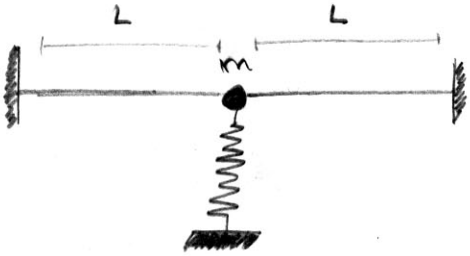
P] Fig. muestra una cuerda de 2l con sus extremos fijos a dos paredes. En el punto medio hay una masa puntual "m" y un resorte de constante "b". Se pide encontrar una ecuación que permite calcular las frecuencias propias del sist. La cuerda es de densidad μ y sometida a una tensión T. $\nexists \vec{g}$



Sea $y_1(x,t) = (A \cdot \mathrm{Sen}(kx) + B \cdot \mathrm{Cos}(kx)) \, \mathrm{Cos}(\omega t + \phi) \; ; \; -L \leq x \leq 0$

$y_2(x,t) = (C \cdot \mathrm{Sen}(kx) + D \cdot \mathrm{Cos}(kx)) \cdot \mathrm{Cos}(\omega t + \phi) \; ; \; 0 \leq x \leq L$

C. B

① $y_1(-l,t) = 0$

② $y_2(l,t) = 0$

③ $y_1(0,t) = y_2(0,t)$ (Continuidad)

④ $T\left(\frac{\partial y_2(0,t)}{\partial x} - \frac{\partial y_1}{\partial x}(0,t)\right) - b\, y_1(0,t) = m \frac{\partial^2 y_1(0,t)}{\partial t^2}$

Así:

① $\Rightarrow \; -A \cdot \mathrm{Sen}(kl) + B \cdot \mathrm{Cos}(kl) = 0 \Rightarrow A \cdot \mathrm{tg}(kl) = B$

② $\Rightarrow \; C \cdot \mathrm{Sen}(kl) + D \cdot \mathrm{Cos}(kl) = 0 \Rightarrow C \cdot \mathrm{tg}(kl) = -D$

③ $\Rightarrow \; B = D$

④ $T(kC - kA) - b \cdot B = -\omega^2 B \cdot m$

pero $A = \frac{B}{\mathrm{tg}(kl)} \; \wedge \; C = -\frac{B}{\mathrm{tg}(kl)}$

$$-m\cancel{B}\omega^2 = -Tk\frac{\cancel{B}}{tg(kl)} - Tk\frac{\cancel{B}}{tg(kl)} - b\cancel{B}$$

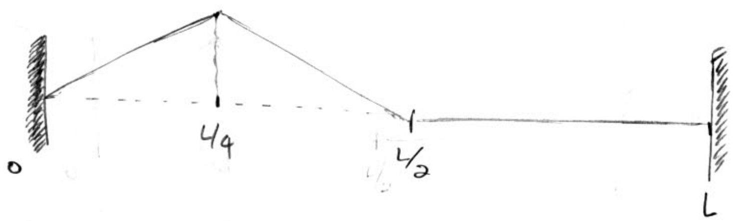
$$m\omega^2 = \frac{2Tk}{tg(kl)} + b \quad , \text{ pero } \quad k = \frac{\omega}{v} = \omega\left(\frac{\mu}{T}\right)^{1/2}$$

$\Rightarrow$

$$m\omega^2 = \frac{2T\omega(\mu/T)^{1/2}}{tg\left(\omega\left(\frac{\mu}{T}\right)^{1/2} l\right)} + b$$

//

P xxx) Análisis de Fourier de una perturbación triangular entre $x=0$ ∧ $x = L/2$ y nula para $L > x > L/2$ cuando $t=0$ y la velocidad tranversal de todos los puntos es nula en $t=0$.



Sea $y(x,t) = (A \cdot \operatorname{Sen}(kx) + B \cdot \operatorname{Cos}(kx)) \cdot \operatorname{Cos}(wt+\phi)$

∧ Extremos Fijos ; $y(0,t) = y(L,t) = 0$

$\Rightarrow y(0,t) = 0 \Rightarrow B = 0$

$y(l,t) = 0 \Rightarrow \operatorname{Sen}(kl) = 0 \Rightarrow k \cdot l = n\pi$

$\Rightarrow \boxed{k_n = \frac{n \cdot \pi}{l}}$

$\Rightarrow y_n(x,t) = A_n \cdot \operatorname{Sen}(k_n x) \operatorname{Cos}(w_n t + \phi_n)$

Asi, $y(x,t) = \sum_n A_n \cdot \operatorname{Sen}(k_n x) \operatorname{Cos}(w_n t + \phi_n)$

tambien $\left.\frac{\partial y}{\partial t}\right|_{t=0} = 0 \Rightarrow \phi_n = 0$

$\Rightarrow y(x,0) = \sum_n A_n \cdot \operatorname{Sen}(k_n \cdot x)$ (1)

$$\int_0^L \operatorname{Sen}(k_n x) \cdot \operatorname{Sen}(k_m \cdot x)\, dx = \delta_{n,m} \cdot \frac{L}{2}$$

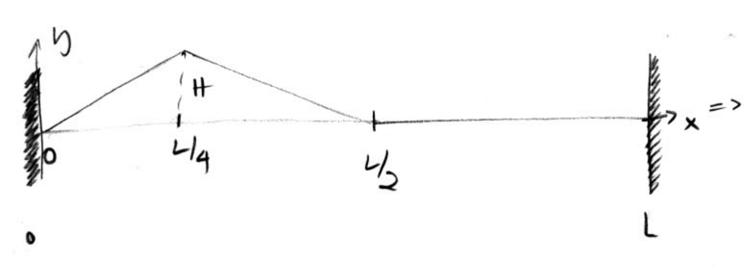
$$\delta_{n,m} = \begin{cases} 1 & n=m \\ 0 & n \neq m \end{cases}$$

Delta de Kronecker.

$$\int_0^L \operatorname{Sen}(k_n \cdot x) \cdot \operatorname{Cos}(k_m \cdot x)\, dx = 0$$

$$\int_0^L \operatorname{Cos}(k_n x) \cdot \operatorname{Cos}(k_m \cdot x)\, dx = \delta_{n,m} L/2$$

Condiciones Iniciales



$\Rightarrow y(x,0) = \begin{cases} y = \frac{4H}{L} x & 0 < x < L/4 \\ y = -\frac{4H}{L} x + 2H & L/4 < x < L/2 \\ y = 0 & L/2 < x < L \end{cases}$

Ahora $\int_0^L \operatorname{sen}(k_m x)\,dx$

$\Rightarrow$

$$\int_0^L y(x,0)\cdot \operatorname{sen}(k_m x)\,dx = A_n \int_0^L \operatorname{sen}(k_n x)\cdot \operatorname{sen}(k_m x)\,dx = A_n \cdot \delta_{n,m}\,\frac{L}{2}$$

$(n=m)$

$$\Rightarrow \quad A_m = \frac{2}{L}\int_0^L y(x,0)\cdot \operatorname{sen}(k_m x)\,dx$$

$$= \frac{2}{L}\left[\int_0^{L/4} \frac{4H}{L}x\cdot \operatorname{sen}(k_m x)\,dx + \int_{L/4}^{L/2}\left(-\frac{4H}{L}x + 2H\right)\operatorname{sen}(k_m x)\,dx\right]$$

Al. Resolver las $\int_s$ $\Rightarrow$ "propuesto"

$$A_n = \frac{8H}{L^2 (k_n)^2}\left[2\cdot \operatorname{sen}(k_n L/4) - \operatorname{sen}(k_n L/2)\right]$$

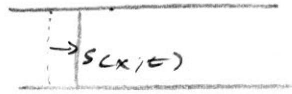
$$\therefore \quad y(x,t) = \sum_n \frac{8H}{L^2 (k_n)^2}\left(2\cdot \operatorname{sen}(k_n L/4) - \operatorname{sen}(k_n L/2)\right)\operatorname{sen}(k_n x)\cdot \cos(\omega_n t)$$

Sonido. - perturbación en la presión o densidad de un Fluido que viaja

- Onda longitudinal (perturbación en el sentido de dirección de propagación)

<u>Def</u>

$s(x,t) =$ desplazamiento del elemento de volumen de un Fluido ubicado en la posición x en t.



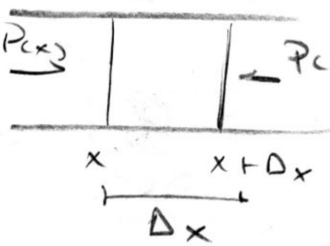
$\rho(x,t) =$ densidad x unidad de Volumen en el punto x en t.

$\rho(x,t) = \rho_0 + \rho_e(x,t) = \rho_0 + \rho_e$

$P(x,t) =$ presión del gas en el pto x en t.

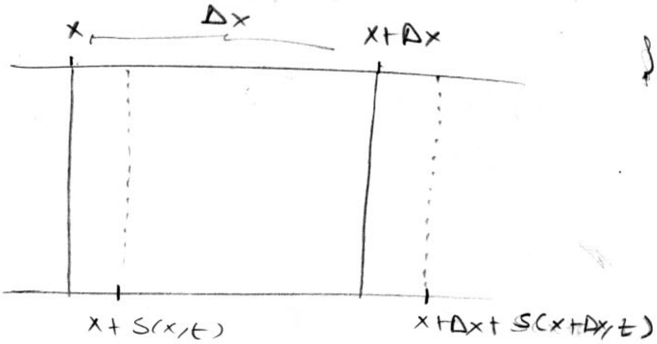
$P(x,t) = P_0 + P_e(x,t) = P_0 + P_e$.

* <u>Newton</u> (1-D) $\Rightarrow \quad P(x)\cdot A - P(x+\Delta x)\cdot A = \rho_0 \cdot \Delta x \cdot A \cdot \dfrac{\partial^2 s}{\partial t^2}$



$$\Rightarrow \boxed{-\frac{\partial P_e}{\partial x} = \rho_0 \cdot \frac{\partial^2 s}{\partial t^2}} \quad (0)$$

* Conservación de la Masa



$$\rho_0 \, \Delta x = \rho(x,t)\left[x + \Delta x + s(x+\Delta x, t) - x - s(x,t)\right]$$

$$\Rightarrow \boxed{\rho_e = -\rho_0 \cdot \frac{\partial s}{\partial x}} \quad (1)$$

$\rho(x,t) = \rho_0 + \rho_e$

$s(x+\Delta x, t) \to$ taylor.

* Ecuación de Estado

$P = P(\rho) \Rightarrow P_0 + P_e = P(\rho_0 + \rho_e) = P(\rho_0) + \left.\dfrac{\partial P}{\partial \rho}\right|_{\rho_0} \cdot \rho_e$

$\cancel{P_0} + P_e = \cancel{P_0} + \left.\dfrac{\partial P}{\partial \rho}\right|_{\rho=\rho_0} \cdot \rho_e$

$\Rightarrow P_e = \left.\dfrac{\partial P}{\partial \rho}\right|_{\rho_0} \rho_e \quad ; \quad c^2 = \left.\dfrac{\partial P}{\partial \rho}\right|_{\rho_0}$

$\Rightarrow \boxed{P_e = c^2 \rho_e}$ (2).

(1) · / $\frac{\partial}{\partial x}$ $\Rightarrow$ $\frac{\partial \rho_e}{\partial x} = -\rho_0 \frac{\partial^2 s}{\partial x^2}$ (1)'

(2) $-\frac{\partial P_e}{\partial x} = \rho_0 \cdot \frac{\partial^2 s}{\partial t^2}$ — $-\frac{\partial c^2 \rho_e}{\partial x} = \rho_0 \cdot \frac{\partial^2 s}{\partial t^2} \Rightarrow -c^2 \frac{\partial \rho_e}{\partial x} = \rho_0 \cdot \frac{\partial^2 s}{\partial t^2}$

pero
(1)' $-c^2\left(-\cancel{\rho_0} \frac{\partial^2 s}{\partial x^2}\right) = \cancel{\rho_0} \cdot \frac{\partial^2 s}{\partial t^2} \Rightarrow \boxed{\frac{\partial^2 s}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 s}{\partial t^2} = 0}$

para (3-D) $\nabla^2 \vec{S} - \frac{1}{c^2} \frac{\partial^2 s}{\partial t^2} = 0$

tambien

$$\frac{\partial^2 s}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 s}{\partial t^2} = 0 \quad / \frac{\partial}{\partial x}$$

$s \in C^3$

$\frac{\partial^2}{\partial x^2}\left(\frac{\partial s}{\partial x}\right) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\left(\frac{\partial s}{\partial x}\right) = 0$ De (1) $\Rightarrow -\frac{\rho_e}{\rho_0} = \frac{\partial s}{\partial x}$

$$\boxed{\frac{\partial^2 \rho_e}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \rho_e}{\partial t^2}} = 0 \Leftrightarrow \boxed{\frac{\partial^2 P_e}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 P_e}{\partial t^2}} = 0$$

$P_e = c^2 \rho_e$

N-S · F
→ Euler
N-S

$$\frac{\partial \vec{v}}{\partial t} + \vec{v}\, \nabla \vec{v} = \frac{\vec{f}}{\rho} - \frac{\nabla P}{\rho}$$

gradiente → Escalar
Divergencia → vector.

Para; $\vec{v} = \vec{v}_e$
$\rho = \rho_0 + \rho_e$
$P = P_0 + P_e$

$$\boxed{\frac{\partial v_e}{\partial t} \cdot \rho_0 = -\nabla P_e}$$