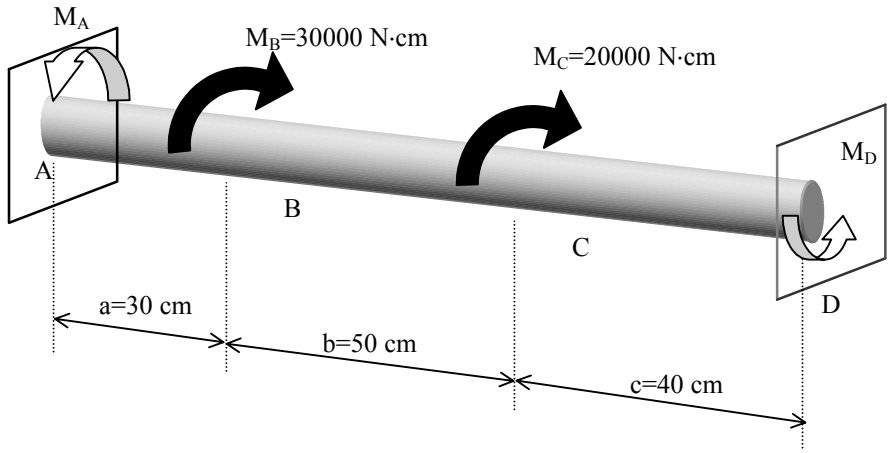
Problema 7.2

Hallar los momentos en los empotramientos M_A y M_D. Dibujar el diagrama de momentos torsores.



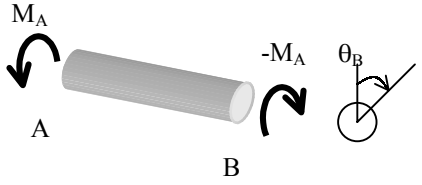
Resolución:

Es un problema hiperestático.

$$\sum M_T = 0 \Rightarrow M_A - M_B - M_C + M_D = 0$$

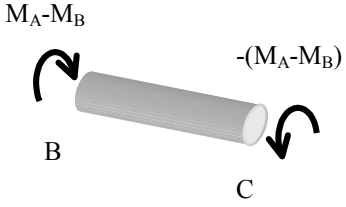
$$M_A + M_D = 30000 + 20000 = 50000 \text{ kg} \cdot \text{cm}$$

Considerando por tramos:

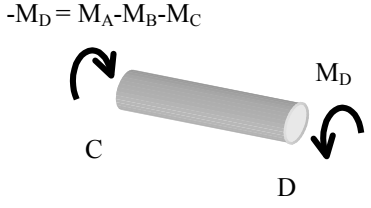


$$\theta_A = 0$$

$$\theta_{BA} = \theta_B - \theta_A = \frac{-M_A \cdot a}{G \cdot I_o} = \frac{-M_A \cdot 30}{G \cdot I_0}$$



$$\theta_{CB} = \theta_C - \theta_B = \frac{-\left(M_A - M_B\right)}{G \cdot I_o} \cdot 50$$



$$\theta_{DC} = \theta_D - \theta_C = \frac{M_D \cdot 40}{G \cdot I_0}$$

$$\theta_D = 0$$

$$\theta_D = \theta_{BA} + \theta_{CB} + \theta_{DC} = 0$$

$$-\frac{M_A \cdot 30}{G \cdot I_o} + \frac{M_D \cdot 40}{G \cdot I_o} - \frac{M_A - M_B}{G \cdot I_o} \cdot 50 = 0$$

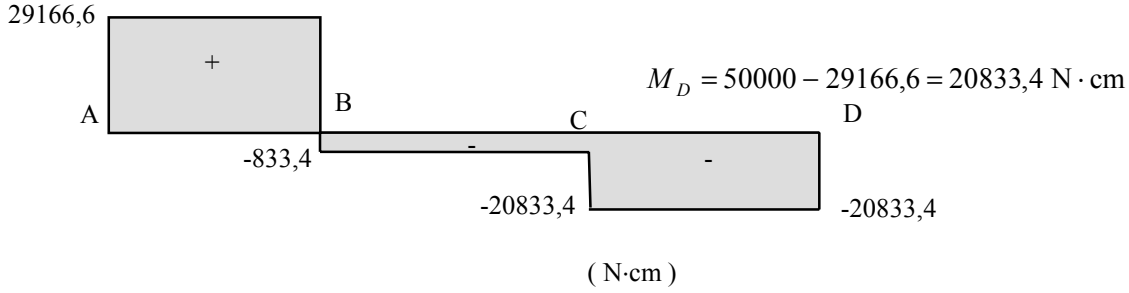
$$-M_A \cdot 30 - \left(M_A - 30000\right) \cdot 50 + M_D \cdot 40 = 0$$

$$-M_A \cdot 30 - M_A \cdot 50 + 1500000 + M_D \cdot 40 = 0$$

$$80 \cdot M_A - 40 \cdot M_D = 1500000$$

$$\begin{cases} M_A + M_D = 50000 \\ 80 \cdot M_A - 40 \cdot M_D = 1500000 \end{cases} \quad \begin{cases} 40 \cdot M_A + 40 \cdot M_D = 2000000 \\ 80 \cdot M_A - 40 \cdot M_D = 1500000 \end{cases} \Rightarrow M_A = \frac{3500000}{120} = 29166{,}6 \text{ N} \cdot \text{cm}$$

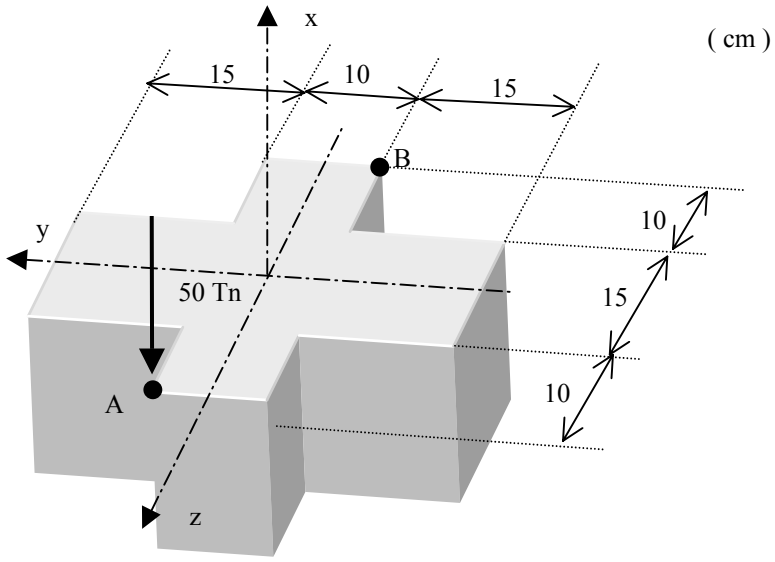
Diagrama de momentos torsores:



$$M_D = 50000 - 29166{,}6 = 20833{,}4 \text{ N} \cdot \text{cm}$$

Problema 6.2

Una columna tiene la sección en cruz indicada en la figura. La fuerza resultante es de compresión (50 Tn) y pasa por el punto A. Hallar la tensión normal en B y dibujar el eje neutro.



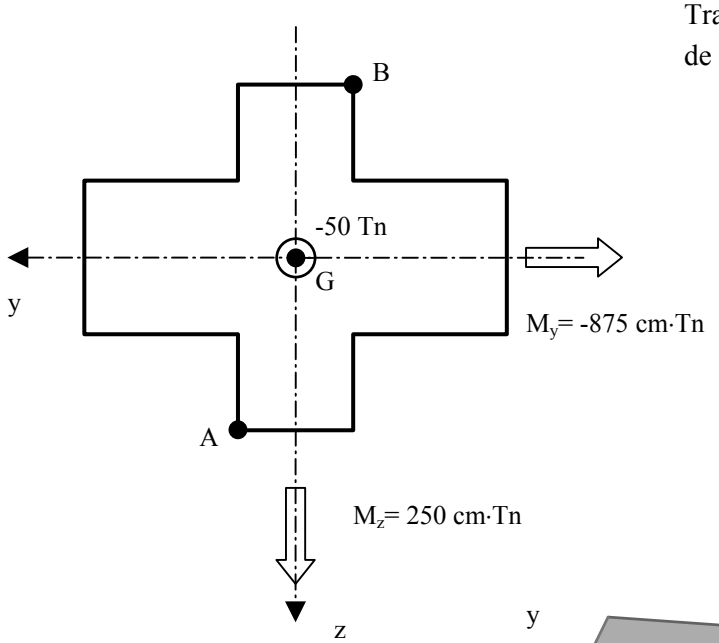
Resolución:

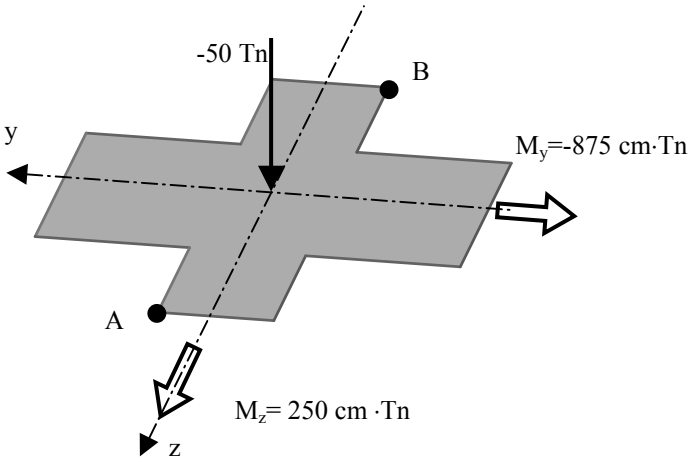
Trasladando la fuerza al centro de gravedad G de la sección, los esfuerzos equivalentes son:

$$N = -50\ \text{Tn}$$

$$M_y = -50\,\text{Tn} \cdot \left(10 + \frac{15}{2}\right)\text{cm} = -875\ \text{cm} \cdot \text{Tn}$$

$$M_z = 50\,\text{Tn} \cdot \frac{10}{2}\text{cm} = 250\ \text{cm} \cdot \text{Tn}$$





$$\sigma_x = \frac{N}{A} - \frac{M_z}{I_z} y + \frac{M_y}{I_y} z$$

$$\sigma_x = \frac{-50\,000}{A} - \frac{250\,000}{I_z} y + \frac{-875\,000}{I_y} z$$

$$A = 10 \cdot 35 + 2 \cdot 15 \cdot 15 = 800 \text{ cm}^2$$

$$I_z = \frac{1}{12} \cdot 15 \cdot (15+10+15)^3 + 2 \cdot \frac{1}{12} \cdot 10 \cdot 10^3 = 81667 \text{ cm}^4$$

$$I_y = \frac{1}{12} \cdot 10 \cdot (10+15+10)^3 + 2 \cdot \frac{1}{12} \cdot 15 \cdot 15^3 = 44167 \text{ cm}^4$$

$$\sigma_x = \frac{-50000}{800} - \frac{250000}{81667} y - \frac{875000}{44167} z \quad \Rightarrow \quad \sigma_x = -62{,}5 - 3{,}06y - 19{,}81z \ \left(\text{kg/cm}^2\right)$$

a) Tensión normal en B

Coordenadas de B $\begin{cases} y = -5 \text{ cm} \\ z = -17{,}5 \text{ cm} \end{cases}$

$$\sigma_{x_B} = -62{,}5 - 3{,}06 \cdot (-5) - 19{,}81 \cdot (-17{,}5) = 299{,}47 \text{ kp/cm}^2$$

b) Eje neutro

$$0 = -62{,}5 - 3{,}06y - 19{,}81z$$

$$y = -\frac{19{,}81}{3{,}06} z - \frac{62{,}5}{3{,}06} \quad \Rightarrow \quad y = -6{,}47z - 20{,}42$$

para $y = 0 \rightarrow z = \dfrac{-20{,}42}{6{,}47} = -3{,}15$

para $z = 0 \rightarrow y = -20{,}42$