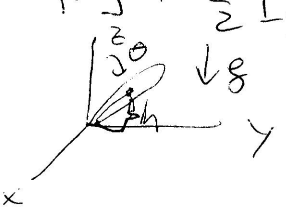
Pauta ej. 10 ~~4~~:

i)
$$w_1 = -\text{sen}\,\beta \cos\gamma\, \dot\alpha + \dot\beta\, \text{sen}\,\gamma$$
$$w_2 = \dot\alpha\, \text{sen}\beta\, \text{sen}\,\gamma + \dot\beta \cos\gamma$$
$$w_3 = \dot\alpha \cos\beta + \dot\gamma$$

$$I_1' = I_1 + mh^2$$
$$I_2' = I_2 + mh^2$$
$$I_3' = I_3$$

$$T = \frac{1}{2}\left(I_1' w_1^2 + I_2' w_2^2 + I_3' w_3^2\right) \qquad (I_1 = I_2)$$
$$= \frac{1}{2} I_1'\left[\dot\alpha^2 \text{sen}^2\beta + \dot\beta^2\right] + \frac{1}{2} I_3'\left[\dot\alpha\cos\beta + \dot\gamma\right]^2$$



$$V = mgh\cos\beta$$

$$L = \frac{1}{2} I_1'\left[\dot\alpha^2 \text{sen}^2\beta + \dot\beta^2\right] + \frac{1}{2} I_3'\left[\dot\alpha\cos\beta + \dot\gamma\right]^2 - mgh\cos\beta$$

ii)
$$L_3 = \frac{\partial L}{\partial \dot\psi} = \frac{\partial L}{\partial \dot\gamma} = I_3'\left[\dot\alpha\cos\beta + \dot\gamma\right]$$

$$L_z = \frac{\partial L}{\partial \dot\varphi} = \frac{\partial L}{\partial \dot\alpha} = I_1'\,\dot\alpha\,\text{sen}^2\beta + I_3'\left[\dot\alpha\cos\beta + \dot\gamma\right]\cos\beta$$

$$L_3 = I_3'\left[\dot\alpha\cos\beta + \dot\gamma\right]$$

$$L_z = I_1'\,\dot\alpha\,\text{sen}^2\beta + L_3\cos\beta$$

$$\boxed{\frac{L_z - L_3\cos\beta}{I_1'\,\text{sen}^2\beta} = \dot\alpha}$$

$$E = \frac{1}{2} I_1' \left[\dot{\alpha}^2 \operatorname{sen}^2\beta + \dot{\beta}^2 \right] + \frac{1}{2} I_3' \left[\dot{\alpha}\cos\beta + \dot{\gamma} \right]^2 + mgh\cos$$

$$= \frac{1}{2} I_1' \left[\frac{(L_z - L_3\cos\beta)^2}{(I_1')^2 \cdot \operatorname{sen}^2\beta} + \dot{\beta}^2 \right] + \frac{1}{2} \frac{L_3^2}{I_3'} + mgh\cos\beta$$

ii) $L_z = L_3 \quad \forall t$ pues se conservan

$$E = \frac{1}{2} \frac{L_z^2 (1-\cos\beta)^2}{I_1' \operatorname{sen}^2\beta} + \frac{1}{2} I_1' \dot{\beta}^2 + \underbrace{\frac{1}{2} \frac{L_z^2}{I_3'}}_{\text{cte (la omitimos)}} + mgh\cos\beta$$

$$E' = \underbrace{\frac{1}{2} \frac{L_z^2 (1-\cos\beta)^2}{I_1' \operatorname{sen}^2\beta} + mgh\cos\beta}_{U_{ef}(\beta)} + \frac{1}{2} I_1' \dot{\beta}^2$$

$$U(\beta) = \frac{1}{2} \frac{L_z^2 (1-\cos\beta)^2}{I_1' \operatorname{sen}^2\beta} + mgh\cos\beta$$

$$= \frac{1}{2} \frac{L_z^2 (1-\cos\beta)^2}{I_1' (1-\cos^2\beta)} + mgh\cos\beta$$

$$= \frac{1}{2} \frac{L_z^2 (1-\cos\beta)^{\cancel{2}}}{I_1' (1+\cos\beta)\cancel{(1-\cos\beta)}} + mgh\cos\beta$$

$$\frac{dU}{d\beta} = \frac{1}{2}\frac{L_z^2}{I_1'} \cdot \frac{(\mathrm{sen}\beta)(1+\cos\beta) + \mathrm{sen}\beta \cdot (1-\cos\beta)}{(1+\cos\beta)^2} + \cancel{mgh\cos\beta} - mgh\,\mathrm{sen}\beta$$

$$= \frac{1}{\cancel{2}}\frac{L_z^2}{I_1'} \cdot \frac{\cancel{2}\,\mathrm{sen}\beta}{(1+\cos\beta)^2} + -mgh\,\mathrm{sen}\beta$$

$$= \frac{L_z^2}{I_1'} \cdot \frac{\mathrm{sen}\beta}{(1+\cos\beta)^2} - mgh\,\mathrm{sen}\beta$$

(siempre es cero para $\beta = 0$).

$$\frac{d^2U}{d\beta^2} = \frac{L_z^2}{I_1'} \cdot \frac{\cos\beta(1+\cos\beta)^2 - \mathrm{sen}\beta\; 2(1+\cos\beta)\cdot(-\mathrm{sen}\beta)}{(1+\cos\beta)^4} - mgh$$

$$= \frac{L_z^2}{I_1'} \cdot \frac{\cos\beta\,(1+\cos\beta)^2 + (1-\cos^2\beta)\cdot 2\,(1+\cos\beta)}{(1+\cos\beta)^4} - mgh$$

$$= \frac{L_z^2}{I_1'} \cdot \frac{\cos\beta(1+\cos\beta)^2 + (1-\cos\beta)(1+\cos\beta)^2\cdot 2}{(1+\cos\beta)^4} - mgh\cos$$

$$= \frac{L_z^2}{I_1'} \cdot \frac{(\cos\beta + 2\cdot(1-\cos\beta))}{(1+\cos\beta)^2} - mgh\cos\beta$$

$$= \frac{L_z^2}{I_1'} \cdot \frac{(-\cos\beta + 2)}{(1+\cos\beta)^2} - mgh\cos\beta$$

$$\Rightarrow \left.\frac{d^2U}{d\beta^2}\right|_{\beta=0} = \frac{L_z^2}{I_1'} \cdot \frac{1}{4} - mgh$$

$$U_{eff}(\beta) \simeq U(0) + \cancel{\left.\frac{\partial U}{\partial \beta}\right|_0}^{\to 0} \cdot \beta + \frac{1}{2}\left.\frac{\partial^2 U}{\partial \beta^2}\right|_0 \cdot \beta^2 .$$

$$U_{eff}'(\beta) = \frac{1}{2}\left.\frac{\partial^2 U}{\partial \beta^2}\right|_0 \cdot \beta^2$$

pequeños oscilaciones $\Rightarrow$ $$\frac{\partial^2 U}{\partial \beta^2} > 0$$

$$\Rightarrow \frac{L_z^2}{I_1'} \cdot \frac{1}{4} - mgh > 0$$

$$\frac{L_z^2}{I_1'} > 4mgh \Rightarrow \boxed{L_z^2 > 4mgh\, I_1'}$$

condición de estabilidad

~~iii)~~ iv) $L \approx \frac{1}{2} I_1' \dot{\beta}^2 - U_{ef}(\beta)$

$$\approx \frac{1}{2} \underbrace{I_1'}_{M} \dot{\beta}^2 - \frac{1}{2} \underbrace{\left(\frac{L_z^2}{I_1'} \cdot \frac{1}{4} - mgh \right)}_{K} \beta^2$$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{L_z^2}{4 I_1'^2} - \frac{mgh}{I_1'}}$$

$$L_z = I_1' \dot{\alpha} \operatorname{sen}^2\beta + I_3' [\dot{\alpha} \cos\beta + \dot{\gamma}] \cos\beta$$

$$L_z(0) = I_3' \dot{\alpha}_0 + I_3' \dot{\gamma}_0$$

$$\omega = \sqrt{\frac{I_3'^2}{I_1'^2 4} \cdot (\dot{\alpha}_0 + \dot{\gamma}_0)^2 - \frac{mgh}{I_1'}} = \sqrt{\left(\frac{I_3'}{I_1'}\right)^2 \left(\frac{\dot{\alpha}_0 + \dot{\gamma}_0}{2}\right)^2}$$

ω será del orden $\dot{\alpha}_0$ y $\dot{\gamma}_0$, según las relaciones entre I_3', I_1, m y h.

Notar que ω se relaciona al promedio entre $\dot{\alpha}_0$ y $\dot{\gamma}_0$.