EXERCISES

"Why," said the Dodo, "the best way to explain it is to do it." —Lewis Carroll, *Alice's Adventures in Wonderland*

2.2 Blackbody Radiation

1. If Planck's constant were smaller than it is, would quantum phenomena be more or less conspicuous than they are now?

2. Express the Planck radiation formula in terms of wavelength.

2.3 Photoelectric Effect

3. Is it correct to say that the maximum photoelectron energy KE_{max} is proportional to the frequency ν of the incident light? If not, what would a correct statement of the relationship between KE_{max} and ν be?

4. Compare the properties of particles with those of waves. Why do you think the wave aspect of light was discovered earlier than its particle aspect?

5. Find the energy of a 700-nm photon.

6. Find the wavelength and frequency of a 100-MeV photon.

7. A 1.00-kW radio transmitter operates at a frequency of 880 kHz. How many photons per second does it emit?

8. Under favorable circumstances the human eye can detect 1.0 $\times 10^{-18}$ J of electromagnetic energy. How many 600-nm photons does this represent?

Figure 1:

9. Light from the sun arrives at the earth, an average of 1.5 $\times 10^{11}$ m away, at the rate of 1.4×10^3 W/m^2 of area perpendicular to the direction of the light. Assume that sunlight is monochromatic with a frequency of 5.0×10^{14} Hz. (*a*) How many photons fall per second on each square meter of the earth's surface directly facing the sun? (*b*) What is the power output of the sun, and how many photons per second does it emit? (*c*) How many photons per cubic meter are there near the earth?

10. A detached retina is being "welded" back in place using 20-ms pulses from a 0.50-W laser operating at a wavelength of 632 nm. How many photons are in each pulse?

11. The maximum wavelength for photoelectric emission in tungsten is 230 nm. What wavelength of light must be used in order for electrons with a maximum energy of 1.5 eV to be ejected?

12. The minimum frequency for photoelectric emission in copper is 1.1×10^{15} Hz. Find the maximum energy of the photoelectrons (in electronvolts) when light of frequency 1.5×10^{15} Hz is directed on a copper surface.

13. What is the maximum wavelength of light that will cause photoelectrons to be emitted from sodium? What will the maximum kinetic energy of the photoelectrons be if 200-nm light falls on a sodium surface?

14. A silver ball is suspended by a string in a vacuum chamber and ultraviolet light of wavelength 200 nm is directed at it. What electrical potential will the ball acquire as a result?

15. 1.5 mW of 400-nm light is directed at a photoelectric cell. If 0.10 percent of the incident photons produce photoelectrons, find the current in the cell.

16. Light of wavelength 400 nm is shone on a metal surface in an apparatus like that of Fig. 2.9. The work function of the metal is 2.50 eV. (*a*) Find the extinction voltage, that is, the retarding voltage at which the photoelectron current disappears. (*b*) Find the speed of the fastest photoelectrons.

17. A metal surface illuminated by 8.5×10^{14} Hz light emits electrons whose maximum energy is 0.52 eV. The same surface illuminated by 12.0×10^{14} Hz hight emits electrons whose maximum energy is 1.97 eV. From these data find Planck's constant and the work function of the surface.

18. The work function of a tungsten surface is 5.4 eV. When the surface is illuminated by light of wavelength 175 nm, the maximum photoelectron energy is 1.7 eV. Find Planck's constant from these data.

19. Show that it is impossible for a photon to give up all its energy and momentum to a free electron. This is the reason why the photoelectric effect can take place only when photons strike bound electrons.

2.5 X-Rays

20. What voltage must be applied to an x-ray tube for it to emit x-rays with a minimum wavelength of 30 pm?

21. Electrons are accelerated in television tubes through potential differences of about 10 kV. Find the highest frequency of the electromagnetic waves emitted when these electrons strike the screen of the tube. What kind of waves are these?

2.6 X-Ray Diffraction

22. The smallest angle of Bragg scattering in potassium chloride (KCl) is 28.4° for 0.30-nm x-rays. Find the distance between atomic planes in potassium chloride.

23. The distance between adjacent atomic planes in calcite ($CaCO_3$) is 0.300 nm. Find the smallest angle of Bragg scattering for 0.030-nm x-rays.

24. Find the atomic spacing in a crystal of rock salt (NaCl), whose structure is shown in Fig. 2.19. The density of rock salt is 2.16×10^3 kg/m^3 and the average masses of the Na and Cl atoms are respectively 3.82×10^{-26} kg and 5.89×10^{-26} kg.

2.7 Compton Effect

25. What is the frequency of an x-ray photon whose momentum is 1.1×10^{-23} kg · m/s?

26. How much energy must a photon have if it is to have the momentum of a 10-MeV proton?

27. In Sec. 2.7 the x-rays scattered by a crystal were assumed to undergo no change in wavelength. Show that this assumption is reasonable by calculating the Compton wavelength of a Na atom and comparing it with the typical x-ray wavelength of 0.1 nm.

28. A monochromatic x-ray beam whose wavelength is 55.8 pm is scattered through 46°. Find the wavelength of the scattered beam.

29. A beam of x-rays is scattered by a target. At 45° from the beam direction the scattered x-rays have a wavelength of 2.2 pm. What is the wavelength of the x-rays in the direct beam?

30. An x-ray photon whose initial frequency was 1.5×10^{19} Hz emerges from a collision with an electron with a frequency of 1.2×10^{19} Hz. How much kinetic energy was imparted to the electron?

31. An x-ray photon of initial frequency 3.0×10^{19} Hz collides with an electron and is scattered through 90°. Find its new frequency.

32. Find the energy of an x-ray photon which can impart a maximum energy of 50 keV to an electron.

33. At what scattering angle will incident 100-keV x-rays leave a target with an energy of 90 keV?

34. (*a*) Find the change in wavelength of 80-pm x-rays that are scattered 120° by a target. (*b*) Find the angle between the directions of the recoil electron and the incident photon. (*c*) Find the energy of the recoil electron.

35. A photon of frequency ν is scattered by an electron initially at rest. Verify that the maximum kinetic energy of the recoil electron is $KE_{max} = (2h^2\nu^2/mc^2)/(1 + 2h\nu/mc^2)$.

36. In a Compton-effect experiment in which the incident x-rays have a wavelength of 10.0 pm, the scattered x-rays at a certain angle have a wavelength of 10.5 pm. Find the momentum (magnitude and direction) of the corresponding recoil electrons.

37. A photon whose energy equals the rest energy of the electron undergoes a Compton collision with an electron. If the electron moves off at an angle of 40° with the original photon direction, what is the energy of the scattered photon?

38. A photon of energy E is scattered by a particle of rest energy E_0. Find the maximum kinetic energy of the recoiling particle in terms of E and E_0.

2.8 Pair Production

39. A positron collides head on with an electron and both are annihilated. Each particle had a kinetic energy of 1.00 MeV. Find the wavelength of the resulting photons.

40. A positron with a kinetic energy of 2.000 MeV collides with an electron at rest and the two particles are annihilated. Two photons are produced; one moves in the same direction as the incoming positron and the other moves in the opposite direction. Find the energies of the photons.

41. Show that, regardless of its initial energy, a photon cannot undergo Compton scattering through an angle of more than 60° and still be able to produce an electron-positron pair. (*Hint:* Start by expressing the Compton wavelength of the electron in terms of the maximum photon wavelength needed for pair production.)

42. (*a*) Verify that the minimum energy a photon must have to create an electron-positron pair in the presence of a stationary nucleus of mass M is $2mc^2(1 + m/M)$, where m is the electron rest mass. (*b*) Find the minimum energy needed for pair production in the presence of a proton.

43. (*a*) Show that the thickness $x_{1/2}$ of an absorber required to reduce the intensity of a beam of radiation by a factor of 2 is given by $x_{1/2} = 0.693/\mu$. (*b*) Find the absorber thickness needed to produce an intensity reduction of a factor of 10.

44. (*a*) Show that the intensity of the radiation absorbed in a thickness x of an absorber is given by $I_0\mu x$ when $\mu x \ll 1$. (*b*) If $\mu x = 0.100$, what is the percentage error in using this formula instead of Eq. (2.25)?

45. The linear absorption coefficient for 1-MeV gamma rays in lead is 78 m^{-1}. Find the thickness of lead required to reduce by half the intensity of a beam of such gamma rays.

46. The linear absorption coefficient for 50-keV x-rays in sea-level air is 5.0×10^{-3} m^{-1}. By how much is the intensity of a beam of such x-rays reduced when it passes through 0.50 m of air? Through 5.0 m of air?

47. The linear absorption coefficients for 2.0-MeV gamma rays are 4.9 m^{-1} in water and 52 m^{-1} in lead. What thickness of water would give the same shielding for such gamma rays as 10 mm of lead?

48. The linear absorption coefficient of copper for 80-keV x-rays is 4.7×10^4 m^{-1}. Find the relative intensity of a beam of 80-keV x-rays after it has passed through a 0.10-mm copper foil.

49. What thickness of copper is needed to reduce the intensity of the beam in Exercise 48 by half?

50. The linear absorption coefficients for 0.05-nm x-rays in lead and in iron are, respectively, 5.8×10^4 m^{-1} and 1.1×10^4 m^{-1}. How thick should an iron shield be in order to provide the same protection from these x-rays as 10 mm of lead?

2.9 Photons and Gravity

51. The sun's mass is 2.0×10^{30} kg and its radius is 7.0×10^8 m. Find the approximate gravitational red shift in light of wavelength 500 nm emitted by the sun.

52. Find the approximate gravitational red shift in 500-nm light emitted by a white dwarf star whose mass is that of the sun but whose radius is that of the earth, 6.4×10^6 m.

53. As discussed in Chap. 12, certain atomic nuclei emit photons in undergoing transitions from "excited" energy states to their "ground" or normal states. These photons constitute gamma rays. When a nucleus emits a photon, it recoils in the opposite direction. (*a*) The $^{57}_{27}Co$ nucleus decays by K capture to $^{57}_{26}Fe$, which then emits a photon in losing 14.4 keV to reach its ground state. The mass of a $^{57}_{26}Fe$ atom is 9.5×10^{-26} kg. By how much is the photon energy reduced from the full 14.4 keV available as a result of having to share energy and momentum with the recoiling atom? (*b*) In certain crystals the atoms are so tightly bound that the entire crystal recoils when a gamma-ray photon is emitted, instead of the individual atom. This phenomenon is known as the **Mössbauer effect.** By how much is the photon energy reduced in this situation if the excited $^{57}_{26}Fe$ nucleus is part of a 1.0-g crystal? (*c*) The essentially recoil-free emission of gamma rays in situations like that of *b* means that it is possible to construct a source of virtually monoenergetic and hence monochromatic photons. Such a source was used in the experiment described in Sec. 2.9. What is the original frequency and the change in frequency of a 14.4-keV gamma-ray photon after it has fallen 20 m near the earth's surface?

54. Find the Schwarzschild radius of the earth, whose mass is 5.98×10^{24} kg.

55. The gravitational potential energy U relative to infinity of a body of mass m at a distance R from the center of a body of mass M is $U = -GmM/R$. (*a*) If R is the radius of the body of mass M, find the escape speed v_e of the body, which is the minimum speed needed to leave it permanently. (*b*) Obtain a formula for the Schwarzschild radius of the body by setting $v_e = c$, the speed of light, and solving for R. (Of course, a relativistic calculation is correct here, but it is interesting to see what a classical calculation produces.)