Probabilidades y Procesos Estocásticos

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FORMULARIO CONTROL 3
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- $f(x) = \frac{\delta F(X)}{\delta X}$.
- $f_Y(y) = f_X(H^{-1})|\frac{\delta H^{-1}(y)}{\delta y}|$ con $Y = H(X)$.
- Esperanza: $\mathbb{E}(X) = \int_{-\infty}^{\infty} x f_X(x) dx$, $\mathbb{E}(X) = \sum_{i=0}^{\infty} i \cdot \mathbb{P}(X = i)$.
- Varianza: $V(X) = \mathbb{E}(X - \mathbb{E}(X))^2 = \mathbb{E}(X^2) - \mathbb{E}(X)^2$.
- $\mathbb{E}(C) = C$, $\mathbb{E}(CX) = C\mathbb{E}(X)$, $\mathbb{E}(\sum_{i=0}^{N} X_i) = \sum_{i=0}^{N} \mathbb{E}(X_i)$.
- $V(C) = 0$, $V(CX) = C^2 V(X)$.
- Binomial: $X \to B(n,p) \Rightarrow \mathbb{P}(x = k) = \binom{n}{k} p^k (1-p)^{n-k}$.
  - $\mathbb{E}(X) = np$
  - $Var(X) = np(1-p)$
  - $M_x(t) = (e^t p + 1 - p)^n$
- Binomial Negativa: $X \to BN(r,p) \Rightarrow \mathbb{P}(x = k) = \binom{k-1}{r-1} p^r (1-p)^{k-r} \quad k = r..\infty$.
  - $\mathbb{E}(X) = \frac{r(1-p)}{p}$
  - $Var(X) = \frac{r(1-p)}{p^2}$
  - $M_x(t) = \frac{pe^t}{1-(1-p)e^t} \quad 0 < (1-p)e^t < 1$
- Uniforme: $X \to U(a,b)$ si $f(x) = \frac{1}{b-a}$ para $x \epsilon (a,b)$.
  - $\mathbb{E}(X) = \frac{a+b}{2}$
  - $Var(X) = \frac{(b-a)^2}{12}$
  - $M_x(t) = \frac{e^{tb} - e^{ta}}{t(b-a)} \quad t \neq 0$
- Función Gamma: $\Gamma(p) = \int_0^{\infty} x^{p-1} \exp(-x) dx$
  - $\Gamma(p) = (p-1)\Gamma(p-1)$
  - $\Gamma(1) = 1$
  - $\Gamma(\frac{1}{2}) = \sqrt{\pi}$
- Distribución Gamma: $X \to G(\alpha, r) \Rightarrow f(x) = \frac{\alpha^r x^{r-1} exp(-\alpha x)}{\Gamma(r)}$ para $x > 0$.

- $\mathbb{E}(X) = \frac{r}{\alpha}$
- $Var(X) = \frac{r}{\alpha^2}$
- $M_x(t) = (\frac{\alpha}{\alpha - t})^r \quad t < \alpha$

- Normal: $X \to N(\mu, \sigma^2) \Rightarrow f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$
  - $\mathbb{E}(X) = \mu$
  - $Var(X) = \sigma^2$
  - $M_x(t) = exp(t\mu + \frac{t^2\sigma^2}{2})$

- $X \to N(\mu, \sigma^2)$ y $Z = \frac{x-\mu}{\sigma} \Rightarrow Z \to N(0,1)$

- $X_1, X_2 ... X_n$ v.a. independientes $X_i \to N(\mu_i, \sigma_i^2) \Rightarrow Z = \frac{\sum(X_i)}{n} \to N(\frac{\sum(\mu_i)}{n}, \frac{\sum(\sigma_i^2)}{n})$

- Poisson: $\mathbb{P}(X = i) = \frac{\lambda^i e^{-\lambda}}{i!}$ para i=0,1..∞.
  - $\mathbb{E}(X) = Var(X) = \lambda$
  - $M_x(t) = exp(\lambda(e^t - 1))$

- Función Generadora de Momentos: $M_x(t) = \mathbb{E}(e^{tx})\ t \in \mathbb{R}$
  - Caso Continuo: $M_x(t) = \int_{-\infty}^{\infty} e^{tx} f_x(x) dx$
  - Caso Discreto: $M_x(t) = \sum_0^{\infty} e^{tx} \mathbb{P}(X = i)$

- $\frac{d^n(M_x(t))}{dt^n}|_{t=0} = \mathbb{E}(X^n)$; $M_{\sum_1^n X_i}(t) = \prod_1^n M_{X_i}(t)$; $M_x(t) = M_y(t) \forall t \Rightarrow$x e y idd.

- $M_x(t) = \sum_0^{\infty} \mathbb{E}(x^j) \frac{t^j}{j!}$; si existe $M_x(t)$ y se tiene $y = a + bx \Rightarrow M_y(t) = e^{at} M_x(t)$

- Desigualdad de Chebyshev: $\mathbb{P}(|x - c| \geq \epsilon) \leq \frac{\mathbb{E}((x-c)^2)}{\epsilon^2}$
  Si c=$\mu \Rightarrow \mathbb{P}(|x - \mu| \geq \epsilon) \leq \frac{Var(x)}{\epsilon^2}$

- Ley de los grandes Números: $\lim_{n\to\infty} \mathbb{P}(|f_n(A) - p| \geq \epsilon) = 0\ \forall \epsilon > 0$; $f_n(A) \underset{n\to\infty}{\overset{P}{\to}} p$

- Teorema Central del Límite: X_i idd. $\Rightarrow \frac{1}{n}\sum_1^n X_i \underset{n\to\infty}{\to} N(\mathbb{E}(X_i), \frac{Var(X_i)}{n})$

- Procesos Estocásticos: X_t : N. de eventos en [0,t] $p_k(t) = \mathbb{P}(X_t = k)$

  - $p_1(\Delta t) = \lambda \Delta t + O(\Delta t) \qquad p_0(\Delta t) = 1 - \lambda \Delta t + O(\Delta t)$
  - $\sum_2^{\infty} p_i(\Delta t) = O(\Delta t) \qquad X_0 = 0$
  - $Y_{t_1,t} = X_{t_1+t} - X_t \Rightarrow Y_{t_1,t} = X_t$
  - $p_0(t) = ke^{-\lambda t} \qquad p_k(t) = e^{-\lambda t}\frac{(\lambda t)^n}{k!} \qquad \lambda = \frac{\mathbb{E}(X_t)}{t}$

- Proceso de Nacimiento y Muerte, en regimen permanente: $p_k = \frac{\prod_0^{k-1} \lambda_i}{\prod_1^k \mu_i} \cdot p_0$

  L=$\overline{\lambda} w$; $\quad L_q = \overline{\lambda} w_q$; $\quad L_s = \overline{\lambda} w_s$; $\quad L_s = L - L_q$