 4\text{ ft} = 48\text{ in} \qquad L_e = 2L = 96\text{ in}$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 (29000)(\pi d^4)}{(64)(96)^2} = 1.52449\, d^4$$

$$r^2 = \frac{I}{A} = \frac{\pi d^4}{64} \cdot \frac{4}{\pi d^2} = \frac{1}{16} d^2 \qquad P = 12\text{ kips}$$

$$\frac{ec}{r^2} = \frac{(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{16}d^2} = 4.0$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right] = \frac{P}{A}\left[1 + 4.0\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

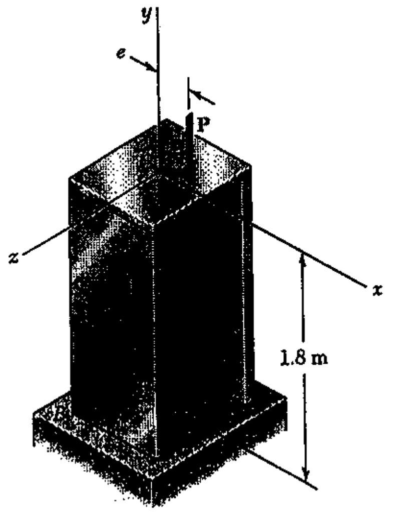
d (in)	A (in²)	P_{cr} (kips)	σ_{max} (ksi)
2.25	3.976	39.07	21.75
3.0	7.068	123.48	9.39
2.5	4.909	59.55	15.28
2.625	5.412	72.38	13.27

Use $d = 2.625$ in.

$\sigma_{max} = 13.27$ ksi < 15 ksi

PROBLEM 10.53

10.53 An axial load of magnitude $P = 220$ kN is applied at a point located on the x axis at a distance $e = 6$ mm from the geometric axis of the wide-flange column BC. Knowing that $E = 200$ GPa, chose the lightest W200 shape that may be used if $\sigma_{all} = 120$ MPa.



SOLUTION

$P = 220 \times 10^3$ N $\qquad L = 1.8$ m $\qquad L_e = 2L = 3.6$ m

$$P_{cr} = \frac{\pi^2 E I_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{3.6^2} = 152.3 \times 10^9 I_y \quad \text{N}$$

$e = 6$ mm $\qquad c = \frac{b_f}{2} \qquad \frac{ec}{r^2} = \frac{e b_f}{2 r_y^2} =$

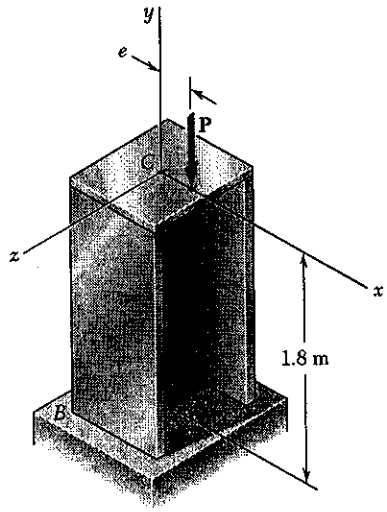
$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2} \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

Shape	$A\,(10^{-6}\,m^2)$	b_f (mm)	$I_y\,(10^{-6}\,m^4)$	r_y (mm)	P_{cr} (kN)	$\frac{ec}{r^2}$	σ_{max} (MPa)
W200 × 41.7	5310	166	9.01	41.2	1372	0.2934	56.5
W200 × 26.6	3390	133	3.30	31.2	502.6	0.4099	117.4
W200 × 22.5	2860	102	1.42	22.3	*216.3	[illegible]	[illegible]

$* < P$

Use W200 × 26.6 ◄ $\qquad \sigma_{max} = 117.4$ MPa

PROBLEM 10.54



10.53 An axial load of magnitude $P = 220$ kN is applied at a point located on the x axis at a distance $e = 6$ mm from the geometric axis of the wide-flange column BC. Knowing that $E = 200$ GPa, chose the lightest W200 shape that may be used if $\sigma_{all} = 120$ MPa.

10.54 Solve Prob. 10.53, assuming that the magnitude of the axial load is $P = 345$ kN.

SOLUTION

$P = 345 \times 10^3$ N $\qquad L = 1.8$ m $\qquad L_e = 2L = 3.6$ m

$$P_{cr} = \frac{\pi^2 E I_y}{L_e^2} = \frac{\pi^2 (200 \times 10^9) I_y}{(3.6)^2} = 152.3 \times 10^9 I_y \quad \text{N}$$

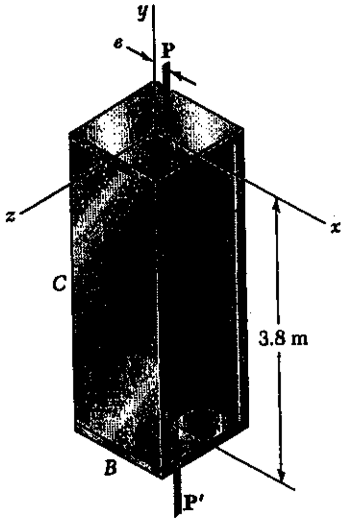
$e = 6$ mm $\qquad c = \frac{b_f}{2} \qquad \frac{ec}{r^2} = \frac{e b_f}{2 r_y^2}$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

Shape	A (10^{-6} m^2)	b_f (mm)	I_y (10^{-6} m^4)	r_y (mm)	P_{cr} (kN)	$\frac{ec}{r^2}$	σ_{max} (MPa)	
W 200 × 41.7	5310	166	9.01	41.2	1372	0.2934	92.0	
W 200 × 26.6	3390	133	3.30	31.2	502.6	0.4099	258	
W 200 × 35.9	4580	165	7.64	40.8	1164	0.2974	109.5	←
W 200 × 31.3	4000	134	4.10	32.0	624.4	0.3926	172.6	

Use W 200 × 35.9 ◄ $\qquad \sigma_{max} = 109.5$ MPa

PROBLEM 10.55



10.55 Axial loads of magnitude $P = 175$ kN are applied to a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W250 × 44.8 rolled-steel column AB. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (*Hint:* Since the factor of safety must be applied to the load **P**, not to the stresses, use Fig. 10.24 to determine P_Y.)

SOLUTION

For W 250 × 44.8 $\quad A = 5720 \text{ mm}^2, \quad r_y = 35.1 \text{ mm}$

$L_e = 3800 \text{ mm} \qquad L_e/r = 108.26$

$c = \frac{b_f}{2} = \frac{148}{2} = 74 \text{ mm} \qquad e = 12 \text{ mm}$

$\frac{ec}{r^2} = \frac{(12)(74)}{(35.1)^2} = 0.72077$

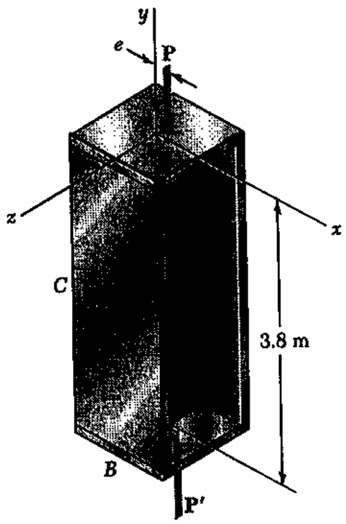
Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{ec}{r^2} = 0.72077$

$P_Y/A = 90.37 \text{ MPa} = 90.37 \times 10^6 \text{ N/m}^2$

$P_Y = A P_Y/A = (5720 \times 10^{-6})(90.37 \times 10^6) = 517 \times 10^3 \text{ N} = 517 \text{ kN}$

$F.S. = \frac{P_Y}{P} = \frac{517}{175} = 2.95$ ◄

PROBLEM 10.56



10.55 Axial loads of magnitude $P = 175$ kN are applied to a point located on the x axis at a distance $e = 12$ mm from the geometric axis of the W250 × 44.8 rolled-steel column AB. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, determine the factor of safety with respect to yield. (*Hint:* Since the factor of safety must be applied to the load **P**, not to the stresses, use Fig. 10.24 to determine P_Y.)

10.56 Solve Prob. 10.55, assuming that $e = 16$ mm and $P = 155$ kN.

SOLUTION

For W 250 × 44.8 $\quad A = 5720 \text{ mm}^2, \quad r_y = 35.1 \text{ mm}$

$L_e = 3800 \text{ mm} \qquad L_e/r = 108.26$

$c = \frac{b_f}{2} = \frac{148}{2} = 74 \text{ mm} \qquad e = 16 \text{ mm}$

$\frac{ec}{r^2} = \frac{(16)(74)}{(35.1)^2} = 0.96103$

Using Fig 10.24 with $L_e/r = 108.26$ and $\frac{ec}{r^2} = 0.96103$

$P_Y/A = 81.17 \text{ MPa} = 81.17 \text{ N/m}^2$

$P_Y = A(P_Y/A) = (5720 \times 10^{-6})(81.17 \times 10^6) = 464 \times 10^3 \text{ N} = 464 \text{ kN}$

$F.S. = \frac{P_Y}{P} = \frac{464}{155} = 3.00$ ◄

PROBLEM 10.57

10.57 Using allowable stress design, determine the allowable centric load for a column of 6.5-m effective length that is made from the following rolled-steel shape: (*a*) W250 × 49.1, (*b*) W250 × 80. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.

SOLUTION

Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.664$

(a) W250×49.1 $\quad A = 6250\times10^{-6}\ \text{m}^2 \quad r_{min} = 49.2\times10^{-3}$ m

$$\frac{L_e}{r} = \frac{6.5}{49.2\times10^{-3}} = 132.11 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(200\times10^9)}{(1.92)(132.11)^2} = 58.9\times10^6\ \text{Pa}$$

$$P_{all} = A\sigma_{all} = (6250\times10^{-6})(58.9\times10^6) = 368\times10^3\ \text{N} = 368\ \text{kN}$$

(b) W250 × 80 $\quad A = 10200\times10^{-6}\ \text{m}^2 \quad r_{min} = 65.0\times10^{-3}$ m

$$\frac{L_e}{r} = \frac{6.5}{65.0\times10^{-3}} = 100 < C_c \qquad \frac{L/r}{C_c} = 0.79577$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.79577) - \frac{1}{8}(0.79577)^3 = 1.90209$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250\times10^6}{1.90209}\left[1 - \frac{1}{2}(0.79577)^2\right] = 89.82\times10^6\ \text{Pa}$$

$$P_{all} = A\sigma_{all} = (10200\times10^{-6})(89.82\times10^6) = 916\times10^3\ \text{N} = 916\ \text{kN}$$

PROBLEM 10.58

10.58 A W8 × 31 rolled-steel shape is used to form a column of 21-ft effective length. Using allowable stress design, determine the allowable centric load if the yield strength of the grade of steel used is (*a*) $\sigma_Y = 36$ ksi, (*b*) $\sigma_Y = 50$ ksi. Use $E = 29\times10^6$ psi.

SOLUTION

Steel: $E = 29000$ ksi $\quad$ W 8×31 $\quad A = 9.13\ \text{in}^2 \quad r_{min} = 2.02$ in

$L_e = 21$ ft $= 252$ in $\quad L_e/r = 124.75$

(a) $\sigma_Y = 36$ ksi $\quad C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29000)}{36}} = 126.10$

$L_e/r < C_c \qquad \dfrac{L/r}{C_c} = 0.98932$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.98982) - \frac{1}{8}(0.98932)^3 = 1.91662$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.91662}\left[1 - \frac{1}{2}(0.98932)^2\right] = 9.59\ \text{ksi}$$

$$P_{all} = \sigma_{all}A = (9.59)(9.13) = 87.6\ \text{kips}$$

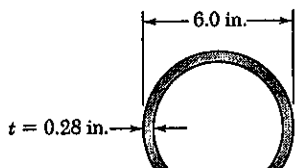
(b) $\sigma_Y = 50$ ksi $\quad C_c = \sqrt{\dfrac{2\pi^2(29000)}{50}} = 107.00$

$L_e/r > C_c \qquad \sigma_{all} = \dfrac{\pi^2 E}{\frac{23}{12}(L/r)^2} = 9.59$ ksi

$$P_{all} = \sigma_{all}A = (9.59)(9.13) = 87.6\ \text{kips}$$

PROBLEM 10.59

10.59 A steel pipe having the cross section shown is used as a column. Using allowable stress design, determine the allowable centric load if the effective length of the column is (*a*) 18 ft, (*b*) 26 ft. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



SOLUTION

$c_o = \frac{d_o}{2} = 3.0$ in. $\quad c_i = c_o - t = 2.72$ in.

$A = \pi(c_o^2 - c_i^2) = 5.0316$ in^2 $\quad r = \sqrt{\frac{I}{A}} = 2.0247$ in

$I = \frac{\pi}{4}(c_o^4 - c_i^4) = 20.627$ in^4

Steel: $E = 29000$ ksi $\quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$

(a) $L_e = 18$ ft $= 216$ in. $\quad L_e/r = 106.68 < C_c \quad \frac{L_e/r}{C_c} = 0.84601$

$F.S. = \frac{5}{3} + \frac{3}{8}(0.84601) - \frac{1}{8}(0.84601)^3 = 1.9082$

$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.9082}\left[1 - \frac{1}{2}(0.84601)^2\right] = 12.11$ ksi

$P_{all} = \sigma_{all} A = (12.11)(5.0316) = 61.0$ kips

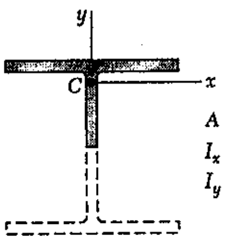
(b) $L_e = 26$ ft $= 312$ in $\quad L_e/r = 154.097 > C_c$

$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(29000)}{(1.92)(154.097)^2} = 6.28$ ksi

$P_{all} = \sigma_{all} A = (6.28)(5.0316) = 31.6$ kips

PROBLEM 10.60

10.60 A column is made from half of a W360 × 216 rolled-steel shape, with the geometric properties as shown. Using allowable stress design, determine the allowable centric load if the effective length of the column is (*a*) 4.0 m, (*b*) 6.5 m. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.



$A = 13.8 \times 10^3$ mm^2
$I_x = 26.0 \times 10^6$ mm^4
$I_y = 142.0 \times 10^6$ mm^4

SOLUTION

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{26.0 \times 10^6}{13.8 \times 10^3}} = 43.406$ mm $= 43.406 \times 10^{-3}$ m

$A = 13.8 \times 10^{-3}$ m^2

Steel $\quad C_c = \frac{2\pi^2 E}{\sigma_Y} = \sqrt{\frac{2\pi^2(200 \times 10^9)}{345 \times 10^6}} = 106.97$

(a) $L_e = 4.0$ m $\quad \frac{L_e}{r} = 92.153 < C_c \quad \frac{L_e/r}{C_c} = 0.86149$

$F.S. = \frac{5}{3} + \frac{3}{8}(0.86149) - \frac{1}{8}(0.86149)^3 = 1.9098$

$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{345 \times 10^6}{1.9098}\left[1 - \frac{1}{2}(0.86149)^2\right] = 113.61 \times 10^6$ Pa

$P_{all} = \sigma_{all} A = (113.61 \times 10^6)(13.8 \times 10^{-3}) = 1568 \times 10^3$ N $= 1568$ kN

(b) $L_e = 6.5$ m $\quad \frac{L_e}{r} = 149.75 > C_c$

$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(200 \times 10^9)}{(1.92)(149.75)^2} = 45.845 \times 10^6$ Pa

$P_{all} = \sigma_{all} A = (45.845 \times 10^6)(13.8 \times 10^{-3}) = 633 \times 10^3$ N $= 633$ kN

PROBLEM 10.61

10.61 A 3.5-m effective length column is made of sawn lumber with a 114 × 140-mm cross section. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain $\sigma_C = 7.6$ MPa and $E = 10$ GPa, determine the maximum allowable centric load for the column.

SOLUTION

Sawn lumber: $c = 0.8$, $\sigma_c = 7.6$ MPa $\quad K_{cE} = 0.3 \quad E = 10000$ MPa

$A = (114)(140) = 15960\ \text{mm}^2 = 15960 \times 10^{-6}\ \text{m}^2$

$d = 114\ \text{mm} = 114 \times 10^{-3}\ \text{m}$

$L/d = 3.5/114 \times 10^{-3} = 30.70$

$\sigma_{cE} = \dfrac{K_{cE}E}{(L/d)^2} = \dfrac{(0.3)(10000)}{(30.70)^2} = 3.1827\ \text{MPa} \qquad \dfrac{\sigma_{cE}}{\sigma_c} = 0.41878$

$U = \dfrac{1 + \sigma_{cE}/\sigma}{2c} = \dfrac{1.41878}{(2)(0.8)} = 0.88673 \qquad V = \dfrac{\sigma_{cE}/\sigma_c}{c} = 0.523475$

$C_p = U - \sqrt{U^2 - V} = 0.37408$

$\sigma_{all} = \sigma_c C_p = (7.6)(0.37408) = 2.84\ \text{MPa}$

$P_{all} = \sigma_{all} A = (2.84 \times 10^6)(15960 \times 10^{-6}) = 45.4 \times 10^3\ \text{N} = 45.4\ \text{kN}$ ◄

PROBLEM 10.62

10.62 A sawn lumber column with a 7.5 × 5.5-in. cross section has a 18-ft effective length. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 1220$ psi and that $E = 1.3 \times 10^6$ psi, determine the maximum allowable centric load for the column.

SOLUTION

Sawn lumber: $c = 0.8$, $\sigma_c = 1220$ psi $\quad E = 1.3 \times 10^6$ psi $\quad K_{cE} = 0.3$

$A = (7.5)(5.5) = 41.25\ \text{in}^2 \qquad d = 5.5\ \text{in.} \qquad L = 18\ \text{ft} = 216\ \text{in}$

$L/d = 216/5.5 = 39.273$

$\sigma_{cE} = \dfrac{K_{cE}E}{(L/d)^2} = \dfrac{(0.3)(1.3 \times 10^6)}{(39.273)^2} = 252.86\ \text{ps} \qquad \dfrac{\sigma_{cE}}{\sigma_c} = 0.20726$

$U = \dfrac{1 + \sigma_{cE}/\sigma_c}{2c} = \dfrac{1.20726}{(2)(0.8)} = 0.754537 \qquad V = \dfrac{\sigma_E/\sigma_c}{c} = 0.259075$

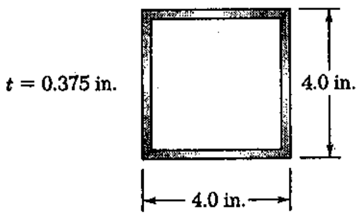
$C_p = U - \sqrt{U^2 - V} = 0.197535$

$\sigma_{all} = \sigma_c C_p = (1220)(0.197535) = 241.0\ \text{psi}$

$P_{all} = \sigma_{all} A = (241.0)(41.25) = 9.94 \times 10^3\ \text{lb.} = 9.94\ \text{kips}$ ◄

PROBLEM 10.63

10.63 A compression member has the cross section shown and an effective length of 5 ft. Knowing that the aluminum alloy used is 2014-T6, determine the allowable centric load.



SOLUTION

$b_o = 4.0 \quad b_i = b_o - 2t = 3.25$ in.

$A = (4.0)^2 - (3.25)^2 = 5.4375$ in^2

$I = \frac{1}{12}[(4.0)^4 - (3.25)^4] = 12.036$ in^4

$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{12.036}{5.4375}} = 1.488$ in. $\qquad L_e = 5$ ft $= 60$ in

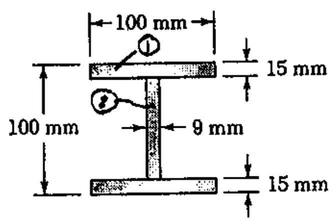
$\frac{L}{r} = \frac{60}{1.488} = 40.33 < 55$ for 2014-T6 aluminum alloy

$\sigma_{all} = 30.7 - 0.23(L/r) = 30.7 - (0.23)(40.33) = 21.42$ ksi

$P_{all} = \sigma_{all} A = (21.42)(5.4375) = 116.5$ kips ◄

PROBLEM 10.64

10.64 A compression member has the cross section shown and an effective length of 1.55 m. Knowing that the aluminum alloy used is 6061-T6, determine the allowable centric load.



SOLUTION

$I_{x1} = \frac{1}{12}(100)(15)^3 + (100)(15)(42.5)^2 = 2.7375 \times 10^6$ mm^4

$I_{x2} = \frac{1}{12}(9)(70)^3 = 257.25 \times 10^3$ mm^4

$I_x = 2I_{x1} + I_{x2} = 5.73225 \times 10^6$ mm^4

$I_y = 2\left[\frac{1}{12}(15)(100)^3\right] + \frac{1}{12}(70)(9)^3 = 2.50425 \times 10^6$ mm^4

$A = 2(100)(15) + (9)(70) = 3630$ mm^2 $= 3630 \times 10^{-6}$ m^2

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.50425 \times 10^6}{3630}} = 26.265$ mm $= 26.265 \times 10^{-3}$ m

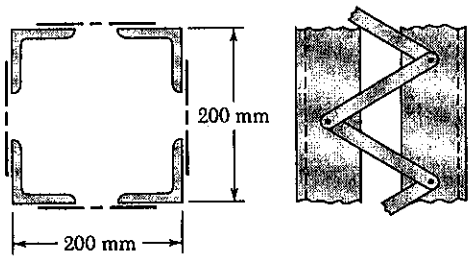
$L_e = 1.55$ m $\qquad L_e/r = 59.01 < 66$ (6061-T6 aluminum)

$\sigma_{all} = 139 - 0.868(L/r) = 139 - (0.868)(59.01) = 87.78$ MPa

$P_{all} = \sigma_{all} A = (87.78 \times 10^6)(3630 \times 10^{-6}) = 319 \times 10^3$ N $= 319$ kN ◄

PROBLEM 10.65

10.65 A column of 6.4-m effective length is obtained by connecting four 89 × 89 × 9.5-mm steel angles with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 345$ MPa and $E = 200$ GPa.



SOLUTION

Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(200\times10^9)}{345\times10^6}} = 106.97$

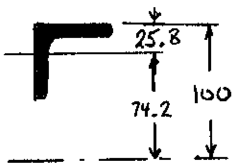
89 × 89 × 9.5 mm angle

$A_L = 1600 \text{ mm}^2$

$x = 25.8$ mm

$I_x = 1.19\times10^6 \text{ mm}^2$

$d = 100 - x = 74.2$ mm



$$I = 4(Ad^2 + I_x) = 4[(1600)(74.2)^2 + 1.19\times10^6]$$
$$= 39.996\times10^6 \text{ mm}^4$$

$$A = 4A_L = 6400 \text{ mm}^2 = 6400\times10^{-6} \text{ m}^2$$

$$r = \sqrt{\frac{I}{A}} = 79.053 \text{ mm} = 79.053\times10^{-3} \text{ m}$$

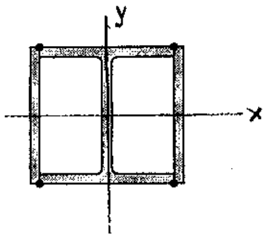
$$\frac{L_e}{r} = \frac{6.4}{79.053\times10^{-3}} = 80.958 < C_c \qquad \frac{L_e/r}{C_c} = 0.75683$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.75683) - \frac{1}{8}(0.75683)^3 = 1.8963$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{345\times10^6}{1.8963}\left[1 - \frac{1}{2}(0.75683)^2\right] = 129.83\times10^6 \text{ Pa}$$

$$P_{all} = \sigma_{all}A = (129.83\times10^6)(6400\times10^{-6}) = 831\times10^3 \text{ N} = 831 \text{ kN}$$

PROBLEM 10.68



10.68 A column of 23-ft effective length is obtained by welding two $\frac{3}{8}$-in. steel plates to a W10 × 33 rolled-steel shape as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

For W10×33 $\quad A = 9.71\ \text{in}^2,\ d = 9.73\ \text{in}\quad b_f = 7.960\ \text{in.}$

$I_x = 170\ \text{in}^4,\ I_y = 36.6\ \text{in}^4$

For column: $\quad A = 9.71 + (2)(\frac{3}{8})(9.73) = 17.0075\ \text{in}^2$

$$I_x = 170 + (2)\tfrac{1}{12}(\tfrac{3}{8})(9.73)^3 = 227.57\ \text{in}^4$$

$$I_y = 36.6 + (2)\left[(\tfrac{3}{8})(9.73)(\tfrac{7.960}{2} + \tfrac{3}{16})^2 + \tfrac{1}{12}(9.73)(\tfrac{3}{8})^3\right]$$

$$= 36.6 + (2)\left[63.37 + 0.043\right] = 163.43\ \text{in}^4$$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{163.43}{17.0075}} = 3.100\ \text{in.}$$

Steel: $\quad C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29000)}{50}} = 107.00$

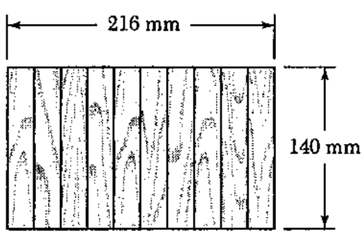
$L_e = 23\ \text{ft} = 276\ \text{in} \qquad \dfrac{L_e}{r} = \dfrac{276}{3.100} = 89.03 < C_c \qquad \dfrac{L_e/r}{C_c} = 0.83208$

$$F.S. = \tfrac{5}{3} + \tfrac{3}{8}(0.83208) - \tfrac{1}{8}(0.83208)^3 = 1.9067$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \tfrac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{50}{1.9067}\left[1 - \tfrac{1}{2}(0.83208)^2\right] = 17.145\ \text{ksi}$$

$$P_{all} = \sigma_{all} A = (17.145)(17.0075) = 292\ \text{kips}$$

PROBLEM 10.69



10.69 A rectangular column with a 4.4-m effective length is made of glued laminated wood. Knowing that for the grade of wood used the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 8.3$ MPa and that $E = 10$ GPa, determine the maximum allowable centric load for the column.

SOLUTION

Glued laminated column $\quad c = 0.9,\ K_{cE} = 0.418$

$\sigma_c = 8.3\ \text{MPa} \qquad E = 10000\ \text{MPa}$

$A = (216)(140) = 30240\ \text{mm}^2 = 30240 \times 10^{-6}\ \text{m}^2$

$d = 140\ \text{mm} = 0.140\ \text{m} \qquad L = 4.4\ \text{m} \qquad L/d = 31.429$

$$\sigma_{cE} = \frac{K_{cE} E}{(L/d)^2} = \frac{(0.418)(10000)}{(31.429)^2} = 4.2318\ \text{MPa} \qquad \sigma_{cE}/\sigma_c = 0.50986$$

$$u = \frac{1 + \sigma_{cE}/\sigma_c}{2c} = \frac{1.50986}{(2)(0.9)} = 0.838811 \qquad v = \frac{\sigma_{cE}/\sigma_c}{c} = 0.566111$$

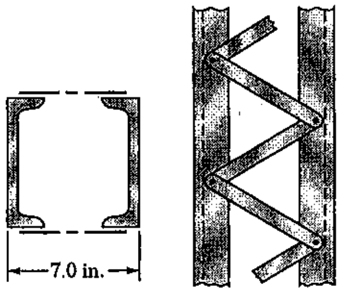
$$C_p = u - \sqrt{u^2 - v} = 0.46801$$

$$\sigma_{all} = \sigma_c C_p = (8.3)(0.46801) = 3.8845\ \text{MPa}$$

$$P_{all} = \sigma_{all} A = (3.8845 \times 10^6)(30240 \times 10^{-6}) = 117.5 \times 10^3\ \text{N} = 117.5\ \text{kN}$$

PROBLEM 10.66

10.66 A column of 21-ft effective length is obtained by connecting two C10 × 20 steel channels with lacing bars as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



SOLUTION

C10×20 $\quad A = 5.88\ \text{in}^2 \quad x = 0.606$ in

$I_x = 78.9\ \text{in}^4 \quad I_y = 2.81\ \text{in}^4$

$d = 3.5 - x = 2.894$ in

For the column: $A = (2)(5.88) = 11.76\ \text{in}^2$

$I_x = (2)(78.9) = 157.8\ \text{in}^4$

$I_y = 2\left[2.81 + (5.88)(2.894)^2\right] = 104.11\ \text{in}^4$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{104.11}{11.76}} = 2.975\ \text{in} \qquad L_e = 21\ \text{ft} = 252\ \text{in.}$$

$$\frac{L_e}{r} = 84.69 \qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$$\frac{L_e}{r} < C_c \qquad \frac{L_e/r}{C_c} = 0.67165$$

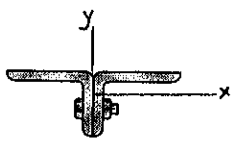
$$F.S. = \frac{5}{3} + \frac{3}{8}(0.67165) - \frac{1}{8}(0.67165)^3 = 1.8807$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.8807}\left[1 - \frac{1}{2}(0.67165)^2\right] = 14.82\ \text{ksi}$$

$$P_{all} = \sigma_{all} A = (14.82)(11.76) = 174.3\ \text{kips}$$

PROBLEM 10.67

10.67 A compression member of 2.3-m effective length is obtained by bolting together two 127 × 76 × 12.7-mm steel angles as shown. Using allowable stress design, determine the allowable centric load for the column. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.



SOLUTION

$$\text{Steel:} \quad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.66$$

L 127×76×12.7 mm $\quad$ Table gives $\quad A_L = 2420\ \text{mm}^2, \quad I_x = 3.93\times10^6\ \text{mm}^4$

$y = 44.4$ mm, $\quad I_y = 1.06\times10^6\ \text{mm}^4, \quad x = 19.0$ mm, $\quad r_y =$

For column $\quad I_x = 2(I_y)_{table} = (2)(1.06\times10^6) = 2.12\times10^6\ \text{mm}^4$

$I_y > I_x \quad \therefore \quad I_{min} = I_x = 2.12\times10^6\ \text{mm}^4 = 2.12\times10^{-6}\ \text{m}^4$

$A = 2A_L = 4840\ \text{mm}^2 = 4840\times10^{-6}\ \text{m}^2$

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.12\times10^{-6}}{4840\times10^{-6}}} = 20.93\times10^{-3}\ \text{m}$$

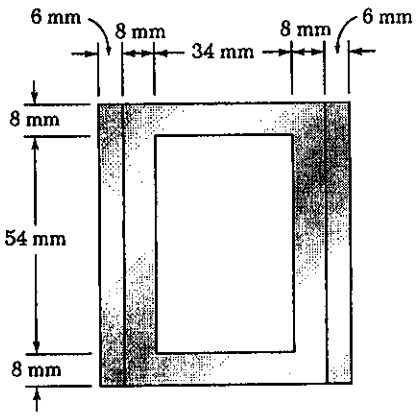
$$\frac{L_e}{r} = \frac{2.3}{20.93\times10^{-3}} = 109.90 < C_c \qquad \frac{L_e/r}{C_c} = 0.87455$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.87455) - \frac{1}{8}(0.87455)^3 = 1.9110$$

$$\sigma_{all} = \frac{\sigma_Y}{1.9110}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250\times10^6}{1.9110}\left[1 - \frac{1}{2}(0.87455)^2\right] = 80.79\times10^6\ \text{Pa}$$

$$P_{all} = \sigma_{all} A = (80.79\times10^6)(4840\times10^{-6}) = 391\times10^3\ \text{N} = 391\ \text{kN}$$

PROBLEM 10.70



10.70 An aluminum structural tube is reinforced by riveting two plates to it as shown for use as a column of 1.7-m effective length. Knowing that all material is aluminum alloy 2014-T6, determine the maximum allowable centric load.

SOLUTION

$b_o = 6 + 8 + 34 + 8 + 6 = 62 \text{ mm}$

$b_i = 34 \text{ mm}$

$h_o = 8 + 54 + 8 = 70 \text{ mm}$

$h_i = 54 \text{ mm}$

$A = b_o h_o - b_i h_i = (62)(70) - (34)(54)$
$= 2.504 \times 10^3 \text{ mm}^2 = 2.504 \times 10^{-3} \text{ m}^2$

$I_x = \frac{1}{12}\left[b_o h_o^3 - b_i h_i^3\right] = \frac{1}{12}\left[(62)(70)^3 - (34)(54)^3\right]$
$= 1.32602 \times 10^6 \text{ mm}^4$

$I_y = \frac{1}{12}\left[h_o b_o^3 - h_i b_i^3\right] = \frac{1}{12}\left[(70)(62)^3 - (54)(34)^3\right] = 1.21337 \times 10^6 \text{ mm}^4 = I_{min}$

$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{1.21337 \times 10^6}{2.504 \times 10^3}} = 22.013 \text{ mm} = 22.013 \times 10^{-3} \text{ m} \qquad L = 1.7 \text{ m}$

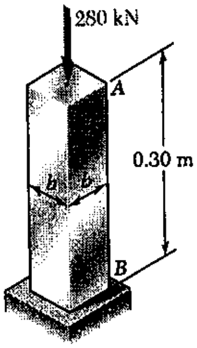
$\frac{L}{r} = \frac{1.7}{22.013 \times 10^{-3}} = 77.23 > 55$ for aluminum alloy 2014-T6

$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{77.23^2} = 62.37 \text{ MPa}$

$P_{all} = \sigma_{all} A = (62.37 \times 10^6)(2.504 \times 10^{-3}) = 156.2 \times 10^3 \text{ N} = 156.2 \text{ kN}$ ◄

PROBLEM 10.71

10.71 A 280-kN centric load is applied to the column shown, that is free at its top A and fixed at its base B. Using aluminum alloy 2014-T6, select the smallest square cross section that can be used.



SOLUTION

$$L_e = 2L = (2)(0.30) = 0.60 \text{ m}$$

$$A = b^2 \qquad I = \frac{1}{12} b^4 \qquad r = \sqrt{\frac{I}{A}} = \frac{b}{\sqrt{12}}$$

$$\frac{L}{r} = \frac{0.60\sqrt{12}}{b} = \frac{2.0785}{b}$$

2014-T6 aluminum alloy

Assume $\frac{L}{r} < 55$ $\qquad \sigma_{all} = 212 - 1.585(L/r) = 212 - (1.585)(2.0785/b)$

$$= \left(212 - \frac{3.294}{b}\right) \text{ MPa} = \left[212 - \frac{3.294}{b}\right](10^6) \text{ Pa}$$

$$P_{all} = \sigma_{all} A = \left[212\, b^2 - 3.294\, b\right](10^6) = 280 \times 10^3$$

$$212 b^2 - 3.294 b - 280 \times 10^{-3} = 0$$

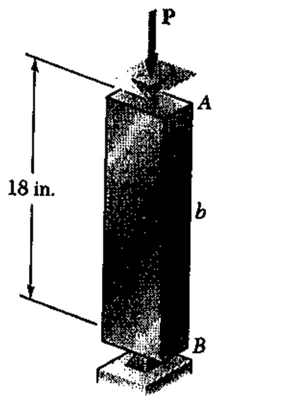
$$b = \frac{3.294 + \sqrt{(3.294)^2 + (4)(212)(280 \times 10^{-3})}}{(2)(212)} = 44.9 \times 10^{-3} \text{ m}$$

$$\frac{L}{r} = \frac{2.0785}{b} = \frac{2.0785}{44.9 \times 10^{-3}} = 46.26 < 55$$

Answer: $b = 44.9 \times 10^{-3}$ m $= 44.9$ mm

PROBLEM 10.72

10.72 A 16-kip centric load must be supported by an aluminum column as shown. Using the aluminum alloy 6061-T6, determine the minimum dimension b that can be used.



SOLUTION

$L_e = L = 18 \text{ in} \qquad A = 2b^2 \qquad I_{min} = \frac{1}{12}(2b)(b)^3 = \frac{1}{6}b^4$

$r = \sqrt{\frac{I_{min}}{A}} = \frac{b}{\sqrt{12}} \qquad \frac{L}{r} = \frac{18\sqrt{12}}{b} = \frac{62.354}{b}$

6061-T aluminum alloy. Assume $\frac{L}{r} < 66$

$\sigma_{all} = 20.2 - 0.126(L/r) = 20.2 - (0.126)\frac{62.354}{b}$

$= 20.2 - \frac{7.8566}{b}$ ksi

$P_{all} = \sigma_{all} A = (20.2 - \frac{7.8566}{b})(2b^2) = 40.4\, b^2 - 15.713\, b$ kip

$40.4\, b^2 - 11.111\, b = 16 \qquad b = \frac{15.713 + \sqrt{(15.713)^2 + (4)(40.4)(16)}}{(2)(40.4)} = 0.853 \text{ in}$

$\frac{L}{r} = \frac{62.354}{b} = \frac{62.354}{0.853} = 73.09 > 66$ Assumption not verified.

Assume $\frac{L}{r} > 66 \qquad \sigma_{all} = \frac{51000}{(L/r)^2} = \frac{51000\, b^2}{(62.354)^2} = 13.117\, b^2$ ksi

$P_{all} = \sigma_{all} A = (13.117\, b^2)(2b^2) = 26.234\, b^4 = 16$ kips

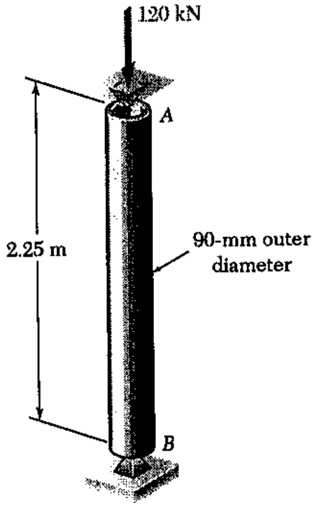
$b = \sqrt[4]{\frac{16}{26.234}} = 0.884 \text{ in.}$

$\frac{L}{r} = \frac{62.354}{0.884} = 70.56 > 66$ Assumption verified.

$b = 0.884 \text{ in.}$

PROBLEM 10.73

10.73 An aluminum tube of 90-mm outer diameter is to carry a centric load of 120 kN. Knowing that the stock of tubes available for use are made of alloy 2014-T6 and with wall thickness in increments of 3 mm from 6 mm to 15 mm, determine the lightest tube that can be used.



SOLUTION

$L = 2250 \text{ mm}, \quad P = 120 \times 10^3 \text{ N} \qquad r_o = 45 \text{ mm}$

$r_i = r_o - t \qquad A = \pi(r_o^2 - r_i^2) \qquad I = \frac{\pi}{4}(r_o^4 - r_i^4)$

$r = \sqrt{I/A}$

For 2014-T6 aluminum alloy

$\sigma_{all} = 212 - 1.585(L/r) \text{ MPa} \quad \text{if } L/r < 55$

$\sigma_{all} = \dfrac{372 \times 10^3}{(L/r)^2} \text{ MPa} \quad \text{if } L/r > 55$

$P_{all} = \sigma_{all} A$

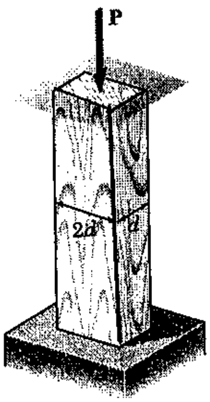
Calculate P_{all} for each thickness.

t mm	r_i mm	A mm^2	I 10^6 mm^4	r mm	L/r	σ_{all} MPa	P_{all} kN
6	39	1583	1.404	29.78	75.56	65.16	103.1
→ 9	36	2290	1.901	28.82	78.08	61.01	139.7 ←
12	33	2941	2.289	27.90	80.65	57.20	168.2
15	30	3534	2.584	27.04	83.20	53.74	189.9

Since P_{all} must be greater than 120 kN, use $t = 9$ mm

PROBLEM 10.74

10.74 A 18-kip centric load is applied to a rectangular sawn lumber column of 22-ft effective length. Using sawn lumber for which the adjusted allowable stress for compression parallel to the grain is $\sigma_C = 1050$ psi and knowing that $E = 10 \times 10^6$ psi. determine the smallest cross section that can be used for the column if $b = 2d$.



SOLUTION

Sawn lumber $c = 0.8$ $K_{cE} = 0.3$

$\sigma_c = 1050$ psi $E = 10 \times 10^6$ psi

$A = 2d^2$ $L = 22$ ft $= 264$ $L/d = \frac{264}{d}$

Assumed $C_p = 0.5$

$$\sigma_{all} = \sigma_c C_p = (1050)(0.5) = 525 \text{ psi}$$

$$P_{all} = \sigma_{all} A = 2\sigma_{all} d^2$$

$$d = \sqrt{\frac{P_{all}}{2\sigma_{all}}} = \sqrt{\frac{18\,000}{2\sigma_{all}}} = \frac{94.868}{\sqrt{\sigma_{all}}} = 4.14 \text{ in.}$$

$L/d = 63.76$ $$\sigma_{cE} = \frac{K_{cE} E}{(L/d)^2} = \frac{(0.3)(10\times10^6)}{(L/d)^2} = \frac{3\times10^6}{(L/d)^2} \quad 737.9 \text{ psi}$$

$$\sigma_{cE}/\sigma_c = 0.7028$$

Checked $$C_p = \frac{1+\sigma_{cE}/\sigma_c}{2c} - \sqrt{\left(\frac{1+(\sigma_{cE}/\sigma_c)}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_c}{c}} = 0.5601$$

Results of similar trials are summarized in the table below.

Assumed C_p	σ_{all} (psi)	d (in)	L/d	σ_{cE} (psi)	σ_{cE}/σ_c	Checked C_p	ΔC_p
0.5	525	4.14	63.76	737.9	0.7028	0.5601	0.0601
0.56	588	3.91	67.48	658.8	0.6275	0.5169	−0.0431
0.535	561.75	4.00	66.00	688.7	0.6559	0.5337	−0.0013
0.5343	561.0	4.005	65.92	690.4	0.6575	0.5346	≈ 0

Answer $d = 4.01$ in.

PROBLEM 10.77

10.77 A column of 5.6-m effective length must carry a centric load of 2750 kN. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use allowable stress design to select the wide-flange shape of 360-mm nominal depth that should be used.

SOLUTION

$$P < \frac{\sigma_Y A}{F.S.}$$

$$A > \frac{(F.S)P}{\sigma_Y} = \frac{(5/3)(2750\times10^3)}{250\times10^6} = 18.33\times10^{-3}\ m^2 = 18330\ mm^2$$

$$P < \frac{\pi^2 E I}{1.92 L^2}$$

$$I > \frac{1.92 P L^2}{\pi^2 E} = \frac{(1.92)(2750\times10^3)(5.6)^2}{\pi^2(200\times10^9)} = 83.9\times10^{-6}\ m^4 = 83.9\times10^6\ mm^4$$

Try W 360 × 216 $\quad A = 27600\ mm^2 = 27600\times10^{-6}\ m^2$ o.k.

$I_{min} = 283\times10^6\ mm^4$ o.k.

$r_y = 101\ mm = 101\times10^{-3}\ m$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{250\times10}} = 125.66$$

$$\frac{L_e}{r} = \frac{5.6}{101\times10^{-3}} = 55.45 < C_c \qquad \frac{L_e/r}{C_c} = 0.44123$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.44123) - \frac{1}{8}(0.44123)^3 = 1.8214$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{250\times10^6}{1.8214}\left[1 - \frac{1}{2}(0.44123)^2\right] = 123.9\times10^6\ Pa$$

$$P_{all} = \sigma_{all} A = (123.9\times10^6)(27600\times10^{-6}) = 3420\times10^3\ N = 3420\ kN$$

3420 kN > 2750 kN $\qquad$ Use W360 × 216 ◄

PROBLEM 10.78

10.78 A column of 4.6-m effective length must carry a centric load of 525 kN. Knowing that $\sigma_Y = 345$ MPa and $E = 200$ GPa, use allowable stress design to select the wide-flange shape of 200-mm nominal depth that should be used.

SOLUTION

$$P < \frac{\sigma_Y A}{F.S.}$$

$$A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(525\times10^3)}{345\times10^6} = 2.54\times10^{-3}\ m^2 = 2540\ mm^2$$

$$P < \frac{\pi^2 EI}{1.92 L_e^2}$$

$$I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(525\times10^3)(4.6)^2}{\pi^2(200\times10^9)} = 10.89\times10^{-6}\ m^4 = 10.89\times10^6\ mm^4$$

Try W 200×46.1 $\quad A = 5860\ mm^2,\quad I_{min} = 15.3\times10^6\ mm^4,\quad r = 51.1\times10^{-3}\ m$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{345\times10^6}} = 106.97$$

$$\frac{L_e}{r} = \frac{4.6}{51.1\times10^{-3}} = 90.02 < C_c \qquad \frac{L_e/r}{C_c} = 0.84154$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.84154) - \frac{1}{8}(0.84154)^3 = 1.9077$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{345\times10^6}{1.9077}\left[1 - \frac{1}{2}(0.84154)^2\right] = 116.8\times10^6\ Pa$$

$$P_{all} = \sigma_{all} A = (116.8\times10^6)(5860\times10^{-6}) = 684\ kN > 525\ kN$$

Use W 200×46.1

PROBLEM 10.79

10.79 A column of 22.5-ft effective length must carry a centric load of 288 kips. Using allowable stress design, select the wide-flange shape of 14-in. nominal depth that should be used. Use $\sigma_Y = 50$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

$$P < \frac{\sigma_Y A}{F.S.} \qquad A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(288)}{50} = 9.6 \text{ in}^2$$

$$L_e = 22.5 \text{ ft} = 270 \text{ in} \qquad E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$P < \frac{\pi^2 E I}{1.92 L_e^2} \qquad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(288)(270)^2}{\pi^2 (29000)} = 140.8 \text{ in}^4$$

Try W 14 × 82 $\quad A = 24.1 \text{ in}^2, \quad I_{min} = 148 \text{ in}^4, \quad r = 2.48 \text{ in}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{50}} = 107.00$$

$$\frac{L_e}{r} = \frac{270}{2.48} = 108.87 > 107.00$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L/r)^2} = \frac{\pi^2(29000)}{(1.92)(108.87)^2} = 12.58 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (12.58)(24.1) = 303 \text{ kips} > 288 \text{ kips}$$

Use W 14 × 82

PROBLEM 10.80

10.80 A column of 17-ft effective length must carry a centric load of 235 kips. Using allowable stress design, select the wide-flange shape of 10-in. nominal depth that should be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

$$P < \frac{\sigma_Y A}{F.S.} \qquad A > \frac{(F.S.)P}{\sigma_Y} = \frac{(5/3)(235)}{36} = 10.88 \text{ in}^2$$

$$L_e = 17 \text{ ft} = 204 \text{ in}^2 \qquad E = 29 \times 10^6 \text{ psi} = 29000 \text{ ksi}$$

$$P < \frac{\pi^2 E I}{1.92 L^2} \qquad I > \frac{1.92 P L_e^2}{\pi^2 E} = \frac{(1.92)(235)(204)^2}{\pi^2 (29000)} = 65.6 \text{ in}^4$$

Try W 10 × 54 $\quad A = 15.8 \text{ in}^2 \quad I_y = 103 \text{ in}^4 \quad r = 2.56 \text{ in}$

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$$\frac{L_e}{r} = \frac{204}{2.56} = 79.69 < C_c \qquad \frac{L_e/r}{C_c} = \frac{79.69}{126.10} = 0.63194$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.63194) - \frac{1}{8}(0.63194)^3 = 1.8721$$

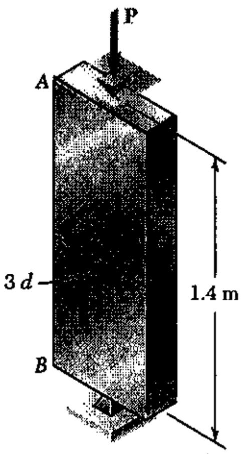
$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.8721}\left[1 - \frac{1}{2}(0.63194)^2\right] = 15.39 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (15.39)(15.8) = 243 \text{ kips} > 235 \text{ kips}$$

Use W 10 × 54

PROBLEM 10.81

10.81 A centric load **P** must be supported by the steel bar AB. Using allowable stress design, determine the smallest dimension d of the cross section that can be used when (*a*) $P = 108$ kN, (*b*) $P = 166$ kN. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.



SOLUTION

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200\times10^9)}{250}} = 125.66$$

$$L_e = L = 1.4 \text{ m}$$

$$A = (3d)(d) = 3d^2$$

$$I = \tfrac{1}{12}(3d)(d)^3 = \tfrac{1}{4}d^4$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} = 0.288675\, d$$

(a) $P = 108\times10^3$ N $\qquad$ Assume $\frac{L_e}{r} > C_c$

$$P_{all} = \frac{\pi^2 E I}{1.92\, L_e^2} \qquad I = \frac{(1.92)\, P_{all}\, L_e^2}{\pi^2 E} = \tfrac{1}{4}d^4$$

$$d^4 = \frac{(4)(1.92)\, P\, L_e^2}{\pi^2 E} = \frac{(4)(1.92)(108\times10^3)(1.4)^2}{\pi^2(200\times10^9)} = 823.59\times10^{-9} \text{ m}^4$$

$d = 30.125\times10^{-3}$ m $\qquad r = 8.696\times10^{-3}$ m

$$\frac{L_e}{r} = \frac{1.4}{8.696\times10^{-3}} = 160.99 > 125.66 \checkmark \qquad d = 30.1 \text{ mm}$$

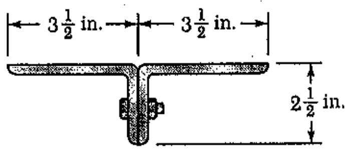
(b) $P = 166\times10^3$ N $\qquad$ Assume $\frac{L_e}{r} > C_c$

$$d^4 = \frac{(4)(1.92)\, P\, L_e^2}{\pi^2 E} = \frac{(4)(1.92)(166\times10^3)(1.4)^2}{\pi^2(200\times10^9)} = 1.26588\times10^{-6} \text{ m}^4$$

$d = 33.543\times10^{-3}$ m $\qquad r = 9.68295\times10^{-3}$

$$\frac{L_e}{r} = \frac{1.4}{9.68295\times10^{-3}} = 144.58 > 125.66 \checkmark \qquad d = 33.5 \text{ mm}$$

PROBLEM 10.82



10.82 Two $3\frac{1}{2} \times 2\frac{1}{2}$ -in. angles are bolted together as shown for use as a column of 8-ft effective length to carry a centric load of 41 kips. Knowing that the angles available have thicknesses of $\frac{1}{4}$ in., $\frac{3}{8}$ in., and $\frac{1}{2}$ in., use allowable stress design to determine the lightest angles that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

Steel: $E = 29000$ ksi $\qquad L_e = 8$ ft $= 96$ in²

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

Try L $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$ in.

$$A = (2)(2.11) = 4.22 \text{ in}^2$$
$$I_x = (2)(1.09) = 2.18 \text{ in}^2 < I_y$$
$$r = \sqrt{\frac{I_x}{A}} = 0.719 \text{ in.}$$

$$\frac{L_e}{r} = \frac{96}{0.719} = 133.52 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(29000)}{1.92\,(133.52)^2} = 8.36 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (8.36)(4.22) = 35.3 \text{ kips} < 41 \text{ kips}$$ Do not use.

Try L $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$ in.

$$A = (2)(2.75) = 5.50 \text{ in}^2$$
$$r = 0.704 \text{ in}$$

$$\frac{L_e}{r} = \frac{96}{0.704} = 136.36 > C_c$$

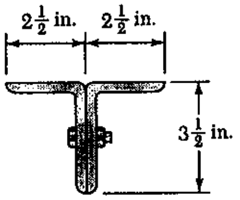
$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(29000)}{(1.92)(136.36)^2} = 8.02 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (8.02)(5.50) = 44.1 \text{ kips} > 41 \text{ kips}$$

Use L $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{2}$ in.

PROBLEM 10.83

10.83 Two $3\frac{1}{2} \times 2\frac{1}{2}$ -in. angles are bolted together as shown for use as a column of 6-ft effective length to carry a centric load of 54 kips. Knowing that the angles available have thicknesses of $\frac{1}{4}$ in., $\frac{3}{8}$ in., and $\frac{1}{2}$ in., use allowable stress design to determine the lightest angles that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.



SOLUTION

Steel: $E = 29000$ ksi $\qquad L_e = 6\text{ ft} = 72$ in

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

Try L $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$ in.

$$A = (2)(2.11) = 4.22 \text{ in}^2$$
$$I_x = (2)(2.56) = 5.12 \text{ in}^4$$
$$I_y = 2\left[1.09 + (2.11)(0.660)^2\right] = 4.018 \text{ in}^4 = I_{min}$$
$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{4.018}{4.22}} = 0.9758$$

$$\frac{L_e}{r} = \frac{72}{0.9758} = 73.78 < C_c \qquad \frac{L_e/r}{C_c} = \frac{73.78}{126.10} = 0.58509$$

$$F.S = \frac{5}{3} + \frac{3}{8}(0.58509) - \frac{1}{8}(0.58509)^3 = 1.8610$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.8610}\left[1 - \frac{1}{2}(0.58509)^2\right] = 16.03 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (16.03)(4.22) = 67.7 \text{ kips} > 54 \text{ kips} \quad \text{(allowed)}$$

Try L $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ in.

$$A = (2)(1.44) = 2.88 \text{ in}^2$$
$$I_x = (2)(1.80) = 3.60 \text{ in}^4$$
$$I_y = (2)\left[0.777 + (1.44)(0.614)^2\right] = 2.6397 \text{ in}^4 = I_{min}$$
$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{2.6397}{2.88}} = 0.97538 \text{ in.}$$

$$\frac{L_e}{r} = \frac{72}{0.95738} = 75.205 < C_c \qquad \frac{L/r}{C_c} = \frac{75.205}{126.10} = 0.59633$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.59633) - \frac{1}{8}(0.59633)^3 = 1.8638$$

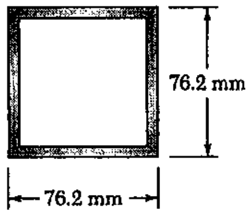
$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \frac{36}{1.8638}\left[1 - \frac{1}{2}(0.59633)^2\right] = 15.88 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (15.88)(2.88) = 45.7 \text{ kips} < 54 \text{ kips} \quad \text{Do not use}$$

Use L $3\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$ in.

PROBLEM 10.84

10.84 A square structural tube having the cross section shown is used as a column of 3.1-m effective length to carry a centric load of 129 kN. Knowing that the tubes available for use are made with wall thicknesses of 3.2 mm, 4.8 mm, 6.4 mm, and 7.9 mm, use allowable stress design to determine the lightest tube that can be used. Use $\sigma_Y = 250$ MPa and $E = 200$ GPa.



SOLUTION

$b_o = 76.2$ mm $\quad b_i = b_o - 2t \quad A = b_o^2 - b_i^2$

$I = \frac{1}{12}(b_o^4 - b_i^4)$

$L_e = 3.1$ m $\quad P = 129$ kN $\quad b_o = 76.2$ mm

Steel: $C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(200\times10^9)}{250\times10^6}} = 125.66$

Try $t = 4.8$ mm $\quad b_i = 76.2 - 9.6 = 66.6$ mm

$A = (76.2)^2 - (66.6)^2 = 1.37088\times10^3\ \text{mm}^2 = 1.37088\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}[(76.2)^4 - (66.6)^4] = 1.17005\times10^6\ \text{mm}^4 = 1.17005\times10^{-6}\ \text{m}^4$

$r = \sqrt{\dfrac{I}{A}} = 29.21\times10^{-3}$ m

$\dfrac{L_e}{r} = \dfrac{3.1}{29.21\times10^{-3}} = 106.11 < C_c \qquad \dfrac{L_e/r}{C_c} = 0.84443$

$F.S. = \frac{5}{3} + \frac{3}{8}(0.84443) - \frac{1}{8}(0.84443)^3 = 1.9081$

$\sigma_{all} = \dfrac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \dfrac{250\times10^6}{1.9081}\left[1 - \frac{1}{2}(0.84443)^2\right] = 84.3\times10^6$ Pa

$P_{all} = \sigma_{all}A = (84.3\times10^6)(1.37088\times10^{-3}) = 115.6\times10^3$ N

$= 115.6$ kN < 129 kN $\qquad$ Do not use.

Try $t = 6.4$ mm $\quad b_i = 76.2 - 12.8 = 63.4$ mm

$A = (76.2)^2 - (63.4)^2 = 1.78688\times10^3\ \text{mm}^2 = 1.78688\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}[(76.2)^4 - (63.4)^4] = 1.46316\times10^6\ \text{mm}^4 = 1.46316\times10^{-6}\ \text{m}^4$

$r = \sqrt{\dfrac{I}{A}} = 28.615\times10^{-3}$ m

$\dfrac{L_e}{r} = \dfrac{3.1}{28.615\times10^{-3}} = 108.33 < C_c \qquad \dfrac{L_e/r}{C_c} = 0.86212$

$F.S. = \frac{5}{3} + \frac{3}{8}(0.86212) - \frac{1}{8}(0.86212)^3 = 1.9099$

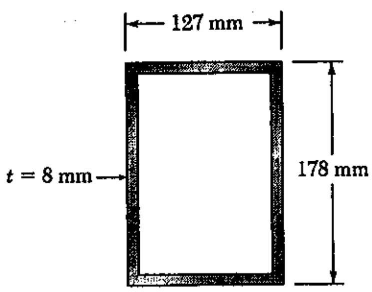
$\sigma_{all} = \dfrac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r}{C_c}\right)^2\right] = \dfrac{250\times10^6}{1.9099}\left[1 - \frac{1}{2}(0.86212)^2\right] = 82.25\times10^6$ Pa

$P_{all} = \sigma_{all}A = (82.25\times10^6)(1.78688\times10^{-3}) = 147.0\times10^3$ N

$= 147.0$ kN > 129 kN

Use $\quad t = 6.4$ mm

PROBLEM 10.85



*10.85 A rectangular tube having the cross section shown is used as a column of 4.5-m effective length. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use load and resistance factor design to determine the largest centric live load that can be applied if the centric dead load is 140 kN. Use a dead load factor $\gamma_D = 1.2$, a live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.

SOLUTION

$h_o = 127$ mm $\quad b_o = 178$ mm $\quad h_i = h_o - 2t = 111$ mm

$b_i = b_o - 2t = 162$ mm

$A = b_o h_o - b_i h_i = (178)(127) - (162)(111)$

$= 4624 \text{ mm}^2 = 4624 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}\left[b_o h_o^3 - b_i h_i^3\right] = \frac{1}{12}\left[(178)(127)^3 - (162)(111)^3\right] = 11.9213 \times 10^6 \text{ mm}^4$

$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{11.9213 \times 10^6}{4624}} = 50.775 \text{ mm} = 50.775 \times 10^{-3} \text{ m}$

$\frac{L}{r} = \frac{4.5}{50.775 \times 10^{-3}} = 88.63$

$\lambda_c = \frac{L}{r\pi}\sqrt{\frac{\sigma_Y}{E}} = \frac{88.63}{\pi}\sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 0.9974 < 1.5$

$\lambda_c^2 = 0.9948$

$P_U = A(0.658)^{\lambda_c^2}\sigma_Y = (4624 \times 10^{-6})(0.658)^{0.9948}(250 \times 10^6) = 762.3 \times 10^3$ N

$= 762.3$ kN

$\gamma_D P_D + \gamma_L P_L = \phi P_U$

$(1.2)(140) + 1.6 P_L = (0.85)(762.3) \qquad P_L = 300$ kN

PROBLEM 10.86

*10.86 A column with a 19.5-ft effective length supports a centric load, with ratio of dead to live load equal to 1.35. The dead load factor is $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$, and the resistance factor $\phi = 0.85$. Use load and resistance factor design to determine the allowable centric dead and live loads if the column is made of the following rolled-steel shape: (*a*) W10 × 39, (*b*) W 14 × 68. Use $E = 29 \times 10^6$ psi and $\sigma_Y = 50$ ksi.

SOLUTION

$L_e = 19.5 \text{ ft} = 234 \text{ in}$

(a) W10×39 $\quad A = 11.5 \text{ in}^2 \quad r_y = 1.98 \text{ in} \quad L_e/r_y = 118.18$

$$\lambda_c = \frac{L_e/r}{\pi}\sqrt{\frac{\sigma_Y}{E}} = \frac{118.18}{\pi}\sqrt{\frac{50}{29000}} = 1.5620 > 1.5$$

$$P_U = A\left(\frac{0.877}{\lambda_c^2}\right)\sigma_Y = \frac{(11.5)(0.877)(50)}{(1.5620)^2} = 206.67 \text{ kips}$$

$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

$$(1.2)(1.35\,P_L) + 1.6\,P_L = (0.85)(206.67)$$

$P_D = 73.7$ kips ◄
$P_L = 54.6$ kips ◄

(b) W 14×68 $\quad A = 20.0 \text{ in}^2 \quad r_y = 2.46 \text{ in} \quad L_e/r_y = 95.12$

$$\lambda_c = \frac{L_e/r}{\pi}\sqrt{\frac{\sigma_Y}{E}} = \frac{95.12}{\pi}\sqrt{\frac{50}{29000}} = 1.2572 < 1.5$$

$$\lambda_c^2 = 1.5806$$

$$P_U = A\,(0.658)^{\lambda_c^2}\sigma_Y = (20.0)(0.658)^{1.5806}(50) = 516 \text{ kips}$$

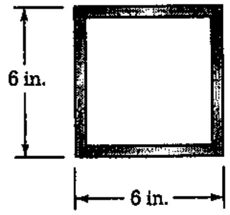
$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

$$(1.2)(1.35\,P_L) + 1.6\,P_L = (0.85)(516)$$

$P_D = 183.9$ kips ◄
$P_L = 136.2$ kips ◄

PROBLEM 10.87

*10.87 The structural tube having the cross section shown is used as a column of 15-ft effective length to carry a centric dead load of 51 kips and a centric live load of 58 kips. Knowing that the tubes available for use are made with wall thicknesses in increments of $\frac{1}{16}$ in. from $\frac{3}{16}$ in. to $\frac{3}{8}$ in., use load and resistance factor design to determine the lightest tube that can be used. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.



SOLUTION

$L_e = 15 \text{ ft} = 180 \text{ in}$

$\gamma_D P_D + \gamma_L P_L = \varphi P_U$

Required $P_U = \dfrac{\gamma_D P_D + \gamma_L P_L}{\varphi} = \dfrac{(1.2)(51) + (1.6)(58)}{0.85} = 181.2$ kips

Try $t = \frac{1}{4}$ in $= 0.25$ in $\quad b_o = 6.0$ in $\quad b_i = b_o - 2t = 5.5$ in

$A = b_o^2 - b_i^2 = (6)^2 - (5.5)^2 = 5.75 \text{ in}^2$

$I = \frac{1}{12}(b_o^4 - b_i^4) = \frac{1}{12}\left[(6)^4 - (5.5)^4\right] = 31.74 \text{ in}^4$

$r = \sqrt{\dfrac{I}{A}} = \sqrt{\dfrac{31.74}{5.75}} = 2.3496$ in $\qquad \dfrac{L_e}{r} = \dfrac{180}{2.3496} = 76.61$

$\lambda_c = \dfrac{L_e/r}{\pi}\sqrt{\dfrac{\sigma_Y}{E}} = \dfrac{76.61}{\pi}\sqrt{\dfrac{36}{29000}} = 0.85916 < 1.5 \qquad \lambda_c^2 = 0.73815$

$P_U = A\,(0.658)^{\lambda_c^2}\sigma_Y = (5.75)(0.658)^{0.73815}(36) = 152.0 \text{ kips} < 181.2 \text{ kips}$

Thickness is to small.

Since P_U is approximately proportional to thickness, the required thickness is approximately

$$\frac{t_{req}}{0.25} \approx \frac{P_{U(req)}}{152} = \frac{181.18}{152} \qquad t_{req} \approx 0.296 \text{ in.}$$

Try $t = \frac{5}{16}$ in $= 0.3125$ in., $\quad b_i = 5.375$

$A = 7.1094 \text{ in}^2$, $I = 38.44 \text{ in}^4$, $r = 2.3254$ in $\qquad \dfrac{L_e}{r} = 77.41$

$\lambda_c = \dfrac{77.41}{\pi}\sqrt{\dfrac{36}{29000}} = 0.86811 < 1.5 \qquad \lambda_c^2 = 0.75361$

$P_U = (7.1094)(0.658)^{0.75361}(36) = 186.7 \text{ kips} > 181.2 \text{ kips}$

Use $t = \frac{5}{16}$ in. ◀

PROBLEM 10.88

*10.88 A column of 5.5-m effective length must carry a centric dead load of 310 kN and a centric live load of 375 kN. Knowing that $\sigma_Y = 250$ MPa and $E = 200$ GPa, use load and resistance factor design to select the wide-flange shape of 310-mm nominal depth that should be used. The dead load factor $\gamma_D = 1.2$, the live load factor $\gamma_L = 1.6$ and the resistance factor $\phi = 0.85$.

SOLUTION

$$\gamma_D P_D + \gamma_L P_L = \varphi P_U$$

Required $P_U = \dfrac{\gamma_D P_D + \gamma_L P_L}{\varphi} = \dfrac{(1.2)(310) + (1.6)(375)}{0.85} = 1143$ kN

Preliminary calculations

$$P_U < \sigma_Y A \quad \therefore \quad A > \frac{P_U}{\sigma_Y} = \frac{1143 \times 10^3}{250 \times 10^6} = 4.572 \times 10^{-3}\ \text{m}^2 = 4572\ \text{mm}^2$$

$$P_U < \frac{\pi^2 EI}{L^2} \quad \therefore \quad I > \frac{P_U L_e^2}{\pi^2 E} = \frac{(1143 \times 10^3)(5.5)^2}{\pi^2 (200 \times 10^9)} = 17.52 \times 10^{-6}\ \text{m}^4 = 17.52 \times 10^6\ \text{mm}^4$$

Try W 310×60 $\quad A = 7590\ \text{mm}^2 = 7590 \times 10^{-6}\ \text{m}^2$

$I_y = 18.3 \times 10^6\ \text{mm}^6$, $\quad r_y = 49.1\ \text{mm} = 49.1 \times 10^{-3}$ m

$$\lambda_c = \frac{L_e}{\pi r}\sqrt{\frac{\sigma_Y}{E}} = \frac{5.5}{\pi (49.1 \times 10^{-3})}\sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 1.2606 < 1.5$$

$$\lambda_c^2 = 1.5892$$

$$P_U = A (0.658)^{\lambda_c^2} \sigma_Y = (7590 \times 10^{-6})(0.658)^{1.5892}(250 \times 10^6)$$

$$= 975 \times 10^3\ \text{N} = 975\ \text{kN} < 1143\ \text{kN}$$

Too light. Do not use.

Try W310 × 74 $\quad A = 9480\ \text{mm}^2 = 9480 \times 10^{-6}\ \text{m}^2$

$r_y = 49.7\ \text{mm} = 49.7 \times 10^{-6}$ m

$$\lambda_c = \frac{5.5}{\pi (49.7 \times 10^{-3})}\sqrt{\frac{250 \times 10^6}{200 \times 10^9}} = 1.2454 \qquad \lambda_c^2 = 1.5510$$

$$P_U = (9480 \times 10^{-6})(0.658)^{1.5510}(250 \times 10^6) = 1238 \times 10^3\ \text{N}$$

$$= 1238\ \text{kN} > 1143\ \text{kN}$$

Use W 310×74 ◄

PROBLEM 10.89

10.89 A sawn lumber column with a 125-mm-square cross section and a 3.6-m effective length is made of a grade of wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 9.2$ MPa and a modulus of elasticity $E = 12$ GPa. Using the allowable-stress method, determine the maximum load **P** that can be safely supported with an eccentricity of 50 mm.

SOLUTION

$d = 125 \text{ mm} = 0.125 \text{ m} \qquad A = d^2 = 15.625 \times 10^{-3} \text{ m}^2 \qquad \frac{L}{d} = \frac{3.6}{0.125} = 28.8$

$\sigma_C = 9.2 \text{ MPa}, \quad E = 12000 \text{ MPa}, \quad$ sawn lumber: $c = 0.8, \; K_E = 0.300$

$$\sigma_{CE} = \frac{K_{CE} E}{(L/d)^2} = \frac{(0.300)(12000)}{(28.8)^2} = 4.34 \text{ MPa} \qquad \sigma_{CE}/\sigma_C = 0.47177$$

$$C_P = \frac{1 + (\sigma_{CE}/\sigma_C)}{2c} - \sqrt{\left(\frac{1 + \sigma_{CE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{CE}/\sigma_C}{c}} = 0.41347$$

$\sigma_{all} = \sigma_C C_C = (9.2)(0.41347) = 3.804 \text{ MPa} \qquad e = 50 \text{ mm} = 0.050 \text{ m}$

$I = \frac{1}{12} d^4 = \frac{1}{12}(0.125)^4 = 20.345 \times 10^{-6} \text{ m}^4 \qquad c = \frac{1}{2} d = 0.0625 \text{ m}$

$$\frac{P}{A} + \frac{Pec}{I} \le \sigma_{all} \qquad \left(\frac{1}{A} + \frac{ec}{I}\right)P < \sigma_{all}$$

$$P < \frac{\sigma_{all}}{\frac{1}{A} + \frac{ec}{I}} = \frac{3.804 \times 10^6}{\frac{1}{15.625 \times 10^{-3}} + \frac{(0.050)(0.0625)}{20.345 \times 10^{-6}}} = 17.48 \times 10^3 \text{ N}$$

$$P < 17.48 \text{ kN}$$

PROBLEM 10.90

10.89 A sawn lumber column with a 125-mm-square cross section and a 3.6-m effective length is made of a grade of wood that has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 9.2$ MPa and a modulus of elasticity $E = 12$ GPa. Using the allowable-stress method, determine the maximum load **P** that can be safely supported with an eccentricity of 50 mm.

10.90 Solve Prob. 10.89 using the interaction method and an allowable stress in bending of 12.8 MPa.

SOLUTION

$d = 125 \text{ mm} = 0.125 \text{ m} \qquad A = d^2 = 15.625 \times 10^{-3} \text{ m}^2 \qquad \frac{L}{d} = \frac{3.6}{0.125} = 28.8$

$\sigma_c = 9.2 \text{ MPa} \qquad E = 12000 \text{ MPa} \qquad$ sawn lumber: $c = 0.8, \; K_{cE} = 0.300$

$$\sigma_{cE} = \frac{K_{cE} E}{(L/d)^2} = \frac{(0.300)(12000)}{(28.8)^2} = 4.34 \text{ MPa} \qquad \sigma_{cE}/\sigma_c = 0.47177$$

$$C_p = \frac{1 + (\sigma_{cE}/\sigma_c)}{2c} - \sqrt{\left(\frac{1+\sigma_{cE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_c}{c}} = 0.41347$$

$$\sigma_{all,c} = \sigma_c C_c = (9.2)(0.41347) = 3.804 \text{ MPa} \qquad e = 50 \text{ mm} = 0.050 \text{ m}$$

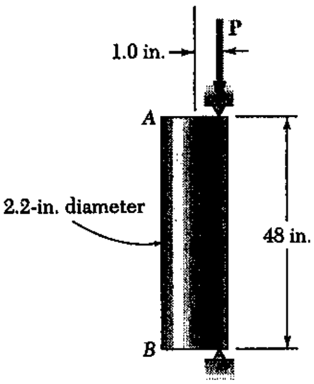
$$I = \frac{1}{12} d^4 = \frac{1}{12}(0.125)^4 = 20.345 \times 10^{-6} \text{ m}^4 \qquad c = \frac{1}{2} d = 0.0625 \text{ m}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,B}} < 1$$

$$P < \frac{1}{\frac{1}{A\sigma_{all,c}} + \frac{ec}{I\sigma_{all,B}}} = \frac{1}{\frac{1}{(15.625\times10^{-3})(3.804\times10^6)} + \frac{(0.050)(0.0625)}{(20.345\times10^{-6})(12.8\times10^6)}}$$

$$= 34.7 \times 10^3 \text{ N} = 34.7 \text{ kN}$$

PROBLEM 10.91



10.91 An eccentric load is applied at a point 1 in. from the geometric axis of a 2.2-in.-diameter rod made of a steel for which $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. Using the allowable-stress method, determine the allowable load **P**.

SOLUTION

$c = \frac{1}{2}d = 1.1 \text{ in.} \qquad A = \pi c^2 = 3.8013 \text{ in}^2$

$I = \frac{\pi}{4}c^4 = 1.1499 \text{ in}^4 \qquad r = \sqrt{\frac{I}{A}} = 0.550 \text{ in}$

$L_e = 48 \text{ in} \qquad L_e/r = 48/0.550 = 87.2724$

$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10 > L_e/r$

$\frac{L_e/r}{C_c} = 0.6921$

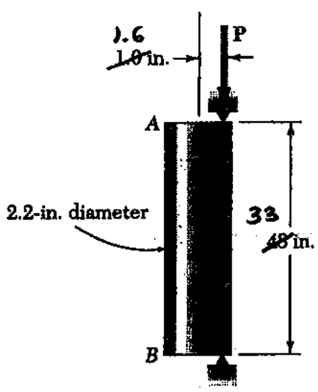
$F.S. = \frac{5}{3} + \frac{3}{8}(0.6921)^2 - \frac{1}{8}(0.6921)^3 = 1.8848$

$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.8848}\left[1 - \frac{1}{2}(0.6921)^2\right] = 14.526 \text{ ksi}$

$\frac{P_{all}}{A} + \frac{P_{all}ec}{I} = \sigma_{all} \qquad \left(\frac{1}{A} + \frac{ec}{I}\right)P_{all} = \sigma_{all} \qquad P_{all} = \sigma_{all}\left[\frac{1}{A} + \frac{ec}{I}\right]^{-1}$

$P_{all} = (14.526)\left[\frac{1}{3.8013} + \frac{(1.0)(1.1)}{1.1499}\right]^{-1} = 11.91 \text{ kips}$ ◄

PROBLEM 10.92



10.91 An eccentric load is applied at a point 1 in. from the geometric axis of a 2.2-in.-diameter rod made of a steel for which $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. Using the allowable-stress method, determine the allowable load **P**.

10.92 Solve Prob. 10.91, assuming that the load is applied at a point 1.6 in. from the geometric axis and that the effective length is 33 in.

SOLUTION

$c = \frac{1}{2}d = 1.1 \text{ in.} \qquad A = \pi c^2 = 3.8013 \text{ in}^2$

$I = \frac{\pi}{4}c^4 = 1.1499 \text{ in}^4 \qquad r = \sqrt{\frac{I}{A}} = 0.550 \text{ in}$

$L_e = 33 \text{ in} \qquad L_e/r = 33/0.550 = 60$

$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$

$\frac{L_e/r}{C_c} = 0.4758 \qquad F.S. = \frac{5}{3} + \frac{3}{8}(0.4758) - \frac{1}{8}(0.4758)^3 = 1.8316$

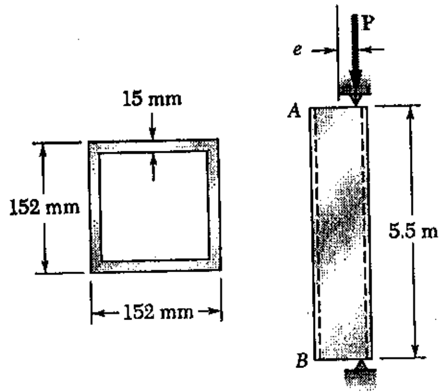
$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.8316}\left[1 - \frac{1}{2}(0.4758)^2\right] = 17.430 \text{ ksi}$

$\frac{P_{all}}{A} + \frac{P_{all}ec}{I} = \sigma_{all} \qquad \left(\frac{1}{A} + \frac{ec}{I}\right)P_{all} = \sigma_{all} \qquad P_{all} = \sigma_{all}\left[\frac{1}{A} + \frac{ec}{I}\right]^{-1}$

$P_{all} = (17.430)\left[\frac{1}{3.8013} + \frac{(1.6)(1.1)}{1.1499}\right]^{-1} = 9.72 \text{ kips}$ ◄

PROBLEM 10.93

10.93 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load **P**, knowing that when the eccentricity is (*a*) $e = 0$, (*b*) $e = 40$ mm.



SOLUTION

$b_o = 152 \text{ mm} \qquad b_i = b_o - 2t = 122 \text{ mm}$

$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8220 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$

$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 56.26 \times 10^{-3} \text{ m}$

$\frac{L}{r} = \frac{5.5}{56.26 \times 10^{-3}} = 97.76 > 55$

$\sigma_{all,c} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(97.76)^2} = 38.92 \text{ MPa}$ for centric loading

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

(a) $e = 0 \qquad P = A\sigma_{all,c} = (8220 \times 10^{-6})(38.92 \times 10^6) = 320 \times 10^3 \text{ N} = 320 \text{ kN}$ ◄

(b) $e = 40 \times 10^{-3} \text{ m} \qquad c = \frac{1}{2}(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$

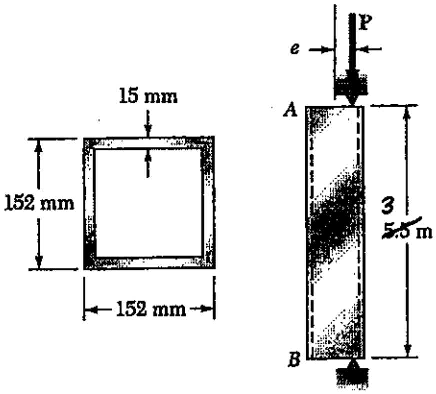
$$\frac{P}{(8220 \times 10^{-6})(38.92 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^{-6})(220 \times 10^6)} = 3.6568 \times 10^{-6} P = 1$$

$P = 273 \times 10^3 \text{ N} = 273 \text{ kN}$ ◄

PROBLEM 10.94

10.93 A column of 5.5-m effective length is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 220 MPa. Using the interaction method, determine the allowable load **P**, knowing that when the eccentricity is (*a*) $e = 0$, (*b*) $e = 40$ mm.

10.94 Solve Prob. 10.93, assuming that the effective length of a column is 3.0 m.



SOLUTION

$b_o = 152$ mm $\qquad b_i = b_o - 2t = 122$ mm

$A = b_o^2 - b_i^2 = 8220 \text{ mm}^2 = 8200 \times 10^{-6} \text{ m}^2$

$I = \frac{1}{12}(b_o^4 - b_i^4) = 26.02 \times 10^6 \text{ mm}^4$

$r = \sqrt{\frac{I}{A}} = 56.26 \text{ mm} = 52.26 \times 10^{-3} \text{ m}$

$$\frac{L}{r} = \frac{3.0}{56.26 \times 10^{-3}} = 53.32 < 55$$

$$\sigma_{all,c} = 212 - 1.585\,(L/r) = 212 - (1.585)(53.32) = 127.5 \text{ MPa}$$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1$$

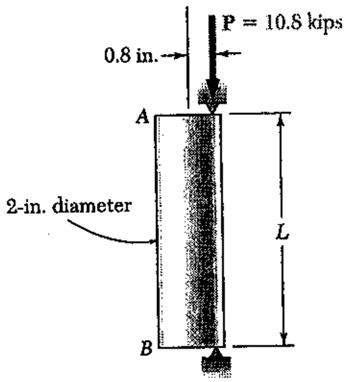
(a) $e = 0 \qquad P = A\sigma_{all} = (8220 \times 10^{-6})(127.5 \times 10^6) = 1048 \times 10^3 \text{ N} = 1048 \text{ kN}$ ◄

(b) $e = 40 \text{ mm} = 40 \times 10^{-3} \text{ m} \qquad c = (\frac{1}{2})(152) = 76 \text{ mm} = 76 \times 10^{-3} \text{ m}$

$$\frac{P}{(8220 \times 10^{-6})(127.5 \times 10^6)} + \frac{P(40 \times 10^{-3})(76 \times 10^{-3})}{(26.02 \times 10^{-6})(220 \times 10^6)} = 1.4852 \times 10^{-6} P = 1$$

$$P = 673 \times 10^3 \text{ N} = 673 \text{ kN}$$ ◄

PROBLEM 10.95



10.95 An eccentric load $P = 10.8$ kips is applied at a point 0.8 in. from the geometric axis of a 2-in.-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 21 ksi, determine the largest allowable effective length L that can be used.

SOLUTION

$c = \frac{1}{2}d = 1.0 \text{ in} \qquad A = \pi c^2 = 3.1416 \text{ in}^2$

$I = \frac{\pi}{4}c^4 = 0.7854 \text{ in}^4 \qquad r = \sqrt{\frac{I}{A}} = 0.5 \text{ in.}$

$e = 0.8 \text{ in} \qquad \sigma_{all,b} = 21 \text{ ksi}$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1 \qquad \frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

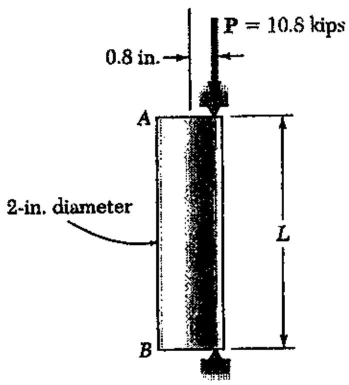
$$\frac{1}{\sigma_{all,c}} = \frac{A}{P}\left(1 - \frac{Pec}{I\sigma_{all,b}}\right) = \frac{3.1416}{10.8}\left[1 - \frac{(10.8)(0.8)(1.0)}{(0.7854)(21)}\right] = 0.1385 \text{ ksi}^{-1}$$

$\sigma_{all,c} = 7.22$ ksi $\qquad$ Assume $L/r > 66$

$$\sigma_{all,c} = \frac{51000}{(L/r)^2} \qquad \frac{L}{r} = \sqrt{\frac{51000}{\sigma_{all,c}}} = 84.05 > 66$$

$$L = 84.05\,r = (84.05)(0.5) = 42.0 \text{ in.}$$

PROBLEM 10.96



10.95 An eccentric load $P = 10.8$ kips is applied at a point 0.8 in. from the geometric axis of a 2-in.-diameter rod made of the aluminum alloy 6061-T6. Using the interaction method and an allowable stress in bending of 21 ksi, determine the largest allowable effective length L that can be used.

10.96 Solve Prob. 10.95, assuming that the aluminum alloy used is 2014-T6 and that the allowable stress in bending is 26 ksi.

SOLUTION

$c = \frac{1}{2}d = 1.0 \text{ in.} \qquad A = \pi c^2 = 3.1416 \text{ in}^2$

$I = \frac{\pi}{4}c^2 = 0.7854 \text{ in}^4 \qquad r = \sqrt{\frac{I}{A}} = 0.5 \text{ in.}$

$e = 0.8 \text{ in.} \qquad \sigma_{all,b} = 26 \text{ ksi}$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = 1 \qquad \frac{P}{A\sigma_{all,c}} = 1 - \frac{Pec}{I\sigma_{all,b}}$$

$$\frac{1}{\sigma_{all,c}} = \frac{A}{P}\left(1 - \frac{Pec}{I\sigma_{all,b}}\right) = \frac{3.1416}{10.8}\left[1 - \frac{(10.8)(0.8)(1.0)}{(0.7854)(26)}\right] = 0.1678 \text{ ksi}^{-1}$$

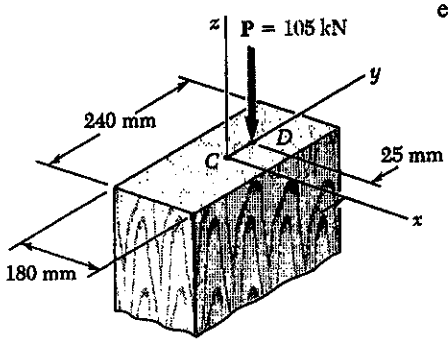
$\sigma_{all,c} = 5.96$ ksi $\qquad$ Assume $L/r > 55$

$$\sigma_{all,c} = \frac{54000}{(L/r)^2} \qquad \frac{L}{r} = \sqrt{\frac{54000}{\sigma_{all,c}}} = \sqrt{\frac{54000}{5.96}} = 95.19 > 55$$

$$L = 95.19\,r = (95.19)(0.5) = 47.6 \text{ in.}$$

PROBLEM 10.97

10.97 A rectangular column is made of sawn lumber that has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 8.3$ MPa and a modulus of elasticity $E = 11.1$ GPa. Using the allowable-stress method, determine the largest allowable effective length L that can used.



SOLUTION

$d = 180 \text{ mm} = 0.180 \text{ m} \qquad b = 240 \text{ mm} = 0.240 \text{ m}$

$A = bd = 43.2 \times 10^{-3} \text{ m}^2 \qquad E = 11100 \text{ MPa}$

$I_x = \frac{1}{12} d b^3 = \frac{1}{12}(0.180)(0.240)^3 = 207.36 \times 10^{-6} \text{ m}^4$

$e = 25 \text{ mm} = 0.025 \text{ m} \qquad c = \frac{b}{2} = 0.120 \text{ m}$

$$\frac{P}{A} + \frac{Pec}{I_x} \leq \sigma_{all} \qquad \sigma_{all} \geq \frac{105 \times 10^3}{43.2 \times 10^{-3}} + \frac{(105 \times 10^3)(0.025)(0.120)}{207.36 \times 10^{-6}} = 3.9496 \times 10^6 \text{ Pa}$$

$$= 3.9496 \text{ MPa}$$

$$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.9496}{8.3} = 0.47586 = y \qquad \text{Let } x = \sigma_{cE}/\sigma_c$$

$$y = \frac{1+x}{c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}} \qquad \text{where } c = 0.8 \text{ for sawn lumber}$$

$$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$$

$$\left(\frac{1+x}{2c} - y\right)^2 = \left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}$$

$$\cancel{\left(\frac{1+x}{2c}\right)^2} - \frac{1+x}{c} y + y^2 = \cancel{\left(\frac{1+x}{2c}\right)^2} - \frac{x}{c}$$

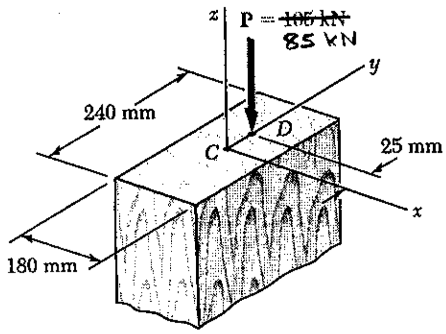
$$x = y\,\frac{1-cy}{1-y} = (0.47586)\frac{1-(0.8)(0.47586)}{1-0.47586} = 0.56227$$

$$\sigma_{cE} = \sigma_c(0.56227) = (8.3)(0.56227) = 4.6668 \text{ MPa}$$

$$\sigma_{cE} = \frac{K_{cE} E}{(L/d)^2} \qquad L^2 = \frac{K_{cE} E d^2}{\sigma_{cE}} \qquad \text{where } K_{cE} = 0.300$$

$$L = d\sqrt{\frac{K_{cE} E}{\sigma_{cE}}} = 0.180\sqrt{\frac{(0.300)(11100)}{4.6668}} = 4.81 \text{ m}$$

PROBLEM 10.98



10.97 A rectangular column is made of sawn lumber that has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 8.3$ MPa and a modulus of elasticity $E = 11.1$ GPa. Using the allowable-stress method, determine the largest allowable effective length L that can used.

10.98 Solve Prob. 10.97, assuming that $P = 85$ kN.

SOLUTION

$d = 180 \text{ mm} = 0.180 \text{ m} \qquad b = 240 \text{ mm} = 0.240 \text{ m}$

$A = bd = 43.2\times10^{-3} \text{ m}^2 \qquad E = 11100 \text{ MPa}$

$I_x = \frac{1}{12} d b^3 = \frac{1}{12}(0.180)(0.240)^3 = 207.36\times10^{-6} \text{ m}^4$

$e = 25 \text{ mm} = 0.025 \text{ m} \qquad c = \frac{b}{2} = 0.120 \text{ m}$

$\frac{P}{A} + \frac{Pec}{I} \le \sigma_{all} \qquad \sigma_{all} \geq \frac{85\times10^3}{43.2\times10^{-3}} + \frac{(85\times10^3)(0.025)(0.120)}{207.36\times10^{-6}} = 3.9496\times10^6 \text{ Pa}$

$= \qquad \text{MPa}$

$C_p = \frac{\sigma_{all}}{\sigma_c} = \frac{3.1973}{8.3} = 0.38522 = y \qquad$ Let $x = \sigma_{cE}/\sigma_c$

$y = \frac{1+x}{2c} - \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$ where $c = 0.8$ for sawn lumber

$\frac{1+x}{2c} - y = \sqrt{\left(\frac{1+x}{2c}\right)^2 - \frac{x}{c}}$

$\cancel{\left(\frac{1+x}{2c}\right)^2} - y\left(\frac{1+x}{c}\right) + y^2 = \cancel{\left(\frac{1+x}{2c}\right)^2} - \frac{x}{c}$

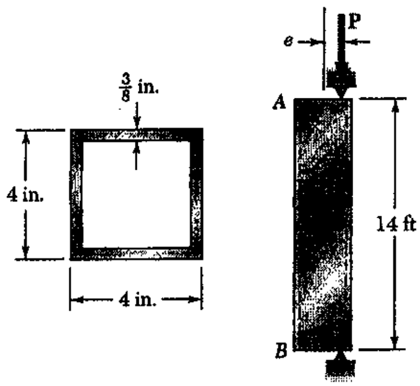
$x = y\frac{(1-cy)}{1-y} = (0.38522)\frac{1-(0.8)(0.38522)}{1-0.38522} = 0.43350$

$\sigma_{cE} = \sigma_c(0.43350) = (8.3)(0.43350) = 3.598 \text{ MPa}$

$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} \qquad L^2 = \frac{K_{cE}Ed^2}{\sigma_{cE}} \qquad$ where $K_{cE} = 0.300$

$L = d\sqrt{\frac{K_{cE}E}{\sigma_{cE}}} = (0.180)\sqrt{\frac{(0.300)(11100)}{3.598}} = 5.48 \text{ m}$

PROBLEM 10.99



10.99 A column of 14-ft effective length consists of a section of steel tubing having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (*a*) $P = 55$ kips, (*b*) $P = 35$ kips. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

SOLUTION

Steel: $\sigma_Y = 36$ ksi $\quad E = 29000$ ksi

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$b_o = 4.0$ in $\quad b_i = b_o - 2t = 3.25$ in. $\quad c = 2.0$ in

$A = b_o^2 - b_i^2 = 5.4375\ \text{in}^2 \quad I = \frac{1}{12}(b_o^4 - b_i^4) = 12.036\ \text{in}^4$

$r = \sqrt{\frac{I}{A}} = 1.4878$ in $\quad L_e = 14$ ft $= 168$ in

$L_e/r = 112.92 < C_c \qquad \frac{L_e/r}{C_c} = 0.89547$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.89547) - \frac{1}{8}(0.89547)^3 = 1.9127$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.9127}\left[1 - \frac{1}{2}(0.89547)^2\right] = 11.275\ \text{ksi}$$

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I} = \sigma_{all} \qquad \frac{P_{all}ec}{I} = \sigma_{all} - \frac{P_{all}}{A} \qquad e = \frac{I}{cP_{all}}\left(\sigma_{all} - \frac{P_{all}}{A}\right)$$

(a) $P_{all} = 55$ kips

$$e = \frac{12.036}{(2.0)(55)}\left[11.275 - \frac{55}{5.4375}\right] = 0.127\ \text{in}$$

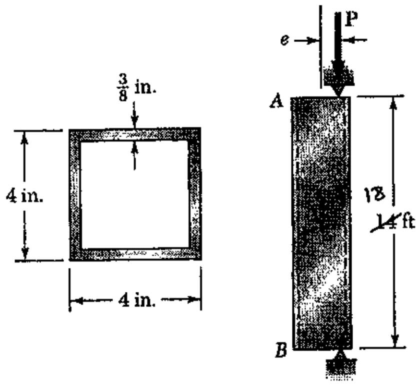
(b) $P_{all} = 35$ kips

$$e = \frac{12.036}{(2.0)(35)}\left[11.275 - \frac{35}{5.4375}\right] = 0.832\ \text{in.}$$

PROBLEM 10.100

10.99 A column of 14-ft effective length consists of a section of steel tubing having the cross section shown. Using the allowable-stress method, determine the maximum allowable eccentricity e if (*a*) $P = 55$ kips, (*b*) $P = 35$ kips. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

10.100 Solve Prob. 11.99, assuming that the effective length of the column is increased to 18 ft and that (*a*) $P = 28$ kips, (*b*) $P = 18$ kips.



SOLUTION

Steel: $\sigma_Y = 36$ ksi $\quad E = 29000$ ksi

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$b_o = 4.0$ in $\quad b_i = b_o - 2t = 3.25$ in $\quad c = 2.0$ in.

$A = b_o^2 - b_i^2 = 5.4375$ in^2 $\quad I = \frac{1}{12}(b_o^4 - b_i^4) = 12.036$ in^4

$r = \sqrt{\frac{I}{A}} = 1.4878 \quad L_e = 18$ ft $= 216$ in $\quad L_e/r = 145.18 > C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(29000)}{(1.92)(145.18)^2} = 7.0726 \text{ ksi}$$

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I} = \sigma_{all} \qquad \frac{P_{all}ec}{I} = \sigma_{all} - \frac{P_{all}}{A} \qquad e = \frac{I}{cP_{all}}\left(\sigma_{all} - \frac{P_{all}}{A}\right)$$

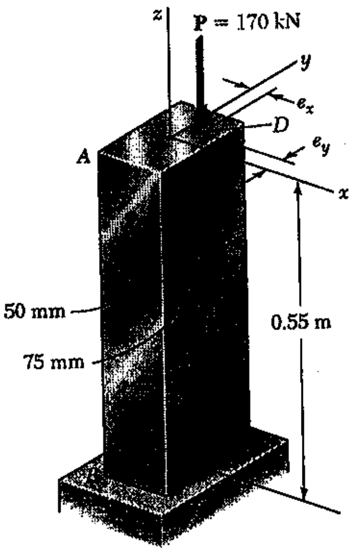
(a) $P_{all} = 28$ kips

$$e = \frac{12.036}{(2.0)(28)}\left[7.0726 - \frac{28}{5.4375}\right] = 0.413 \text{ in.}$$

(b) $P_{all} = 18$ kips

$$e = \frac{12.036}{(2.0)(18)}\left[7.0726 - \frac{18}{5.4375}\right] = 1.258 \text{ in.}$$

PROBLEM 10.101



10.101 The compression member AB is made of a steel for which $\sigma_Y = 250$ MPa and $E = 200$ GPa. It is free at its top A and fixed at its base B. Using the allowable-stress method, determine the largest allowable eccentricity e_x, knowing that (*a*) $e_y = 0$, (*b*) $e_y = 8$ mm.

SOLUTION

Steel: $\sigma_Y = 250$ MPa $\quad E = 200000$ MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2\ 200000}{250}} = 125.66 \text{ MPa}$$

$$A = (75\times10^{-3})(50\times10^{-3}) = 3750\times10^{-6}\ m^2$$

$$I_y = \tfrac{1}{12}(75\times10^{-3})(50\times10^{-3})^3 = 781.25\times10^{-9}\ m^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = 14.434\times10^{-3}\ m = r_{min}$$

$$I_x = \tfrac{1}{12}(50\times10^{-3})(75\times10^{-3})^3 = 1.7578\times10^{-6}\ m^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = 21.651\times10^{-6}\ m$$

$L_e = 2L = (2)(0.55) = 1.10$ m $\qquad L_e/r_{min} = 1.10/14.434\times10^{-3} = 76.21 < C_c$

$$\frac{L_e/r_{min}}{C_c} = \frac{76.21}{125.66} = 0.6065 \qquad F.S. = \tfrac{5}{3} + \tfrac{3}{8}(0.6065) - \tfrac{1}{8}(0.6065)^3 = 1.8662$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r_{min}}{C_c}\right)^2\right] = \frac{250}{1.8662}\left[1 - \tfrac{1}{2}(0.6065)^2\right] = 109.32 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pe_x}{S_y} + \frac{Pe_y}{S_x} = \sigma_{all} \qquad \frac{Pe_x}{S_y} = \sigma_{all} - \frac{P}{A} - \frac{Pe_y}{S_x}$$

$$e_x = \frac{S_y}{P}\left[\sigma_{all} - \frac{P}{A} - \frac{Pe_y}{S_x}\right] = S_y\left[\frac{\sigma_{all}}{P} - \frac{1}{A} - \frac{e_y}{S_x}\right]$$

$$S_y = \frac{I_y}{x} = \frac{781.25\times10^{-9}}{25\times10^{-3}} = 31.25\times10^{-6}\ m^3$$

$$S_x = \frac{I_x}{y} = \frac{1.7578\times10^{-6}}{37.5\times10^{-3}} = 46.875\times10^{-6}\ m^3$$

$$P = 170\times10^3\ N$$

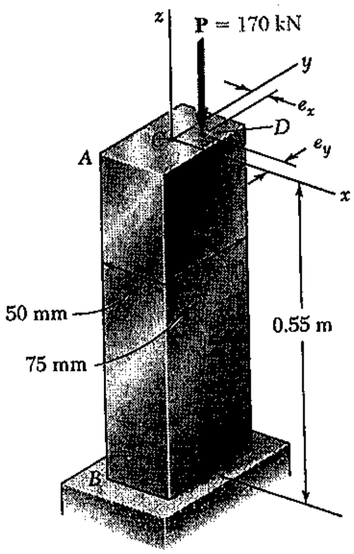
(a) $e_y = 0$ $\qquad e_x = 31.25\times10^{-6}\left[\frac{109.32\times10^6}{170\times10^3} - \frac{1}{3750\times10^{-6}} - 0\right]$

$= 11.76\times10^{-3}$ m $= 11.76$ mm ◀

(b) $e_y = 8\times10^{-3}$ m $\qquad e_x = 31.25\times10^{-6}\left[\frac{109.32\times10^6}{170\times10^3} - \frac{1}{3750\times10^{-6}} - \frac{8\times10^{-3}}{46.875\times10^{-6}}\right]$

$= 6.43\times10^{-3}$ m $= 6.43$ mm ◀

PROBLEM 10.102



10.102 The compression member AB is made of a steel for which $\sigma_Y = 250$ MPa and $E = 200$ GPa. It is free at its top A and fixed at its base B. Using the interaction method with an allowable bending stress equal to 120 MPa and knowing that the eccentricities e_x and e_y are equal, determine the largest allowable common value.

SOLUTION

Steel: $\sigma_Y = 250$ MPa $\quad E = 200000$ MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$$

$$A = (75\times10^{-3})(50\times10^{-3}) = 3750\times10^{-6}\ m^2$$

$$I_y = \tfrac{1}{12}(75\times10^{-3})(50\times10^{-3})^3 = 781.25\times10^{-9}\ m^4$$

$$r_y = \sqrt{\frac{I_y}{A}} = 14.434\times10^{-3}\ m$$

$$I_x = \tfrac{1}{12}(50\times10^{-3})(75\times10^{-3}) = 1.7578\times10^{-6}\ m^4$$

$$r_x = \sqrt{\frac{I_x}{A}} = 21.651\times10^{-6}\ m$$

$L_e = 2L = (2)(0.55) = 1.10$ m $\qquad L_e/r_{min} = 1.10/14.434\times10^{-3} = 76.21 < C_c$

$$\frac{L_e/r_{min}}{C_c} = \frac{76.21}{125.66} = 0.6065 \qquad F.S. = \tfrac{5}{3} + \tfrac{3}{8}(0.6065) - \tfrac{1}{8}(0.6065)^3 = 1.8662$$

$$\sigma_{all\,(centric)} = \frac{\sigma_Y}{F.S.}\left[1 - \tfrac{1}{2}\left(\frac{L_e/r_{min}}{C_c}\right)^2\right] = \frac{250}{1.8662}\left[1 - \tfrac{1}{2}(0.6065)^2\right] = 109.32\ \text{MPa}$$

$$\sigma_{all\,(bending)} = 120\ \text{MPa}$$

$$\frac{P}{A\sigma_{all\,(centric)}} + \frac{Pe_x y}{I_x \sigma_{all\,(bending)}} + \frac{Pe_y x}{I_y \sigma_{all\,(bending)}} = 1 \qquad \text{with } e_x = e_y$$

$$\frac{P}{\sigma_{all\,(bending)}}\left(\frac{y}{I_x} + \frac{x}{I_y}\right)e = 1 - \frac{P}{A\sigma_{all\,(centric)}}$$

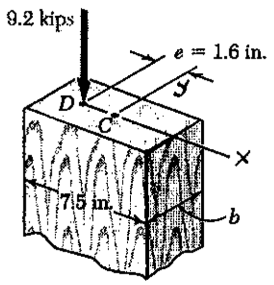
$$\frac{170\times10^3}{120\times10^6}\left(\frac{37.5\times10^{-3}}{1.7578\times10^{-6}} + \frac{25\times10^{-3}}{781.25\times10^{-9}}\right)e = 1 - \frac{170\times10^3}{(3750\times10^{-6})(109.32\times10^6)}$$

$$75.556\,e = 1 - 0.41468$$

$$e = 7.75\times10^{-3}\ m = 7.75\ mm$$

PROBLEM 10.103

10.103 A sawn lumber column of rectangular cross section has a 7.2-ft effective length and supports a 9.2 kip load as shown. The sizes available for use have b equal to 3.5 in., 5.5 in., 7.5 in. and 9.5 in. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 1180$ psi and $E = 1.2 \times 10^6$ psi. Use the allowable-stress method to determine the lightest section that can be used.



SOLUTION

Sawn lumber: $\sigma_c = 1180$ psi $\quad E = 1.2\times10^6$ psi
$c = 0.8 \quad K_{cE} = 0.300$

$L_e = 7.2$ ft $= 86.4$ in

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all} \qquad P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{ec}{I}}$$

$e = 1.6$ in $\qquad c = \frac{1}{2}(7.5) = 3.75$ in. $\qquad A = 7.5\, b$

$$I_x = \frac{1}{12} b (7.5)^3 = 35.156\, b$$

$$\frac{1}{\frac{1}{A} + \frac{ec}{I_x}} = \frac{1}{\frac{1}{7.5b} + \frac{(1.6)(3.75)}{35.156\, b}} = 3.2895\, b \qquad P_{all} = 3.2895\, b\sigma_{all}$$

$d = 7.5$ in. or b, whichever is smaller.

$$\sigma_{cE} = \frac{K_{cE} E}{(L/d)^2} = \frac{K_{cE} E d^2}{L^2} = \frac{(0.300)(1.2\times10^6)\, d^2}{(86.4)^2} = 48.225\, d^2 \text{ (psi)}$$

$$\sigma_{cE}/\sigma_c = (48.225 d^2)/1180 = 0.04087\, d^2$$

$$C_p = \frac{1 + \sigma_{cE}/\sigma_c}{2c} - \sqrt{\left(\frac{1 + \sigma_{cE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_c}{c}}$$

$$\sigma_{all} = \sigma_c C_p = 1180\, C_p$$

$$P_{all} = (3.2895)\, b\, (1180\, C_p) = 3882\, b\, C_p \quad \text{(lb.)}$$

Calculate P_{all} for all four values of b. See table below.

b (in)	d (in.)	σ_{cE}/σ_c	C_p	P_{all} (lb.)
3.5	3.5	0.5007	0.4341	5900
5.5	5.5	1.2363	0.7588	16200
7.5	7.5	2.299	0.8882	25900
9.5	7.5	2.299	0.8882	32800

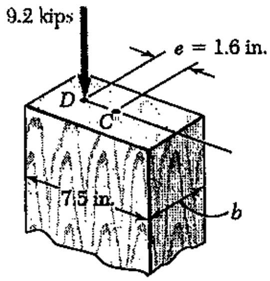
← P = 9200 lb.

Use $b = 5.5$ in.

PROBLEM 10.104

10.103 A sawn lumber column of rectangular cross section has a 7.2-ft effective length and supports a 9.2 kip load as shown. The sizes available for use have b equal to 3.5 in., 5.5 in., 7.5 in. and 9.5 in. The grade of wood has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 1180$ psi and $E = 1.2 \times 10^6$ psi. Use the allowable-stress method to determine the lightest section that can be used.

10.104 Solve Prob. 10.103, assuming that $e = 3.2$ in.



SOLUTION

Sawn lumber: $\sigma_C = 1180$ psi $\quad E = 1.2 \times 10^6$ psi

$c = 0.8 \quad K_{cE} = 0.300$

$L_e = 7.2$ ft $= 86.4$ in.

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all} \qquad P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{ec}{I_x}}$$

$e = 3.2$ in $\qquad c = \frac{1}{2}(7.5) = 3.75$ in $\qquad A = 7.5\,b$

$$I_x = \frac{1}{12} b (7.5)^3 = 35.156\, b$$

$$\frac{1}{\frac{1}{A} + \frac{ec}{I_x}} = \frac{1}{\frac{1}{7.5b} + \frac{(3.2)(3.75)}{35.156\,b}} = 2.1067\, b \qquad P_{all} = 2.1067\, b\, \sigma_{all}$$

$d = 7.5$ in. or b, whichever is smaller

$$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} = \frac{K_{cE}Ed^2}{L^2} = \frac{(0.300)(1.2\times 10^6)d^2}{(86.4)^2} = 48.225\, d^2 \text{ (psi)}$$

$$\sigma_{cE}/\sigma_C = 48.225\, d^2/1180 = 0.04087\, d^2$$

$$C_p = \frac{1 + \sigma_{cE}/\sigma_C}{2c} - \sqrt{\left(\frac{1+\sigma_{cE}/\sigma_C}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_C}{c}}$$

$$\sigma_{all} = \sigma_C C_p = 1180\, C_p$$

$$P_{all} = (2.1067)\, b\, (1180\, C_p) = 2486\, b\, C_p$$

Calculate P_{all} for all four values of b. See table below.

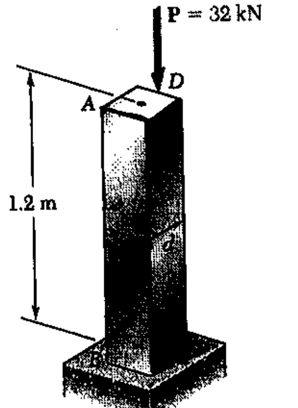
b (in.)	d (in)	σ_{cE}/σ_C	C_p	P_{all} (lb)
3.5	3.5	0.5007	0.4341	3780
5.5	5.5	1.2363	0.7588	10370
7.5	7.5	2.299	0.8882	16560
9.5	7.5	2.299	0.8882	20100

← P = 9200 lb.

Use b = 5.5 in.

PROBLEM 10.105

10.105 A 32-kN vertical load **P** is applied at the midpoint of one edge of the square cross section of the aluminum compression member *AB* that is free at its top *A* and fixed at its base *B*. Knowing that the alloy used is 6061-T6, use the allowable-stress method to determine the smallest allowable dimension *d*.



$L_e = 2L = 2.4$ m

SOLUTION

$$A = d^2 \qquad I = \tfrac{1}{12} d^4 \qquad r = \sqrt{\frac{I}{A}} = \frac{1}{\sqrt{12}} d \qquad c = \tfrac{1}{2} d \qquad e = \tfrac{1}{2} d$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{P}{d^2} + \frac{P(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{12}d^4} = \frac{4P}{d^2} = \sigma_{all}$$

Assume $L/r > 66$ $\qquad \sigma_{all} = \dfrac{B}{(L/r)^2}$ $\qquad B = 351 \times 10^9$ Pa

$$\frac{Br^2}{L^2} = \frac{B d^2}{12 L^2} = \frac{4P}{d^2} \qquad d^4 = \frac{48 PL^2}{B} \qquad 2$$

$$d = \sqrt[4]{\frac{48 PL^2}{B}} = \sqrt[4]{\frac{(48)(32 \times 10^3)(2.4)^2}{351 \times 10^9}} = 70.9 \times 10^{-3} \text{ m}$$

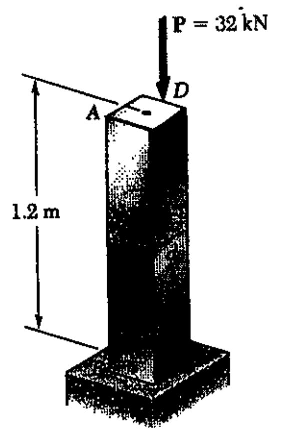
$$r = \frac{70.9 \times 10^{-3}}{\sqrt{12}} = 20.45 \times 10^{-3} \text{ m} \qquad \frac{L_e}{r} = \frac{2.4}{20.45 \times 10^{-3}} = 117.3 > 66$$

answer $\quad d = 70.9$ mm

PROBLEM 10.106

10.105 A 32-kN vertical load **P** is applied at the midpoint of one edge of the square cross section of the aluminum compression member *AB* that is free at its top *A* and fixed at its base *B*. Knowing that the alloy used is 6061-T6, use the allowable-stress method to determine the smallest allowable dimension *d*.

10.106 Solve Prob. 10.105, assuming that the vertical load **P** is applied at a corner of the square cross section of the compression member *AB*.



$L_e = 2L = 2.4$ m

SOLUTION

$$A = d^2, \quad I = \tfrac{1}{12} d^4, \quad r = \sqrt{\frac{I}{A}} = \frac{d}{\sqrt{12}} \qquad x = y = \tfrac{1}{2} d$$

$$e_x = e_y = \tfrac{1}{2} d$$

$$\frac{P}{A} + \frac{Pe_x x}{I_y} + \frac{Pe_y y}{I_x} = \frac{P}{d^2} + \frac{P(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{12}d^4} + \frac{P(\frac{1}{2}d)(\frac{1}{2}d)}{\frac{1}{12}d^4}$$

$$= \frac{7P}{d^2} = \sigma_{all}$$

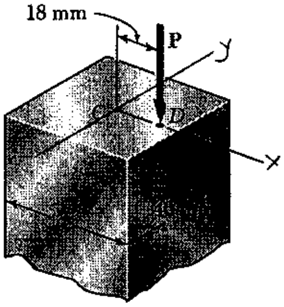
Assume $L/r > 66$ $\qquad \sigma_{all} = \dfrac{B}{(L_e/r)^2}$ $\qquad B = 351 \times 10^9$ Pa

$$\frac{Br^2}{L_e^2} = \frac{Bd^2}{12 L_e^2} = \frac{7P}{d^2} \qquad d^4 = \frac{84 PL_e^2}{B}$$

$$d = \sqrt[4]{\frac{84 PL_e^2}{B}} = \sqrt[4]{\frac{(84)(32 \times 10^3)(2.4)^2}{351 \times 10^9}} = 81.5 \times 10^{-3} \text{ m} \qquad d = 81.5 \text{ mm}$$

$$r = \frac{d}{\sqrt{12}} = 23.5 \times 10^{-3} \text{ m} \qquad \frac{L_e}{r} = \frac{2.4}{23.5 \times 10^{-3}} \quad 102.0 > 66$$

PROBLEM 10.107



10.107 A compression member made of steel has a 720-mm effective length and must support the 198-kN load **P** as shown. For the material used $\sigma_Y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

SOLUTION

Using dimensions in meters

$A = 40\times10^{-3}\ d \qquad L_e = 720\text{ mm} = 0.720\text{ m}$

$I_x = \frac{1}{12}(40\times10^{-3})^3 d = 5.3333\times10^{-6}\ d$

$I_y = \frac{1}{12}(40\times10^{-3})\ d^3 = 3.3333\times10^{-3}\ d^3$

$|x| = \frac{d}{2}, \quad |y| = 20\text{ mm} = 0.020\text{ m} \qquad |e_x| = 18\text{ mm} = 18\times10^{-3}\text{ m}$

Steel: $\sigma_Y = 250$ MPa $\quad E = 200000$ MPa $\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$

$$C_c = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$$

Assume $d > 40\text{ mm} = 40\times10^{-3}\text{ m}$. Then $I_{min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333\times10^{-6}\ d}{40\times10^{-3}\ d}} = 11.547\times10^{-3}\text{ m}, \quad \frac{L_e}{r} = 62.35 < C_c$$

$$\frac{L_e/r}{C_c} = 0.49621 \qquad F.S. = \frac{5}{3} + \frac{3}{8}(0.49621) - \frac{1}{8}(0.49621)^3 = 1.83747$$

$$\sigma_{all,\,centric} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{250}{1.83747}\left[1 - \frac{1}{2}(0.49621)^2\right] = 119.31\text{ MPa}$$

$$\sigma_{all,\,bending} = 150\text{ MPa}$$

$$\frac{P}{A\,\sigma_{all,\,centric}} + \frac{P e_x x}{I_y\,\sigma_{all,\,bending}} = 1$$

$$\frac{198\times10^3}{(40\times10^{-3}d)(119.31\times10^6)} + \frac{(198\times10^3)(18\times10^{-3})(\frac{1}{2}d)}{(3.3333\times10^{-3}d^3)(150\times10^6)} = 1$$

$$\frac{41.489\times10^{-3}}{d} + \frac{3.5640\times10^{-3}}{d^2} = 1$$

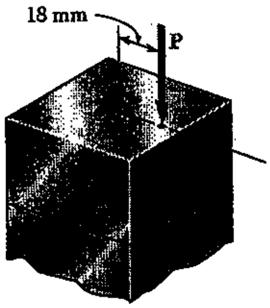
$$d^2 - 41.489\times10^{-3}\,d - 3.5640\times10^{-3} = 0$$

$$d = \frac{1}{2}\left\{41.489\times10^{-3} + \sqrt{(41.489\times10^{-3})^2 + (4)(3.5640\times10^{-3})}\right\}$$

$$= 83.9\times10^{3}\text{ m} \quad > 40\times10^{-3}\text{ m}$$

$d = 83.9$ mm ◄

PROBLEM 10.108



10.107 A compression member made of steel has a 720-mm effective length and must support the 198-kN load **P** as shown. For the material used $\sigma_Y = 250$ MPa and $E = 200$ GPa. Using the interaction method with an allowable bending stress equal to 150 MPa, determine the smallest dimension d of the cross section that can be used.

10.108 Solve Prob. 10.107, assuming that the effective length is 1.62 m and that the magnitude P of the eccentric load is 128 kN.

SOLUTION

Using dimensions in meters

$A = 40\times10^{-3}\, d \qquad L_e = 1.62 \text{ m}$

$I_x = \frac{1}{12}(40\times10^{-3})^3 d = 5.3333\times10^{-6}\, d$

$I_y = \frac{1}{12}(40\times10^{-3}) d^3 = 3.3333\times10^{-3}\, d^3$

$|x| = \frac{1}{2}d, \qquad |y| = 20 \text{ mm} = 20\times10^{-3} \text{ m} \qquad |e_x| = 18 \text{ mm} = 18\times10^{-3} \text{ m}$

Steel: $\sigma_Y = 250$ MPa $\quad E = 200000$ MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$$

Assume $d > 40 \text{ mm} = 40\times10^{-3}$ m $\qquad$ Then $I_{min} = I_x$

$$r = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{5.3333\times10^{-6} d}{3.3333\times10^{-3} d}} = 11.547\times10^{-3} \text{ m} \qquad \frac{L_e}{r} = 140.29 > C_c$$

$$\sigma_{all,centric} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(200000)}{(1.92)(140.29)^2} = 52.236 \text{ MPa} \qquad \sigma_{all,bending} = 150 \text{ MPa}$$

$$\frac{P}{A\sigma_{all,centric}} + \frac{Pe_x x}{I_y \sigma_{all,bending}} = 1$$

$$\frac{128\times10^3}{(40\times10^{-3} d)(52.236\times10^6)} + \frac{(128\times10^3)(18\times10^{-3})(\frac{1}{2}d)}{(3.3333\times10^{-3} d^3)(150\times10^6)} = 1$$

$$\frac{61.260\times10^{-3}}{d} + \frac{2.304\times10^{-3}}{d^2} = 1$$

$$d^2 - 61.260\times10^{-3} d - 2.304\times10^{-3} = 0$$

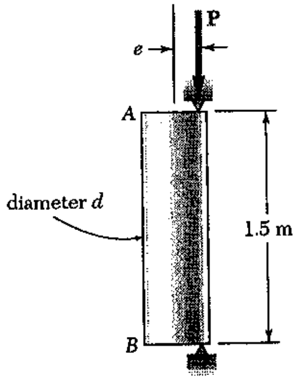
$$d = \frac{1}{2}\left\{61.260\times10^{-3} + \sqrt{(61.260\times10^{-3})^2 + (4)(2.304\times10^{-3})}\right\}$$

$$= 87.6\times10^{-3} \text{ m} > 40\times10^{-3} \text{ m}$$

$$d = 87.6 \text{ mm}$$

PROBLEM 10.109

10.109 The eccentric load **P** has a magnitude of 85 kN and is applied at a point located at a distance $e = 30$ mm from the geometric axis of a rod made of the aluminum alloy 6016-T6. Use the interaction method with a 140-MPa allowable stress in bending to determine the smallest diameter d that can be used.



SOLUTION

Assume $L/r > 66$ $\qquad \sigma_{all} = \dfrac{B}{(L/r)^2} \qquad B = 351 \times 10^9$ Pa

$c = \dfrac{d}{2} \qquad A = \pi c^2 = \dfrac{\pi}{4} d^2 \qquad I = \dfrac{\pi}{4} c^4 = \dfrac{\pi}{64} d^4$

$r = \sqrt{\dfrac{I}{A}} = \dfrac{1}{4} d$

$$\frac{P}{A\sigma_{all,centric}} + \frac{Pec}{I\sigma_{all,bending}} = 1$$

$$\frac{PL^2}{ABr^2} + \frac{Ped}{2I\sigma_{all,bending}} = 1$$

$$\frac{64\,PL^2}{\pi B d^4} + \frac{32\,Ped}{\pi\sigma_{all,bending}} = 1$$

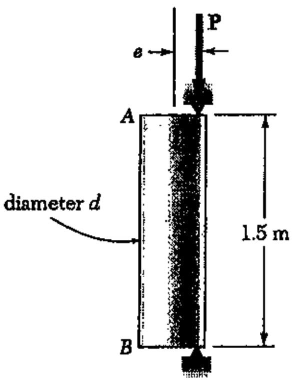
$$\frac{(64)(85 \times 10^3)(1.5)^2}{\pi(351 \times 10^9)d^4} + \frac{(32)(85 \times 10^3)(30 \times 10^{-3})}{\pi(140 \times 10^6)d^3} = 1 \qquad \text{Let } x = \frac{1}{d}$$

$11.1 \times 10^{-6} x^4 + 185.53 \times 10^{-6} x^3 = 1 \qquad$ Solving $x = 14.2725\ \text{m}^{-1}$

$d = \dfrac{1}{x} = 70.0 \times 10^{-3}$ m $\qquad d = 70.0$ mm ◄

$r = \dfrac{d}{4} = 17.50 \times 10^{-3}$ m $\qquad \dfrac{L}{r} = \dfrac{1.5}{17.5 \times 10^{-3}} = 85.7 > 66$

PROBLEM 10.110



10.109 The eccentric load **P** has a magnitude of 85 kN and is applied at a point located at a distance $e = 30$ mm from the geometric axis of a rod made of the aluminum alloy 6016-T6. Use the interaction method with a 140-MPa allowable stress in bending to determine the smallest diameter d that can be used.

10.110 Solve Prob. 10.109, using the allowable-stress method and assuming that the aluminum alloy used is 2014-T6.

SOLUTION

Assume $L/r > 55 \qquad \sigma_{all} = \frac{B}{(L/r)^2} \qquad B = 372\times10^9$ Pa

$c = \frac{d}{2} \qquad A = \pi c^2 = \frac{\pi}{4}d^2 \qquad I = \frac{\pi}{4}c^4 = \frac{\pi}{64}d^4$

$r = \sqrt{\frac{I}{A}} = \frac{1}{4}d$

$$\frac{P}{A} + \frac{Pec}{I} = \sigma_{all} = \frac{Br^2}{L^2}$$

$$\frac{PL^2}{ABr^2} + \frac{PL^2 e \frac{1}{2}d}{IBr^2} = 1 \qquad \frac{64\,PL^2}{\pi d^4 B} + \frac{32\,PL^2}{\pi d^3 B} = 1 \qquad \text{Let } x = \frac{1}{d}$$

$$\frac{64\,PL^2}{\pi B}x^4 + \frac{(16)(64)\,PL^2 e}{2\pi B}x^5 = 1$$

$$\frac{(64)(85\times10^3)(1.5)^2}{\pi(372\times10^9)}x^4 + \frac{(16)(64)(85\times10^3)(1.5)^2(30\times10^{-3})}{2\pi(372\times10^9)} = 1$$

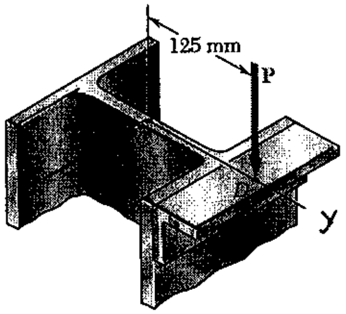
$10.473\times10^{-6}\,x^4 + 2.5136\times10^{-6}\,x^5 = 1 \qquad x = 12.441\ \text{m}^{-1}$

$d = \frac{1}{x} = 80.4\times10^{-3}$ m $\qquad d = 80.4$ mm ◀

$r = \frac{d}{4} = 20.1\times10^{-3}$ m $\qquad \frac{L}{r} = \frac{1.5}{20.1\times10^{-3}} = 74.5 > 55$

PROBLEM 10.111

10.111 A steel compression member of 5.8-m effective length is to support a 296-kN eccentric load **P**. Using the interaction method, select the wide-flange shape of 200-mm nominal depth that should be used. Use $E = 200$ GPa, $\sigma_Y = 250$ MPa and $\sigma_{all} = 150$ MPa in bending.



SOLUTION

Steel: $E = 200000$ MPa $\quad \sigma_Y = 250$ MPa

$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.66$

$L_e = 5.8$ m $\quad \sigma_{all,\,bending} = 150$ MPa

For 200 mm nominal depth wide flange section

$r_x \approx 88$ mm $= 88\times10^{-3}$ m, $\quad y \approx \frac{210}{2} = 105$ mm $= 105\times10^{-3}$ m

$r_y \approx 48$ mm $= 48\times10^{-3}$ m $\quad \frac{L}{r_y} \approx \frac{5.8}{48\times10^{-3}} = 121 \quad \frac{L/r_y}{C_c} \approx 0.96$

$F.S. \approx \frac{5}{3} + \frac{3}{8}(0.96) - \frac{1}{8}(0.96)^3 = 1.916$

$\sigma_{all} \approx \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L/r_y}{C_c}\right)^2\right] = \frac{250}{1.916}\left[1 - \frac{1}{2}(0.96)^2\right] = 70$ MPa

$$\frac{P}{A\,\sigma_{all,\,centric}} + \frac{P e_y y}{I_x\,\sigma_{all,\,bending}} = \frac{1}{A}\left[\frac{P}{\sigma_{all,\,centric}} + \frac{P e_y y}{r_x^2\,\sigma_{all,\,bending}}\right] = 1$$

$$A = \frac{P}{\sigma_{all,\,centric}} + \frac{P e_y y}{r_x^2\,\sigma_{all,\,bending}}$$

$$= \frac{296\times10^3}{70\times10^6} + \frac{(296\times10^3)(125\times10^{-3})(105\times10^{-3})}{(88\times10^{-3})^2(150\times10^6)} = 7.573\times10^{-3}\ \text{m} = 7573\ \text{mm}^2$$

Try W 200 × 59 $\quad A = 7560\times10^{-6}$, $\quad y = 105\times10^{-3}$ m, $I_x = 61.1\times10^{6}$ m^4

$r_y = 51.9\times10^{-3}$ m, $\quad L_e/r_y = 111.75 < C_c$

$\frac{L_e/r_y}{C_c} = 0.8893 \quad F.S. = 1.9122 \quad \sigma_{all,\,centric} = 79.04$ MPa

$$\frac{P}{A\,\sigma_{all,\,centric}} + \frac{P e_y y}{I_x\,\sigma_{all,\,bending}}$$

$$= \frac{296\times10^3}{(7560\times10^{-6})(79.04\times10^6)} + \frac{(296\times10^3)(125\times10^{-3})(105\times10^{-3})}{(61.1\times10^{-6})(150\times10^6)}$$

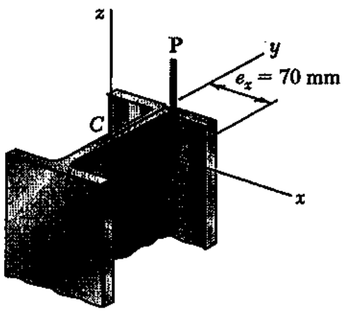
$$= 0.4954 + 0.4239 = 0.9193 < 1 \quad \text{(allowed)}$$

Trying W 200 × 52 leads to

$$\frac{P}{A\,\sigma_{all,\,centric}} + \frac{P e_y y}{I_x\,\sigma_{all,\,bending}} = 1.047 > 1 \quad \text{(not allowed)}$$

Use W 200 × 59

PROBLEM 10.112



10.112 A steel column of 7.2-m effective length is to support an 83-kN eccentric load **P** at a point *D* located on the *x* axis as shown. Using the allowable-stress method, select the wide-flange shape of 250-mm nominal depth that should be used. Use $E = 200$ GPa, $\sigma_Y = 250$ MPa.

SOLUTION

Steel: $E = 200000$ MPa $\quad \sigma_Y = 250$ MPa

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$$

$L_e = 7.2$ m

Try W 250×49.1 $\quad A = 6250 \times 10^{-6}$

$b_f = 202 \times 10^{-3}$ m, $\quad c = 101 \times 10^{-3}$ m, $\quad I_y = 15.1 \times 10^{-6}$ m^4, $\quad r_y = 49.2 \times 10^{-3}$ m

$$\frac{L_e}{r_y} = \frac{7.2}{49.2 \times 10^{-3}} = 146.34 > C_c$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92 (L/r_y)^2} = \frac{\pi^2 (200000)}{(1.92)(146.34)^2} = 48.01 \text{ MPa}$$

$$\frac{P}{A} + \frac{P e_x c}{I_y} = \frac{83 \times 10^3}{6250 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})(101 \times 10^{-3})}{15.1 \times 10^{-6}}$$

$$= 13.28 \times 10^6 + 38.86 \times 10^6 = 52.14 \text{ MPa} > 48.01 \text{ MPa}$$

(not allowed)

Required area $\quad A \approx \left(\frac{52.14}{48.01}\right)(6250 \text{ mm}^2) = 6788 \text{ mm}^2$

Try W 250×58

$$\frac{L_e}{r_y} = \frac{7.2}{50.3 \times 10^{-3}} = 143.14 \qquad \sigma_{all} = \frac{\pi^2 (200000)}{(1.92)(143.14)^2} = 50.18 \text{ MPa}$$

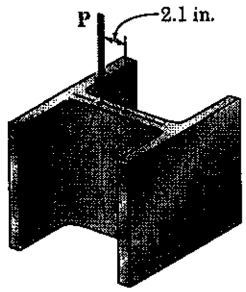
$$\frac{P}{A} + \frac{P e_x c}{I_y} = \frac{83 \times 10^3}{7420 \times 10^{-6}} + \frac{(83 \times 10^3)(70 \times 10^{-3})(101.5 \times 10^{-3})}{18.8 \times 10^{-6}}$$

$$= 11.19 \times 10^6 + 31.37 \times 10^6 = 42.56 \text{ MPa} < 50.18 \text{ MPa}$$

Use W250×58

PROBLEM 10.113

10.113 A steel column of 21-ft effective length must carry a load of 82 kips with an eccentricity of 2.1 in. as shown. Using the interaction method, select the wide-flange shape of 12-in. nominal depth that should be used. Use $E = 29 \times 10^6$ psi, $\sigma_Y = 36$ ksi. and $\sigma_{all} = 22$ ksi in bending.



SOLUTION

Steel: $E = 29000$ ksi $\quad C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29000)}{36}} = 126.10$

$L_e = 21$ ft $= 252$ in.

Try W 12×35 $\quad r_y = 1.54$ in $\quad \dfrac{L_e}{r_y} = 163.64 > C_c$

$$\sigma_{all,\,centric} = \frac{\pi^2 E}{1.92(L/r)^2} = \frac{\pi^2(29000)}{(1.92)(163.64)^2} = 5.57 \text{ ksi}$$

$$\frac{P}{A\sigma_{all,\,centric}} + \frac{Pec}{I_x \sigma_{all,\,bending}} = \frac{82}{(10.3)(5.57)} + \frac{(82)(2.1)(\frac{1}{2}\cdot 12.50)}{(285)(22)}$$

$$= 1.429 + 0.172 = 1.601 \quad \text{(not allowed)}$$

Approximate required $A = (1.596)(10.3) = 16.4 \text{ in}^2$

Try W 12×50 $\quad r_y = 1.96$ in. $\quad \dfrac{L_e}{r_y} = 128.57 > C_c$

$$\sigma_{all,\,centric} = \frac{\pi^2 E}{1.92(L_e/r)^2} = \frac{\pi^2(29000)}{(1.92)(128.57)^2} = 9.02 \text{ ksi}$$

$$\frac{P}{A\sigma_{all,\,centric}} + \frac{Pec}{I_x \sigma_{all,\,bending}} = \frac{82}{(14.7)(9.02)} + \frac{(82)(2.1)(\frac{1}{2}\cdot 12.19)}{(394)(22)}$$

$$= 0.618 + 0.121 = 0.739 \quad \text{(allowed)}$$

Try W 12×40 $\quad r_y = 1.93$ in $\quad \dfrac{L_e}{r_y} = \dfrac{252}{1.93} = 130.57 > C_c$

$$\sigma_{all,\,centric} = \frac{\pi^2 E}{(1.92)(L_e/r_y^2)} = \frac{\pi^2(29000)}{(1.92)(130.57)^2} = 8.74 \text{ ksi}$$

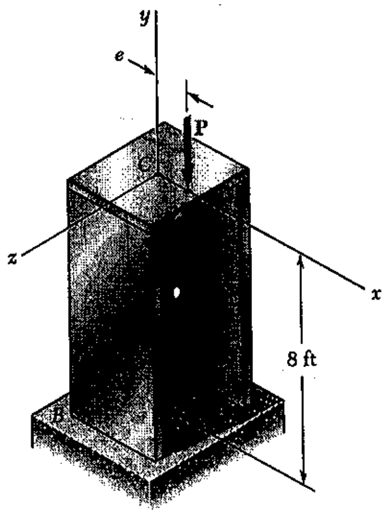
$$\frac{P}{A\sigma_{all,\,centric}} + \frac{Pec}{I_x \sigma_{all,\,bending}} = \frac{82}{(11.8)(8.74)} + \frac{(82)(2.1)(\frac{1}{2}\cdot 11.94)}{(310)(22)}$$

$$= 0.795 + 0.151 = 0.946 \quad \text{(allowed)} \leftarrow$$

Use W 12×40

PROBLEM 10.114

10.114 A 43-kip axial load **P** is applied to the rolled-steel column BC at a point on the x axis at a distance $e = 2.5$ in. from the geometric axis of the column. Using the allowable-stress method, select the wide-flange shape of 8-in. nominal depth that should be used. Use $E = 29 \times 10^6$ psi. and $\sigma_Y = 36$ ksi.



SOLUTION

Steel: $E = 29000$ ksi $\quad \sigma_Y = 36$ ksi

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.10$$

$L = 8$ ft $= 96$ in. $\quad L_e = 2L = 192$ in

Try $\underline{W\ 8\times31}$: $\quad r_y = 2.02$ in, $\quad \frac{L_e}{r_y} = 95.05 < C_c$

$$\frac{L_e/r_y}{C_c} = 0.754$$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.754) - \frac{1}{8}(0.754)^3 = 1.896$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r_y}{C_c}\right)^2\right] = \frac{36}{1.896}\left[1 - \frac{1}{2}(0.754)^2\right]$$

$$= 13.59 \text{ ksi}$$

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{9.13} + \frac{(43)(2.5)(\frac{1}{2}\cdot 7.995)}{37.1} = 4.71 + 11.58 = 16.29 \text{ ksi}$$

> 13.59 ksi (not allowed)

Approximate required area $\left(\frac{16.29}{13.59}\right)(9.13) = 10.9$ in^2

Try $\underline{W\ 8\times35}$ $\quad r_y = 2.03 \quad \frac{L_e}{r_y} = 94.58 < C_c \quad \frac{L_e/r_y}{C_c} = 0.750$

F.S. $= 1.895 \quad \sigma_{all} = 13.65$ ksi

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{10.3} + \frac{(43)(2.5)(\frac{1}{2}\cdot 8.020)}{42.6} = 14.29 \text{ ksi} > 13.65 \text{ ksi} \quad \text{(not allowed)}$$

Try $\underline{W8\times40}$ $\quad r_y = 2.04 \quad \frac{L_e}{r_y} = 94.12 < C_c \quad \frac{L_e/r_y}{C_c} = 0.746$

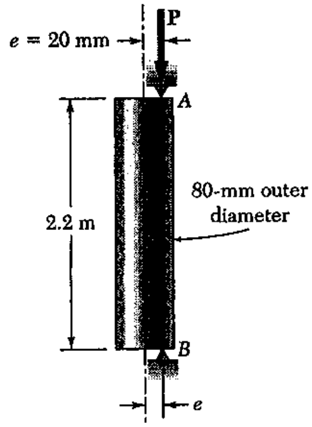
F.S. $= 1.895 \quad \sigma_{all} = 13.71$ ksi

$$\frac{P}{A} + \frac{Pec}{I_y} = \frac{43}{11.7} + \frac{(43)(2.5)(\frac{1}{2}\cdot 8.07)}{49.1} = 12.51 \text{ ksi} < 13.71 \text{ ksi} \quad \text{(allowed)}$$

Use W 8×40

PROBLEM 10.115

10.115 A steel tube of 80-mm outer diameter is to carry a 93-kN load **P** with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable-stress method, determine the lightest tube that can be used. Assume $E = 200$ GPa, $\sigma_Y = 250$ MPa.



SOLUTION

$r_o = \frac{1}{2} d_o = 40$ mm, $\quad r_i = r_o - t$

$A = \pi(r_o^2 - r_i^2), \quad I = \frac{\pi}{4}(r_o^4 - r_i^4) \quad r = \sqrt{\frac{I}{A}}$

t mm	r_i mm	A mm^2	I 10^6 mm^4	r mm
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$L_e = 2.2$ m

$P = 93 \times 10^3$ N

Steel: $E = 200000$ MPa $\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$

Try $t = 9$ mm $\qquad \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < C_c \qquad \frac{L_e/r}{C_c} = 0.6917$

$$F.S. = \frac{5}{3} + \frac{3}{8}(0.6917) - \frac{1}{8}(0.6917)^3 = 1.885$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{250}{1.885}\left[1 - \frac{1}{2}(0.6917)^2\right] = 100.9 \text{ MPa}$$

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2007 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.285 \times 10^{-6}} = 104.2 \text{ MPa} > 100.9 \text{ MPa}$$

(not allowed)

Approximate required area $\left(\frac{104.2}{100.9}\right)(2007 \times 10^{-6}) = 2073 \times 10^{-6}$ m^2 $= 2073$ mm^2

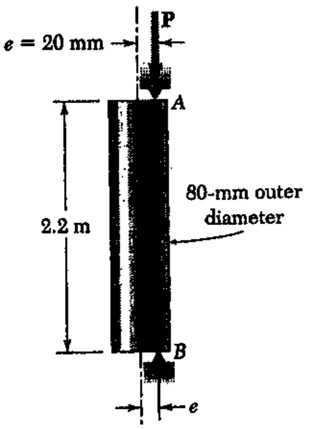
For $t = 12$ mm $\qquad \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < C_c \qquad \frac{L_e/r}{C_c} = 0.7172$

$F.S. = 1.890 \qquad \sigma_{all} = 98.3$ MPa

$$\frac{P}{A} + \frac{Pec}{I} = \frac{93 \times 10^3}{2564 \times 10^{-6}} + \frac{(93 \times 10^3)(20 \times 10^{-3})(40 \times 10^{-3})}{1.528 \times 10^{-6}} = 85.0 \text{ MPa} < 98.3 \text{ MPa}$$

Use $t = 12$ mm ◄

PROBLEM 10.116



10.115 A steel tube of 80-mm outer diameter is to carry a 93-kN load **P** with an eccentricity of 20 mm. The tubes available for use are made with wall thicknesses in increments of 3 mm from 6 mm to 15 mm. Using the allowable-stress method, determine the lightest tube that can be used. Assume $E = 200$ GPa, $\sigma_Y = 250$ MPa.

10.116 Solve Prob. 10.115, using the interaction method with $P = 165$ kN, $e = 15$ mm, and an allowable stress in bending of 150 MPa.

SOLUTION

$r_o = \frac{1}{2} d_o = 40 \text{ mm} \qquad r_i = r_o - t$

$A = \pi (r_o^2 - r_i^2) \qquad I = \frac{\pi}{4}(r_o^4 - r_i^4) \qquad r = \sqrt{\frac{I}{A}}$

t (mm)	r_i (mm)	A (mm²)	I (10^6 mm⁴)	r (mm)
3	37	726	0.539	27.24
6	34	1395	0.961	26.25
9	31	2007	1.285	25.31
12	28	2564	1.528	24.41
15	25	3063	1.704	23.59

$L_e = 2.2 \text{ m}$

$P = 165 \times 10^3 \text{ N}$

$\sigma_{all,\,bending} = 150 \text{ MPa}$

Steel: $E = 200000$ MPa $\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(200000)}{250}} = 125.66$

Try $\underline{t = 9 \text{ mm}}$ $\qquad \frac{L_e}{r} = \frac{2.2}{25.31 \times 10^{-3}} = 86.92 < C_c \qquad \frac{L_e/r}{C_c} = 0.6917$

$F.S. = \frac{5}{3} + \frac{3}{8}(0.6917) - \frac{1}{8}(0.6917)^3 = 1.885$

$\sigma_{all,\,centric} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{250}{1.885}\left[1 - \frac{1}{2}(0.6917)^2\right] = 100.9 \text{ MPa}$

$$\frac{P}{A\sigma_{all,\,centric}} + \frac{Pec}{I\sigma_{all,\,bending}} = \frac{165 \times 10^3}{(2007 \times 10^{-6})(100.9 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.285 \times 10^{-6})(150 \times 10^6)}$$

$= 0.815 + 0.514 = 1.329 > 1$ $\qquad$ (not allowed)

Approximate required area $\qquad A = (1.329)(2007) = 2667 \text{ mm}^2$

For $\underline{t = 12 \text{ mm}}$ $\qquad \frac{L_e}{r} = \frac{2.2}{24.41 \times 10^{-3}} = 90.12 < C_c \qquad \frac{L_e/r}{C_c} = 0.7172$

$F.S = 1.890 \qquad \sigma_{all,\,centric} = 98.3 \text{ MPa}$

$$\frac{P}{A\sigma_{all,\,centric}} + \frac{Pec}{I\sigma_{all,\,bending}} = \frac{165 \times 10^3}{(2564 \times 10^{-6})(98.3 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.528 \times 10^{-6})(150 \times 10^6)}$$

$= 0.655 + 0.432 = 1.087 > 1$ $\qquad$ (not allowed)

Try $\underline{t = 15 \text{ mm}}$ $\qquad \frac{L_e}{r} = \frac{2.2}{23.59 \times 10^{-3}} = 93.26 < C_c \qquad (L_e/r)/C_c = 0.7422$

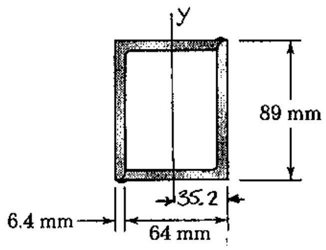
$F.S. = 1.894 \qquad \sigma_{all,\,centric} = 95.64 \text{ MPa}$

$$\frac{P}{A\sigma_{all,\,centric}} + \frac{Pec}{I\sigma_{all,\,bending}} = \frac{165 \times 10^3}{(3063 \times 10^{-6})(95.64 \times 10^6)} + \frac{(165 \times 10^3)(15 \times 10^{-3})(40 \times 10^{-3})}{(1.704 \times 10^{-6})(150 \times 10^6)}$$

$= 0.563 + 0.387 = 0.950 < 1$ $\qquad$ (allowed)

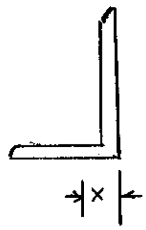
Use $t = 15$ mm ◄

PROBLEM 10.117



10.117 A column of 3.5-m effective length is made by welding together two 89 × 64 × 6.4-mm angles as shown. Using $E = 200$ GPa, determine the allowable centric load if a factor of safety of 2.8 is required.

SOLUTION



One angle $\quad x = 15.8 \text{ mm}$

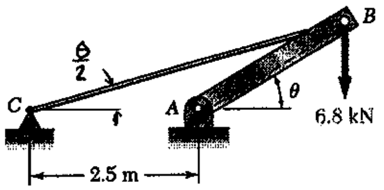
$$I_y = \bar{I}_y + A\,(35.2 - 15.8)^2$$
$$= 0.333 \times 10^6 + (938)(19.4)^2$$
$$= 0.686 \times 10^3 \text{ mm}^6$$

Two angles $\quad I_y = (2)(0.686 \times 10^3) = 1.372 \times 10^6 \text{ mm}^4 = 1.372 \times 10^{-6} \text{ m}^4$

$$P_{all} = \frac{P_{cr}}{F.S.} = \frac{\pi^2 E I_y}{(F.S.)\,L_e^2} = \frac{\pi^2 (200 \times 10^9)(1.372 \times 10^{-6})}{(2.8)(3.5)^2} = 79.0 \times 10^3 \text{ N}$$
$$= 79.0 \text{ kN}$$

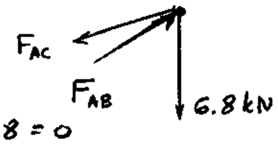
PROBLEM 10.118

10.118 Member *AB* consists of a single C130 × 10.4 steel channel of length 2.5 m. Knowing that the pins at *A* and *B* pass through the centroid of the cross section of the channel, determine the factor of safety for the load shown with respect to buckling in the plane of the figure when $\theta = 30°$. Use Euler's formula with $E = 200$ GPa.



SOLUTION

Since $AB = 2.5$ m, triangle ABC is isosoles.



$$\Sigma F_x = 0$$

$$-F_{AC}\cos 15° + F_{AB}\cos 30° = 0$$

$$F_{AC} = \frac{F_{AB}\cos 30°}{\cos 15°}$$

$$+\uparrow \Sigma F_y = 0$$

$$F_{AB}\sin 30° - F_{AC}\sin 15° - 6.8 = 0$$

$$F_{AB}\left(\sin 30° - \frac{\sin 15° \cos 30°}{\cos 15°}\right) = 0.26795\,F_{AB} = 6.8 \qquad F_{AB} = 25.378 \text{ kN}$$

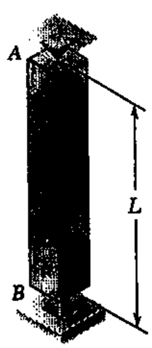
C 130×10.4 $\qquad I_{min} = 0.229 \times 10^6 \text{ mm}^4 = 0.229 \times 10^6 \text{ m}^4$

$$P_{cr} = \frac{\pi^2 E I_{min}}{L_{AB}^2} = \frac{\pi^2 (200\times10^9)(0.229\times10^{-6})}{(2.5)^2} = 72.324 \times 10^3 \text{ N} = 72.324 \text{ kN}$$

$$\text{F.S.} = \frac{P_{cr}}{F_{AB}} = \frac{72.324}{25.378} = 2.85$$

PROBLEM 10.119

10.119 Supports *A* and *B* of the pin-ended column shown are at a fixed distance *L* from each other. Knowing that at a temperature T_0 the force in the column is zero and that buckling occurs when the temperature is $T_1 = T_0 + \Delta T$, express ΔT in terms of *b*, *L*, and the coefficient of thermal expansion α.



SOLUTION

Let P be the compressive force in the column.

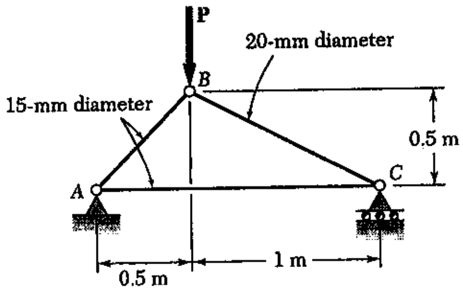
$$L\alpha(\Delta T) - \frac{PL}{EA} = 0 \qquad P = EA\alpha(\Delta T)$$

$$P_{cr} = \frac{\pi^2 E I}{L^2} = P = EA\alpha(\Delta T)$$

$$\Delta T = \frac{\pi^2 E I}{L^2 E A \alpha} = \frac{\pi^2 E\, b^4/12}{L^2 E\, b^2} = \frac{\pi^2 b^2}{12 L^2 \alpha}$$

PROBLEM 10.120

10.120 Knowing that a factor of safety of 2.6 is required, determine the largest load **P** that can be applied to the structure shown. Use $E = 200$ GPa and consider only buckling in the plane of the structure.



SOLUTION

BC: $L_{BC} = \sqrt{1^2 + 0.5^2} = 1.1180$ m

$I = \frac{\pi}{64} d_{BC}^4 = \frac{\pi}{64}(20)^4 = 7.854 \times 10^3 \text{ mm}^4 = 7.854 \times 10^{-9} \text{ m}^4$

$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(200\times10^9)(7.854\times10^{-9})}{(1.1180)^2}$

$= 12.403 \times 10^3 \text{ N} = 12.403 \text{ kN}$

$F_{BC,all} = \frac{P_{cr}}{F.S.} = \frac{12.403}{2.6} = 4.770 \text{ kN}$

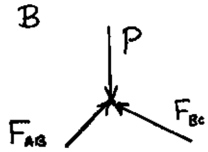
AB. $L_{AB} = \sqrt{0.5^2 + 0.5^2} = 0.70711$ m

$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(15)^4 = 2.485 \times 10^3 \text{ mm}^4 = 2.485 \times 10^{-9} \text{ m}^4$

$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2(200\times10^9)(2.485\times10^{-9})}{(0.70711)^2} = 9.8106 \times 10^3 \text{ N} = 9.8106 \text{ kN}$

$F_{AB,all} = \frac{P_{cr}}{F.S.} = \frac{9.8106}{2.6} = 3.773 \text{ kN}$

Joint B



$\xrightarrow{+} \Sigma F_x = 0 \qquad \frac{0.5}{0.70711} F_{AB} - \frac{1.0}{1.1180} F_{BC} = 0$

$F_{BC} = 0.79057 F_{AB}$

$+\uparrow \Sigma F_y = 0 \qquad \frac{.5}{0.70711} F_{AB} + \frac{0.5}{1.1180} F_{BC} + P = 0$

$0.70711 F_{AB} + (0.44721)(0.79057 F_{AB}) - P = 0$

$P = 1.06066 F_{AB}$

$P = (1.06066)\frac{F_{BC}}{0.79057} = 1.3416 F_{BC}$

Allowable value for P.

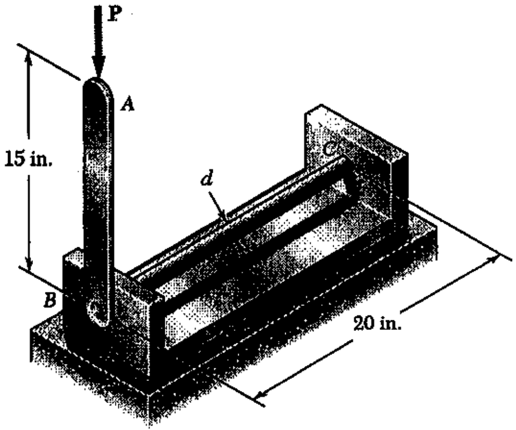
$P < 1.06066 F_{AB,all} = (1.06066)(3.773) = 4.00 \text{ kN}$

$P < 1.3416 F_{BC,all} = (1.3416)(4.770) = 6.40 \text{ kN}$

$P_{all} = 4.00 \text{ kN}$

PROBLEM 10.121

10.121 The steel rod BC is attached to the rigid bar AB and to the fixed support at C. Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of rod BC for which the critical load P_{cr} of the system is 80 lb.



SOLUTION

Look at torsion spring BC

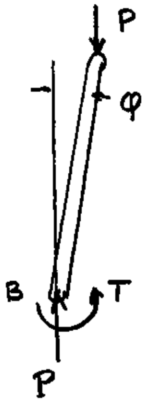
$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ}{L}\varphi = K\varphi$$

$$G = 11.2 \times 10^6 \text{ psi}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}\left(\frac{d}{2}\right)^4 = \frac{\pi d^4}{32}$$

$$L = 20 \text{ in}$$

$$K = \frac{(11.2\times10^6)\,\pi d^4}{(20)(32)} = 54978\, d^4$$



$$\circlearrowleft \Sigma M_B = 0$$

$$T - P\ell \sin\varphi = 0$$

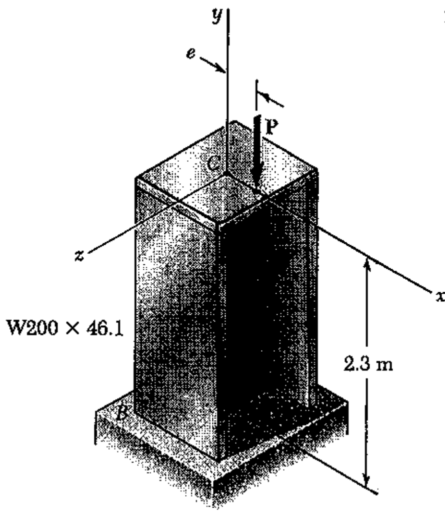
$$K\varphi - P\ell\sin\varphi = 0$$

$$P = \frac{K\varphi}{\ell \sin\varphi} \qquad P_{cr} = \frac{K}{\ell}$$

$$K = 54978\, d^4 = P_{cr}\ell = (80)(15) = 1200$$

$$d = \sqrt[4]{\frac{1200}{54978}} = 0.384 \text{ in.}$$

PROBLEM 10.122



10.122 An axial load **P** of magnitude 560 kN is applied at a point on the x axis at a distance $e = 8$ mm from the geometric axis of the W 200 × 46.1 rolled-steel column BC. Using $E = 200$ GPa, determine (*a*) the horizontal deflection of end C, (*b*) the maximum stress in the column.

SOLUTION

$$L_e = 2L = (2)(2.3) = 4.6 \text{ m} \qquad e = 8\times10^{-3} \text{ m}$$

W 200 × 46.1 $\quad A = 5860 \text{ mm}^2 = 5860\times10^{-6} \text{ m}^2$

$$I_y = 15.3\times10^{6} \text{ mm}^4 = 15.3\times10^{-6} \text{ m}^4$$

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2(200\times10^9)(15.3\times10^{-6})}{(4.6)^2}$$

$$= 1.42727\times10^{6} \text{ N}$$

$$\frac{P}{P_{cr}} = \frac{560\times10^3}{1.42727\times10^6} = 0.39236$$

$$y_m = e\left[\sec\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} - 1\right] = (8\times10^{-3})\left[\sec\left(\frac{\pi}{2}\sqrt{0.39236}\right) - 1\right]$$

$$= (8\times10^{-3})[\sec(0.98393) - 1] = (8\times10^{-3})[1.8058 - 1]$$

$$= 6.447\times10^{-3} \text{ m} \qquad = 6.45 \text{ mm}$$

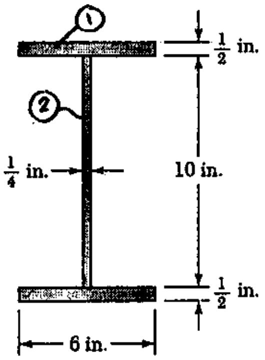
$$M_{max} = P(y_m + e) = (560\times10^3)(8\times10^{-3} + 6.447\times10^{-3}) = 8.090\times10^3 \text{ N}\cdot\text{m}$$

$$S_y = 151\times10^3 \text{ mm}^3 = 151\times10^{-6} \text{ m}^3$$

$$\sigma_{max} = \frac{P}{A} + \frac{M}{S_y} = \frac{560\times10^3}{5860\times10^{-6}} + \frac{8.090\times10^3}{151\times10^{-6}} = 149.1\times10^6 \text{ Pa} = 149.1 \text{ MPa}$$

PROBLEM 10.123

10.123 A column with the cross section shown has a 13.5-ft effective length. Knowing that $\sigma_Y = 36$ ksi, and $E = 29 \times 10^6$ psi., use the AISC allowable stress design formulas to determine the largest centric load that can be applied to the column.



SOLUTION

$$A = 2A_1 + A_2 = (2)(\tfrac{1}{2})(6) + (10)(\tfrac{1}{4}) = 8.5 \text{ in}^2$$

$$I_y = 2I_1 + I_2 = (2)(\tfrac{1}{12})(\tfrac{1}{2})(6)^3 + (\tfrac{1}{12})(10)(\tfrac{1}{4})^3 = 18.013 \text{ in}^4$$

$$r_y = \sqrt{\frac{I_Y}{A}} = \sqrt{\frac{18.013}{8.5}} = 1.4557 \text{ in.}$$

$$L_e = 13.5 \text{ ft} = 162 \text{ in} \qquad \frac{L_e}{r} = 111.29 < C_c$$

Steel: $E = 29000$ ksi, $\sigma_Y = 36$ ksi $\quad C_c = \sqrt{\dfrac{2\pi^2 E}{\sigma_Y}} = \sqrt{\dfrac{2\pi^2(29000)}{36}} = 126.10$

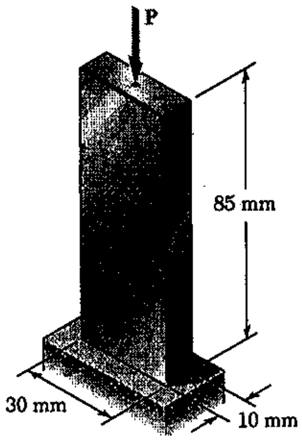
$$\frac{L_e/r}{C_c} = 0.8826 \qquad F.S. = \frac{5}{3} + \frac{3}{8}(0.8826) - \frac{1}{8}(0.8826)^3 = 1.912$$

$$\sigma_{all} = \frac{\sigma_Y}{F.S.}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] = \frac{36}{1.912}\left[1 - \frac{1}{2}(0.8826)^2\right] = 11.49 \text{ ksi}$$

$$P_{all} = \sigma_{all} A = (11.49)(8.5) = 97.7 \text{ kips}$$

PROBLEM 10.125

10.125 Bar *AB* is free at its end *A* and fixed at its base *B*. Determine the allowable centric load **P** if the aluminum alloy is (*a*) 6061-T6, (*b*) 2014-T6.



SOLUTION

$A = (30)(10) = 300 \text{ mm}^2 = 300 \times 10^{-6} \text{ m}^2$

$I_{min} = \frac{1}{12}(30)(10)^3 = 2.50 \times 10^3 \text{ mm}^4$

$r_{min} = \sqrt{\frac{I}{A}} = \sqrt{\frac{2.50 \times 10^3}{300}} = 2.887 \text{ mm}$

$L_e = 2L = (2)(85) = 170 \text{ mm} \qquad \frac{L_e}{r_m} = 58.88$

(a) 6061-T6 $\qquad L/r < 66$

$\sigma_{all} = 139 - 0.868(L/r) = 139 - (0.868)(58.88)$
$= 87.9 \text{ MPa}$

$P_{all} = \sigma_{all} A = (87.9 \times 10^6)(300 \times 10^{-6}) = 26.4 \times 10^3 \text{ N}$
$= 26.4 \text{ kN}$ ◄

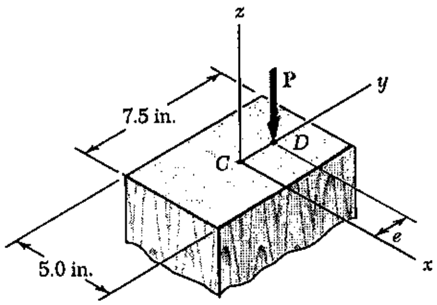
(b) 2014-T6 $\qquad L/r > 55$

$\sigma_{all} = \frac{372 \times 10^3}{(L/r)^2} = \frac{372 \times 10^3}{(58.88)^2} = 107.3 \text{ MPa}$

$P_{all} = \sigma_{all} A = (107.3 \times 10^6)(300 \times 10^{-6}) = 32.2 \times 10^3 \text{ N} = 32.2 \text{ kN}$ ◄

PROBLEM 10.126

10.126 A sawn lumber column of 5.0 × 7.5-in. cross section has an effective length of 8.5 ft. The grade of wood used has an adjusted allowable stress for compression parallel to the grain $\sigma_C = 1180$ psi and a modulus of elasticity $E = 1.2 \times 10^6$ psi. Using the allowable-stress method, determine the largest eccentric load **P** that can be applied when (*a*) $e = 0.5$ in., (*b*) $e = 1.0$ in.



SOLUTION

Sawn lumber: $\sigma_c = 1180$ psi $\quad E = 1.2\times10^6$ psi

$c = 0.8 \quad K_{cE} = 0.300$

$L_e = 8.5 \text{ ft} = 102 \text{ in}$

$b = 7.5 \text{ in}, \quad d = 5.0 \text{ in}, \quad c = \frac{b}{2} = 3.75 \text{ in}$

$A = bd = (7.5)(5.0) = 37.5 \text{ in}^2 \qquad I_x = \frac{1}{12}(5.0)(7.5)^3 = 175.78 \text{ in}^4$

$$\sigma_{cE} = \frac{K_{cE}E}{(L/d)^2} = \frac{K_{cE}Ed^2}{L^2} = \frac{(0.300)(1.2\times10^6)(5.0)^2}{(102)^2} = 865 \text{ psi}$$

$$\sigma_{cE}/\sigma_c = 865/1180 = 0.7331$$

$$C_p = \frac{1+\sigma_{cE}/\sigma_c}{2c} - \sqrt{\left(\frac{1+\sigma_{cE}/\sigma_c}{2c}\right)^2 - \frac{\sigma_{cE}/\sigma_c}{c}} = 0.5763$$

$$\sigma_{all} = \sigma_c C_p = (1180)(0.5763) = 680 \text{ psi}$$

$$\frac{P_{all}}{A} + \frac{P_{all}ec}{I_x} = \sigma_{all}$$

$$P_{all} = \frac{\sigma_{all}}{\frac{1}{A} + \frac{ec}{I_x}}$$

(a) $e = 0.5$ in

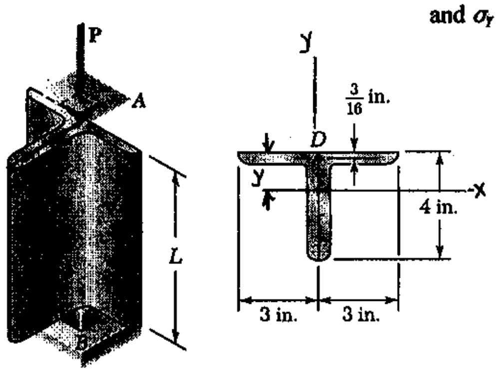
$$P_{all} = \frac{680}{\frac{1}{37.5} + \frac{(0.5)(3.75)}{175.78}} = 18210 \text{ lb.} = 18.21 \text{ kips}$$

(b) $e = 1.0$ in

$$P_{all} = \frac{680}{\frac{1}{37.5} + \frac{(1.0)(3.75)}{175.78}} = 14170 \text{ lb.} = 14.17 \text{ kips}$$

PROBLEM 10.127

10.127 Two $4 \times 3 \times \frac{3}{8}$-in. steel angles are welded together to form the column AB. An axial load **P** of magnitude 14 kips is applied at point D. Using the allowable-stress method, determine the largest allowable length L. Assume $E = 29 \times 10^6$ psi. and $\sigma_Y = 36$ ksi.



SOLUTION

One angle $L\ 4 \times 3 \times \frac{3}{8}$ $\quad A = 2.48\ \text{in}^2$

$I_x = 3.96\ \text{in}^4,\ S_x = 1.46\ \text{in}^3,\ r_x = 1.26\ \text{in.}\quad y = 1.28\ \text{in.}$

$I_y = 1.92\ \text{in}^4,\quad r_y = 0.879\ \text{in},\ x = 0.782\ \text{in}$

Two angles $\quad A = (2)(2.48) = 4.96\ \text{in}^2$

$I_x = (2)(3.96) = 7.92\ \text{in}^4,\quad S_x = (2)(1.46) = 2.92\ \text{in}^3,\ r_x = 1.26,\ y = 1.28\ \text{in.}$

$I_y = 2\left[I_{y(1)} + A x^2\right] = (2)\left[1.92 + (2.48)(0.782)^2\right] = 6.873\ \text{in}^4 = I_{min}$

$r_{min} = \sqrt{\frac{I_{min}}{A}} = 1.177\ \text{in.}\qquad e = y - \frac{3}{16} = 1.28 - \frac{3}{16} = 1.0925\ \text{in}$

$P = 14\ \text{kips}\qquad \sigma_{all} = \frac{P}{A} + \frac{Pey}{I_x} = \frac{14}{4.96} + \frac{(14)(1.0925)(1.28)}{7.92} = 5.294\ \text{ksi}$

$E = 29000\ \text{ksi}\qquad C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = \sqrt{\frac{2\pi^2(29000)}{36}} = 126.1$

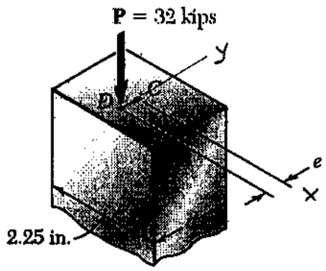
Assume $\frac{L_e}{r} > C_c\qquad \sigma_{all} = \frac{\pi^2 E}{1.92\,(L/r_{min})^2}\qquad \left(\frac{L}{r_{min}}\right)^2 = \frac{\pi^2 E}{1.92\,\sigma_{all}}$

$\frac{L}{r_{min}} = \sqrt{\frac{\pi^2 E}{1.92\,\sigma_{all}}} = \sqrt{\frac{\pi^2(29000)}{(1.92)(5.294)}} = 167.8 > C_c$

$L = 167.8\ r_{min} = (167.8)(1.177) = 197.5\ \text{in.} = 16.46\ \text{ft}$ ◄

PROBLEM 10.128

10.128 A compression member of rectangular cross section has an effective length of 36 in. and is made of the aluminum alloy 2014-T6 for which the allowable stress in bending is 24 ksi. Using the interaction method, determine the smallest dimension d of the cross section that can be used when $e = 0.4$ in.



SOLUTION

$A = 2.25\, d \qquad c = \frac{1}{2} d \qquad e = 0.4 \text{ in} \qquad L_e = 36 \text{ in}$

$\sigma_{all,b} = 24 \text{ ksi} \qquad P = 32 \text{ kips}$

$I_x = \frac{1}{12}(2.25) d^3 \qquad r_x = \frac{d}{\sqrt{12}}$

Assume $r_x = r_{min}$, i.e. $d < 2.25$

$L_e / r_{min} = \sqrt{12}\, L_e / d$

Assume $L_e/r_{min} > 55$. $\quad \sigma_{all,c} = \frac{54000}{(L_e/r_x)^2} = \frac{54000\, d^2}{12\, L_e^2} = \frac{54000}{(12)(36)^2} d^2 = 3.47222\, d^2$

$$\frac{P}{A\sigma_{all,c}} + \frac{Pec}{I\sigma_{all,b}} = \frac{32}{(2.25 d)(3.47222 d^2)} + \frac{(12)(32)(0.4)(\frac{1}{2} d)}{(2.25 d^3)(24)} = 1$$

$\frac{4.096}{d^3} + \frac{1.42222}{d^2} = 1 \qquad$ Let $x = \frac{1}{d} \qquad 4.096\, x^3 + 1.42222\, x^2 = 1$

Solving for x, $\quad x = 0.528118, \quad d = \frac{1}{x} = 1.894$ in. < 2.25 in.

$L/r_x = (\sqrt{12})(36)/1.894 = 65.8 > 66 \qquad d = 1.894 \text{ in.}$

PROBLEM 10.C1

10.C1 A solid steel rod having an effective length of 500 mm is to be used as a compression strut to carry a centric load **P**. For the grade of steel used $E = 200$ GPa and $\sigma_Y = 245$ MPa. Knowing that a factor of safety of 2.8 is required and using Euler's formula, write a computer program and use it to calculate the allowable centric load P_{all} for values of the radius of the rod from 6 mm to 24 mm, using 2-mm increments.

SOLUTION

ENTER RADIUS RAD, EFFECTIVE LENGTH L_e AND FACTOR OF SAFETY FS

COMPUTE RADIUS OF GYRATION

$$A = \pi \, RAD^2$$

$$I = \frac{1}{4} \pi \, RAD^4$$

$$r = \sqrt{\frac{I}{A}}$$

DETERMINE ALLOWABLE CENTRIC LOAD

CRITICAL STRESS:

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

LET σ EQUAL SMALLER OF σ_{cr} AND σ_Y

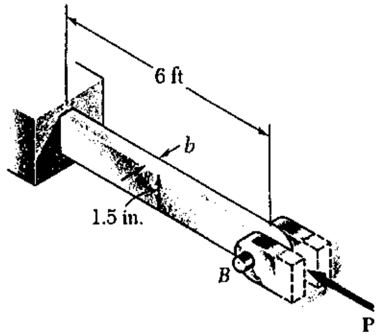
$$P_{all} = \frac{\sigma A}{FS}$$

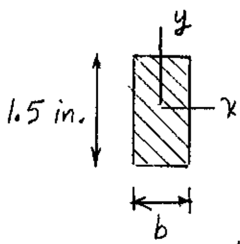
PROGRAM OUTPUT

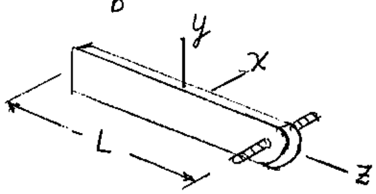
Radius of rod m	Critical stress MPa	Allowable load kN
.006	71.1	2.87
.008	126.3	9.07
.010	197.4	22.15
- - - - - -	- - - - - -	- - - - - -
.012	284.2	39.58
.014	386.9	53.88
.016	505.3	70.37
.018	639.6	89.06
.020	789.6	109.96
.022	955.4	133.05
.024	1137.0	158.34

Below the dashed line we have:
critical stress > yield strength

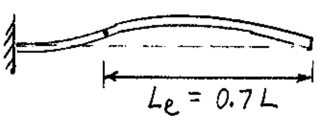
PROBLEM 10.C2



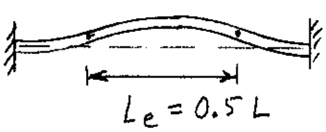




BUCKLING IN yz PLANE



BUCKLING IN xz PLANE



10.C2 An aluminum bar is fixed at end A and supported at end B so that it is free to rotate about a horizontal axis through the pin. Rotation about a vertical axis at end B is prevented by the brackets. Knowing that $E = 10.1 \times 10^6$ psi, use Euler's formula with a factor of safety of 2.5 to determine the allowable centric load **P** for values of b from 0.75 in. to 1.5 in., using 0.125-in. increments.

SOLUTION

ENTER E, LENGTH L AND FACTOR OF SAFETY FS

FOR b = 0.75 TO 1.5 WITH 0.125 INCREMENTS

COMPUTE RADIUS OF GYRATION

$$A = 1.5\, b$$

$$I_x = \frac{1}{12}\, b\, 1.5^3 \qquad r_x = \sqrt{\frac{I_x}{A}}$$

$$I_y = \frac{1}{8}\, b^3 \qquad r_y = \sqrt{\frac{I_y}{A}}$$

COMPUTE CRITICAL STRESSES

$$(\sigma_{cr})_x = \frac{\pi^2 E}{(0.7L/r_x)^2}$$

$$(\sigma_{cr})_y = \frac{\pi^2 E}{(0.5L/r_y)^2}$$

LET σ_{cr} EQUAL SMALLER STRESS

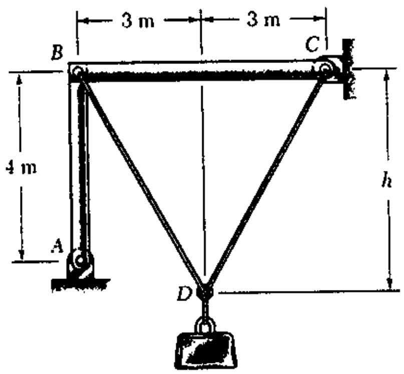
COMPUTE ALLOWABLE CENTRIC LOAD

$$P_{all} = \frac{\sigma_{cr}\, A}{FS}$$

PROGRAM OUTPUT

b in.	Critical stress x axis ksi	Critical stress y axis ksi	Allowable load kips
.750	7.358	3.6	1.62
.875	7.358	4.9	2.58
1.000	7.358	6.4	3.85
1.125	7.358	8.1	4.97
1.250	7.358	10.0	5.52
1.375	7.358	12.1	6.07
1.500	7.358	14.4	6.62

PROBLEM 10.C3



10.C3 The pin-ended members *AB* and *BC* consist of sections of aluminum pipe of 120-mm outer diameter and 10-mm wall thickness. Knowing that a factor of safety of 3.5 is required, determine the mass *m* of the largest block that can be supported by the cable arrangement shown for values of *h* from 4 m to 8 m, using 0.25-m increments. Use $E = 70$ GPa and consider only buckling in the plane of the structure.

SOLUTION

COMPUTE MOMENT OF INERTIA

$$I = \frac{\pi}{4}\left(0.06^4 - 0.05^4\right)$$

FOR $h = 4$ TO 8 USING 0.25 INCREMENTS

COMPUTE ALLOWABLE LOADS FOR MEMBERS

$$\left(F_{AB}\right)_{cr} = \frac{\pi^2 EI}{3.5(4)^2}; \quad \left(F_{BC}\right)_{cr} = \frac{\pi^2 EI}{3.5(6)^2}$$

DETERMINE ALLOWABLE W

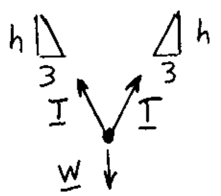
$$\left(W_{all}\right)_1 = 2\left(F_{AB}\right)_{cr}; \quad \left(W_{all}\right)_2 = \frac{h}{1.5}\left(F_{BC}\right)_{cr}$$

W_{all} EQUALS SMALLER VALUE

COMPUTE MASS m

$$m = \frac{W_{all}}{9.81}$$

JOINT D:



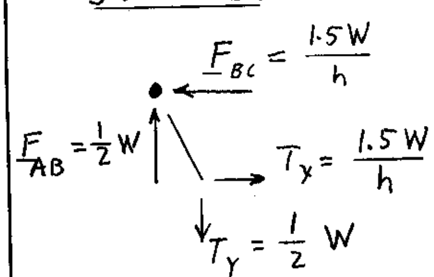
$\Sigma F_y = 0$ YIELDS

$$T_Y = \frac{1}{2} W$$

$$\frac{T_x}{T_y} = \frac{3}{h} \text{ YIELDS}$$

$$T_x = \frac{1.5W}{h}$$

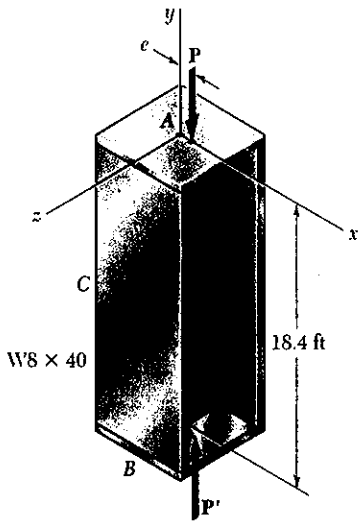
JOINT B:



PROGRAM OUTPUT

h m	Weight critical stress AB kN	Weight critical stress BC kN	mass kg
4.00	455.11	269.7	7854.88
4.25	455.11	286.6	8345.80
4.50	455.11	303.4	8836.74
4.75	455.11	320.3	9327.66
5.00	455.11	337.1	9818.59
5.25	455.11	354.0	10309.52
5.50	455.11	370.8	10800.45
5.75	455.11	387.7	11291.38
6.00	455.11	404.5	11782.31
6.25	455.11	421.4	12273.24
6.50	455.11	438.3	12764.17
6.75	455.11	455.1	13255.10
7.00	455.11	472.0	13255.10
7.25	455.11	488.8	13255.10
7.50	455.11	505.7	13255.10
7.75	455.11	522.5	13255.10
8.00	455.11	539.4	13255.10

PROBLEM 10.C4



10.C4 An axial load **P** is applied at a point located on the x axis at a distance $e = 0.5$ in. from the geometric axis of the W8 × 40 rolled-steel column AB. Using $E = 29 \times 10^6$ psi, write a computer program and use it to calculate for values of P from 25 to 75 kips, using 5-kip increments, (*a*) the horizontal deflection at the midpoint C, (*b*) the maximum stress in the column.

SOLUTION

ENTER LENGTH L, ECCENTRICITY e

ENTER PROPERTIES A, I_y, r_y, b_f

COMPUTE CRITICAL LOAD

$$P_{cr} = \frac{\pi^2 E I_y}{L^2}$$

FOR P = 25 TO 75 IN INCREMENTS OF 5

COMPUTE HORIZONTAL DEFLECTION AT C

$$y_c = e\left(\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1.0\right)$$

COMPUTE MAXIMUM STRESS

$$\sigma_{max} = \frac{P}{A}\left(1 + \frac{e\,b_f}{2\,r_y^2}\sec\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)$$

PROGRAM OUTPUT

Load kip	maximum deflection in.	maximum stress kips
25.0	.059	3.29
30.0	.072	3.99
35.0	.086	4.69
40.0	.100	5.41
45.0	.115	6.14
50.0	.130	6.88
55.0	.146	7.65
60.0	.163	8.43
65.0	.181	9.22
70.0	.199	10.04
75.0	.219	10.88

PROBLEM 10.C5

10.C5 A column of effective length L is made from a rolled-steel shape and carries a centric axial load **P**. The yield strength for the grade of steel used is denoted by σ_Y, the modulus of elasticity by E, the cross-sectional area of the selected shape by A, and its smallest radius of gyration by r. Using the AISC design formulas for allowable stress design, write a computer program that can be used with either SI or U.S. customary units to determine the allowable load **P**. Use this program to solve (*a*) Prob. 10.57, (*b*) Prob. 10.58, (*c*) Prob. 10.60.

SOLUTION

ENTER L, E, σ_Y

ENTER PROPERTIES A, r_y

DETERMINE ALLOWABLE STRESS

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$$

IF $L/r_y \geq C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r_y)^2}$$

IF $L/r_y < C_c$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L/r_y}{C_c}\right) - \frac{1}{8}\left(\frac{L/r_y}{C_c}\right)^3$$

$$\sigma_{all} = \frac{\sigma_Y}{FS}\left(1 - \frac{(L/r_y)^2}{2C_c^2}\right)$$

CACULATE ALLOWABLE LOAD:

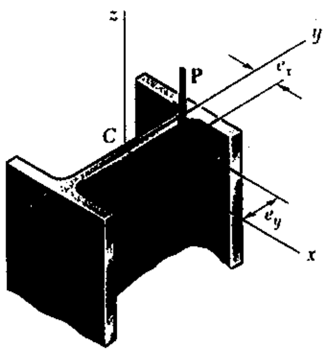
$$P_{all} = \sigma_{all} A$$

CONTINUED

PROGRAM OUTPUT

```
Problem 10.57 (a)

  Effective Length =  6.50  m
                 A =  6250.0 mm**2
                ry =  49.2 mm
    Yield strength =  250.0 MPa
                 E =  200 GPa
 -----------------------------------------
  Allowable centroid load: P =  368.139 kN

Problem 10.57 (b)

  Effective Length =  6.50  m
                 A =  10200.0 mm**2
                ry =  65.0 mm
    Yield strength =  250.0 MPa
                 E =  200 GPa
 -----------------------------------------
  Allowable centroid load: P =  916.148 kN

Problem 10.58 (a)

  Effective Length =  21.00  ft
                 A =  9.130 in**2
                ry =  2.020 in.
    Yield strength =  36.0 ksi
                 E =  29000 ksi
 -----------------------------------------
  Allowable centroid load: P =  87.566 kips

Problem 10.58 (b)

  Effective Length =  21.00  ft
                 A =  9.130 in**2
                ry =  2.020 in.
    Yield strength =  50.0 ksi
                 E =  29000 ksi
 -----------------------------------------
  Allowable centroid load: P =  87.452 kips

Problem 10.60 (a)

  Effective Length =  4.00  m
                 A =  13800.0 mm**2
                ry =  43.4 mm
    Yield strength =  345.0 MPa
                 E =  200 GPa
 -----------------------------------------
  Allowable centroid load: P = 1567.879 kN

Problem 10.60 (b)

  Effective Length =  6.50  m
                 A =  13800.0 mm**2
                ry =  43.4 mm
    Yield strength =  345.0 MPa
                 E =  200 GPa
 -----------------------------------------
  Allowable centroid load: P =  632.667 kN
```

PROBLEM 10.C6



10.C6 A column of effective length L is made from a rolled-steel shape and is loaded eccentrically as shown. The yield strength of the grade of steel used is denoted by σ_Y, the allowable stress in bending by σ_{all}, the modulus of elasticity by E, the cross-sectional area of the selected shape by A, and its smallest radius of gyration by r. Write a computer program that can be used with either SI or U.S. customary units to determine the allowable load **P**, using either the allowable-stress method or the interaction method. Use this program to check the given answer for (*a*) Prob. 10.111, (*b*) Prob. 10.112, (*c*) Prob. 10.113.

SOLUTION

ENTER $L, E, \sigma_Y, (\sigma_{all})_{bending}, e_x, e_y$

ENTER PROPERTIES A, S_x, S_y, r_y

DETERMINE ALLOWABLE STRESS

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}}$$

IF $L/r_y \geq C_c$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L/r)^2}$$

IF $L/r_y < C_c$

$$FS = \frac{5}{3} + \frac{3}{8}\left(\frac{L/r_y}{C_c}\right) - \frac{1}{8}\left(\frac{L/r_y}{C_c}\right)^3$$

$$\sigma_{all} = \frac{\sigma_Y}{FS}\left(1 - \frac{(L/r_y)^2}{2C_c^2}\right)$$

FOR ALLOWABLE-STRESS METHOD

$$COEF = \frac{1}{A} + \frac{e_x}{S_x} + \frac{e_y}{S_y}$$

$$P_{all} = \frac{\sigma_{all}}{COEF}$$

FOR INTERACTION METHOD

$$COFF = \frac{1}{A\,\sigma_{all}} + \frac{(e_x/S_x) + (e_y/S_y)}{(\sigma_{all})_{bending}}$$

$$P_{all} = \frac{1.0}{COEF}$$

CONTINUED

PROBLEM 10.C6 CONTINUED

PROGRAM OUTPUT

```
Problem 10.111

  Effective Length = 5.80 m
                 A = 7560.0 mm**2
                ry = 51.900 mm
                Sx = 582000.0 mm**3
    Yield strength = 250.0 MPa
                 E = 200 GPa
 ----------------------------------------
 Using Interaction Method
 Allowable load: P = 322.022 kN
 ----------------------------------------

Problem 10.112

  Effective Length = 7.20 m
                 A = 7420.0 mm**2
                ry = 50.300 mm
                Sy = 185000.0 mm**3
    Yield strength = 250.0 MPa
                 E = 200 GPa
 ----------------------------------------
 Using Allowable-Stress Method
 Allowable load: P = 97.781 kN
 ----------------------------------------

Problem 10.113

  Effective Length = 21.00 ft
                 A = 11.800 in**2
                ry = 1.930 in.
                Sx = 51.90 in**3
    Yield strength = 36.0 ksi
                 E = 29 x 10^3 ksi
 ----------------------------------------
 Using Interaction Method
 Allowable load: P = 86.722 kips
 ----------------------------------------
```