Cardioide

30 de julio de 2007

Teníamos la parametrización:

$$\vec{r}\,(t) = a\left(\cos(t) - \cos^2(t)\,, \sin(t) - \cos(t)\sin(t)\right)$$

Para sacar la longitud de arco notamos que $\cos(t)\sin(t) = \frac{\sin(2t)}{2}$. Entonces,

$$\begin{aligned}\vec{r}'\,(t) &= a\left(-\sin(t) + 2\sin(t)\cos(t)\,, \cos(t) - \cos(2t)\right)\\ &= a\left(-\sin(t) + \sin(2t)\,, \cos(t) - \cos(2t)\right)\end{aligned}$$

Luego,

$$\begin{aligned}\|\vec{r}'\,(t)\|^2 &= a^2\left(\sin^2(t) - 2\sin(t)\sin(2t) + \sin^2(2t) + \cos^2(t) - 2\cos(t)\cos(2t) + \cos^2(2t)\right)\\ &= 2a^2\left(1 - \left(\sin(t)\sin(2t) + \cos(t)\cos(2t)\right)\right)\\ &= 2a^2\left(1 - \cos(t)\right)\end{aligned}$$

La última igualdad se tiene de $\cos(2t - t) = \cos(2t)\cos(t) + \sin(2t)\sin(t)$. Recordando que $\sin\left(\frac{t}{2}\right) = \sqrt{\frac{1-\cos(t)}{2}}$, $\forall t \in [0, 2\pi]$, tenemos que

$$\|\vec{r}'\,(t)\| = 2a\sin\left(\frac{t}{2}\right)$$

Finalmente,

$$\begin{aligned}L\left(\Gamma\right) &= \int_0^{2\pi} 2a\sin\left(\frac{t}{2}\right)dt\\ &= 2a\left(-2\cos\left(\frac{t}{2}\right)\Big|_0^{2\pi}\right)\\ &= 2a\left(-2\cos(\pi) + 2\cos(0)\right)\\ &= 2a(2+2)\\ &= 8a\end{aligned}$$

Aprovecho de ponerles la fotito con $a = 1$:

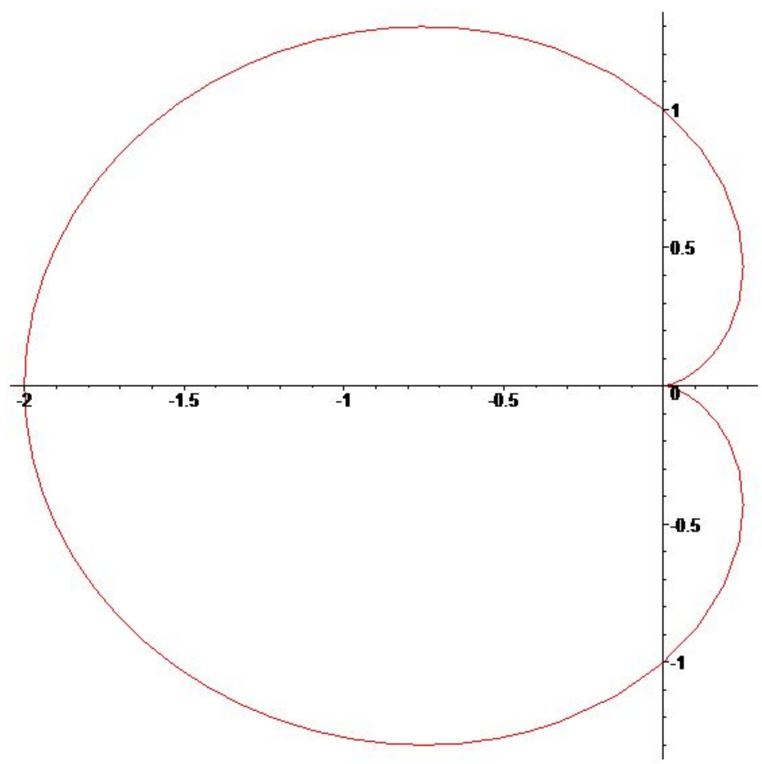

Figura 1: Cardioide con $a = 1$