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PAUTA P3

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(a) $f(t) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cdot \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \cdot \operatorname{sen}\left(\frac{n x \pi}{L}\right)$

$$* \quad a_0 = \frac{\omega}{\pi} \cdot \int_0^{\pi/\omega} \epsilon \operatorname{sen}(\omega x)\, dx = \frac{2\epsilon}{\pi} \Rightarrow \frac{a_0}{2} = \frac{\epsilon}{\pi}$$

$$a_n = \frac{\omega}{\pi}\int_0^{\pi/\omega} \epsilon \operatorname{sen}(\omega x) \cdot \cos(n\omega x)\, dx$$

Usando la indicación

$$= \frac{\omega\epsilon}{2\pi}\int_0^{\pi/\omega} \left[\operatorname{sen}(1+n)\omega x + \operatorname{sen}(1-n)\omega x\right] dx$$

$$= \frac{\omega\epsilon}{2\pi}\left[-\frac{\cos(1+n)\omega x}{(1+n)\omega} - \frac{\cos(1-n)\omega x}{(1-n)\omega}\right]_0^{\pi/\omega}$$

$$= \frac{\epsilon}{2\pi}\left[\frac{-\cos(1+n)\pi + 1}{(1+n)} + \frac{-\cos(1-n)\pi + 1}{(1-n)}\right]$$

Si n impar $\Rightarrow a_n = 0$; si n par se tiene:

$$* \quad a_n = \frac{\epsilon}{2\pi}\left[\frac{2}{(1+n)} + \frac{2}{(1-n)}\right] = \frac{-2\epsilon}{(n-1)(n+1)\pi} \quad \forall n = 2, 4, \ldots$$

$$b_n = \frac{\omega}{\pi}\int_0^{\pi/\omega} \epsilon \operatorname{sen}(\omega x) \cdot \operatorname{sen}(\omega n x)\, dx$$

$$= \frac{\omega\epsilon}{2\pi}\int_0^{\pi/\omega} \left[\cos(n+1)\omega x + \cos(1-n)\omega x\right] dx \quad \text{para } n \neq 1$$

$$= \frac{\omega\epsilon}{2\pi}\left[\frac{\operatorname{sen}(1+n)\omega x}{(1+n)\omega} + \frac{\operatorname{sen}(1-n)\omega x}{(1-n)\omega}\right]_0^{\pi/\omega}.$$

$$* \quad b_n = \frac{\omega\epsilon}{2\pi}\left[\underbrace{\frac{\operatorname{sen}(1+n)\pi}{(1+n)\omega}}_{\emptyset} + \underbrace{\frac{\operatorname{sen}(1-n)\pi}{(1-n)\omega}}_{\emptyset}\right] = \emptyset$$

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Para $n=1$: $\quad b_1 = \dfrac{\omega}{\pi}\displaystyle\int_0^{\pi/\omega} \epsilon \,\mathrm{sen}^2(\omega x)\, dx = \dfrac{\epsilon}{2}$

$$\Rightarrow f(x) = \frac{\epsilon}{\pi} + \frac{\epsilon}{2}\,\mathrm{sen}(\omega x) - \frac{2\epsilon}{\pi} \sum_{k\geq 1} \frac{\cos(2k\omega x)}{(2k+1)(2k-1)}$$

Evaluando en $t = \dfrac{\pi}{2\omega}$

$$\mathrm{sen}\left(\omega \frac{\pi}{2\omega}\right) = 1 \quad ; \quad \sum_{k\geq 1} \frac{\cos\left(2k\omega \frac{\pi}{2\omega}\right)}{(2k+1)(2k-1)} = \sum_{k\geq 1} \frac{(-1)^k}{(2k+1)(2k-1)}$$

Despejando $\displaystyle\sum_{k\geq 1} \frac{(-1)^k}{(2k+1)(2k-1)}$ se obtiene.

$$\epsilon = \frac{\epsilon}{\pi} + \frac{\epsilon}{2} - \frac{2\epsilon}{\pi} \sum_{k\geq 1} \frac{(-1)^k}{(2k+1)(2k-1)}$$

$$\Rightarrow \sum_{k\geq 1} \frac{(-1)^k}{(2k+1)(2k-1)} = \left(\frac{1}{2} + \frac{1}{\pi} - 1\right)\cdot \frac{\pi}{2} = \frac{1}{2} - \frac{\pi}{4}$$

Consultas en el Reclamo.

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(b)

$$\hat{f}(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{-isx} \, dx$$

Caso $s = 0$

$$\hat{f}(s) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} \frac{1}{x^2+a^2} \cdot dx = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{|a|} \left[\operatorname{arcotg} \right]_{-\infty}^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{|a|} \cdot \left[\frac{\pi}{2} - \frac{-\pi}{2} \right] = \frac{\pi}{\sqrt{2\pi}} \cdot \frac{1}{|a|} = \boxed{\sqrt{\frac{\pi}{2}} \cdot \frac{1}{|a|}}$$

Caso $s < 0$

$$\hat{f}(s) = \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} \frac{1}{x^2+a^2} \cdot e^{\overbrace{-isx}^{>0}} \, dx$$

$$\int_{-\infty}^{\infty} \frac{1}{x^2+a^2} e^{-isx} \, dx = 2\pi i \sum_{i=1} \operatorname{Res}\left(f(x) e^{-isx}, p_i \right)$$

$f(x)$ tiene 2 polos simples $p_1 = i|a|$; $p_2 = -i|a|$. , en este caso la curva solo encierra a p_1 , entonces:

$$\operatorname{Res}(f \cdot e^{-isx}, i|a|) = \lim_{x \to i|a|} \cancel{(x - i|a|)} \cdot \frac{e^{-isx}}{\cancel{(x-i|a|)}(x+i|a|)}$$

$$= \frac{e^{s|a|}}{2i|a|}$$

$$\Rightarrow \hat{f}(s) = \frac{1}{\sqrt{2\pi}} \cdot \cancel{2}\pi \cancel{i} \cdot \frac{e^{s|a|}}{\cancel{2}\cancel{i}|a|} = \boxed{\sqrt{\frac{\pi}{2}} \cdot \frac{e^{s|a|}}{|a|}}$$

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Caso $s>0$

$$\hat{f}(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{x^2+a^2} \cdot e^{\overbrace{-isx}^{<0}} dx$$

Hacemos un cambio de variable $x = -w$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{1}{w^2+a^2} \cdot e^{\overbrace{isw}^{>0}} dw$$

Ahora sí podemos usar el Teorema de los Residuos de Cauchy.

$$\int_{-\infty}^{\infty} \frac{1}{w^2+a^2} \cdot e^{isw} dw = 2\pi i \cdot \sum \left(f \cdot e^{isw}, p \right)$$

El polo que se usa el mismo del caso anterior

$$\operatorname{Res}\left(f \cdot e^{isw}, i|a|\right) = \lim_{w \to i|a|} \frac{e^{isw}}{(w + i|a|)} = \frac{e^{-s|a|}}{2i|a|}$$

$$\Rightarrow \hat{f}(s) = \frac{1}{\sqrt{2\pi}} \cdot \cancel{2}\pi \cancel{i} \cdot \frac{e^{-s|a|}}{\cancel{2}\cancel{i}|a|} = \boxed{\sqrt{\frac{\pi}{2}} \cdot \frac{e^{-s|a|}}{|a|}}$$

En general $\hat{f}(s) = \sqrt{\frac{\pi}{2}} \cdot \frac{e^{-|a||s|}}{|a|}$