a) sabemos que $\Delta s = \dfrac{\lambda}{T}$ con λ = calor latente de cambio de fase

y T : temperatura absoluta a la que se produce.

luego, $\lambda_{vap} = T\,\Delta s = (100+273)(7.106 - 1.295,8) = \underline{2,17 \cdot 10^6\ J/kg}$.

b) Como P es constante,

$$\lambda_{vap} = \Delta h \quad \therefore \quad \Delta h = h_{vap} - h_{agua} = 2,17 \cdot 10^6\ J/kg = 2.170\ kJ/kg$$

$$\therefore \quad h_{agua} = (2.675,2 - 2.170)\ kJ/kg$$

$$\therefore \quad \boxed{h_{agua} = 505,2\ kJ/kg}$$

c) La integración de la ec. de Clausius-Clapeyron para el caso de la vaporización, con las aproximaciones $V_{vap} >> V_{agua}$ $\therefore$ $V_{vap} - V_{agua} \simeq V_{vap}$ y luego considerando el vapor como gas ideal : $V_{vap} = \dfrac{RT}{P}$, conduce a

$$\operatorname{Ln} P = -\frac{\lambda_{vap}}{RT} + cte$$

Expresemos λ_{vap} en J/mol : 1 mol de agua = 18 g de agua $\therefore$ $1g = \frac{1}{18}$ mol

$$\therefore \quad \lambda_{vap} = 2,17 \cdot 10^6\ J/kg = 2,17 \cdot 10^3\ J/g = 2,17 \cdot 10^3 \cdot 18\ J/mol$$

$$\therefore \quad \boxed{\lambda_{vap} = 39.060\ J/mol}$$

$$\therefore \quad \frac{\lambda_{vap}}{R} = \frac{39.060}{8,31} \simeq 4.700 \quad \longrightarrow \quad \operatorname{Ln} P = -\frac{4.700}{T} + cte$$

Determinamos la constante : $\operatorname{Ln} 1 = -\dfrac{4.700}{373} + cte \longrightarrow \boxed{cte = 12,6}$

Luego, $\boxed{\operatorname{Ln} P = -\dfrac{4700}{T} + 12,6}$

d) $\operatorname{Ln} \frac{1}{2} = -\dfrac{4700}{T} + 12,6 \longrightarrow -0,7 - 12,6 = -\dfrac{4700}{T} \longrightarrow T = 353,4\ K$

$\boxed{t = 80,4\ °C}$

$\operatorname{Ln} 2 = -\dfrac{4700}{T} + 12,6 \longrightarrow T \simeq 395\ K \longrightarrow \boxed{t = 122\ °C}$

2. a) $\langle \epsilon_c \rangle = \frac{1}{2} kT$ (término cuadrático contribuye con $\frac{1}{2}kT$)

b) $$\langle \epsilon_p \rangle = \frac{\int_{-\infty}^{\infty} bx^4 e^{-\beta k x^4} dx}{\int_{-\infty}^{\infty} e^{-\beta k x^4} dx} = \frac{-\frac{\partial}{\partial\beta}\int_{-\infty}^{\infty} e^{-\beta b x^4} dx}{\int_{-\infty}^{\infty} e^{-\beta b x^4} dx} = -\frac{\partial}{\partial \beta} \operatorname{Ln} \int_{-\infty}^{\infty} e^{-\beta b x^4} dx$$

$$\therefore \langle \epsilon_p \rangle = -\frac{\partial}{\partial\beta} \operatorname{Ln} \text{cte}\cdot\beta^{-\frac{1}{4}} = -\frac{\partial}{\partial\beta}\operatorname{Ln}(\text{cte}) + \frac{\partial}{\partial\beta}\left[\frac{1}{4}\operatorname{Ln}\beta\right]$$

$$= \frac{1}{4}\,\frac{1}{\beta}$$

$$\therefore \boxed{\langle \epsilon_p \rangle = \frac{1}{4} kT}$$

c) Energía media total: $\langle \epsilon \rangle = \langle \epsilon_c \rangle + \langle \epsilon_p \rangle$

$$= \frac{1}{2}kT + \frac{1}{4}kT$$

$$\therefore \boxed{\langle \epsilon \rangle = \frac{3}{4} kT}$$

d) Para 1 mol (N_0 osciladores): $u = \frac{3}{4} N_0 k T$

$$\therefore C_v = \left(\frac{\partial u}{\partial T}\right)_v = \frac{3}{4} N_0 k \longrightarrow \boxed{C_v = \frac{3}{4} R}$$

3.- $N_i = \frac{N}{Z} g_i e^{-\beta\epsilon_i}$ (1) ; $Z = \sum_i g_i e^{-\beta\epsilon_i}$; $\beta = \frac{1}{kT}$

a) En nuestro sistema, $g_1 = g_2 = 1$. Estados no degenerados.

$$\therefore Z = e^{-\beta\epsilon_1} + e^{-\beta\epsilon_2} = e^{-\beta\epsilon_1}\left(1 + e^{-\beta(\epsilon_2 - \epsilon_1)}\right).$$

Si hacemos $\epsilon = \epsilon_2 - \epsilon_1 > 0 \longrightarrow \boxed{Z = e^{-\beta\epsilon_1}(1 + e^{-\beta\epsilon})}$ (2)

Luego, $N_1 = \frac{N \cancel{e^{-\beta\epsilon_1}}}{\cancel{e^{-\beta\epsilon_1}}(1 + e^{-\beta\epsilon})}$ y $N_2 = \frac{N e^{-\beta\epsilon_2}}{e^{-\beta\epsilon_1}(1 + e^{-\beta\epsilon})} = \frac{N e^{-\beta\epsilon}}{1 + e^{-\beta\epsilon}}$

$\therefore \boxed{P_1 = \frac{N_1}{N} = \frac{1}{1 + e^{-\beta\epsilon}}}$ y $\boxed{P_2 = \frac{N_2}{N} = \frac{e^{-\beta\epsilon}}{1 + e^{-\beta\epsilon}}}$ $P_1 + P_2 = 1$. ✓

b) $U = N_1\epsilon_1 + N_2\epsilon_2 = \frac{N(\epsilon_1 + \epsilon_2 e^{-\beta\epsilon})}{1 + e^{-\beta\epsilon}} = \frac{N[\epsilon_1 + (\epsilon_1 + \epsilon)e^{-\beta\epsilon}]}{1 + e^{-\beta\epsilon}}$

$$\therefore U = \frac{N\epsilon_1 \cancel{(1 + e^{-\beta\epsilon})}}{\cancel{(1 + e^{-\beta\epsilon})}} + \frac{N\epsilon e^{-\beta\epsilon}}{1 + e^{-\beta\epsilon}}$$

ó $U = N\epsilon_1 + \frac{N e^{-\beta\epsilon}\epsilon}{1 + e^{-\beta\epsilon}} \Longrightarrow \boxed{U = N\left[\epsilon_1 + \frac{\epsilon}{1 + e^{\beta\epsilon}}\right]}$ (3)

Alternativamente puede obtenerse de $U = -N\frac{\partial \ln Z}{\partial\beta}$

c) $C_v = \left(\frac{\partial U}{\partial T}\right)_v = \frac{\partial U}{\partial\beta}\frac{\partial\beta}{\partial T} = -\frac{1}{kT^2}\frac{\partial U}{\partial\beta} = k\beta^2\frac{\partial U}{\partial\beta}$

$\therefore C_v = \frac{k\beta^2 N\epsilon^2 e^{\beta\epsilon}}{(1 + e^{\beta\epsilon})^2}$ ó $\boxed{C_v = Nk\frac{\beta^2\epsilon^2 e^{\beta\epsilon}}{(1 + e^{\beta\epsilon})^2}}$ (4)