P.1] Considere una variable $X(t)$ que satisface la ecuación de un oscilador armónico amortiguado. Suponga que $\omega_0 = 1 \left[\frac{rad}{seg}\right]$ y que se tiene las condiciones iniciales $X_0 = 2[m]$; $V_0 = 0 \left[\frac{m}{seg}\right]$

a) Solución si $\eta = \frac{\omega_0}{2}$

b) Lo mismo de a) si $X_0 = 0$ y $V_0 = 50$

Sol]

$$X(t) = A e^{-\eta t} \cos(\omega_d t + \phi)$$

$$\dot{X}(t) = A\left[-\eta e^{-\eta t} \cos(\omega_d t + \phi) - e^{-\eta t} \cdot \omega_d \operatorname{sen}(\omega_d t + \phi)\right]$$

a)

$$X(0) = A\cos(\phi)$$

$$\dot{X}(0) = A\left[-\eta \cos(\phi) - \omega_d \operatorname{sen}(\phi)\right]$$

$$\omega_d = \sqrt{|\eta^2 - \omega_0^2|}$$

$$\omega_0 = 1 \qquad \eta = \frac{1}{2} \longrightarrow \omega_d = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$X(0) = A\cos(\phi) = 2$$

$$\dot{X}(0) = A\left[-\frac{\cos(\phi)}{2} - \frac{1}{\sqrt{2}}\operatorname{sen}(\phi)\right] = 0$$

$$\frac{\operatorname{sen}(\phi)}{\cos(\phi)} = Tg(\phi) = -\frac{\sqrt{2}}{2} \longrightarrow \phi = Tg^{-1}\left(-\frac{\sqrt{2}}{2}\right) \approx -0.61\ldots$$

$$A\cos(\phi) = 2 \rightarrow A = \frac{2}{\cos\left(Tg^{-1}\left(\frac{-\sqrt{21}}{2}\right)\right)} \approx -3{,}625$$

$$X_{(t)} = -3{,}625\, e^{-\frac{t}{2}} \cos\left(\frac{1}{\sqrt{21}} t - 9{,}015\right)$$

b) $$A\cos(\phi) = X_0 = 0 \rightarrow \phi = \frac{\pi}{2}$$

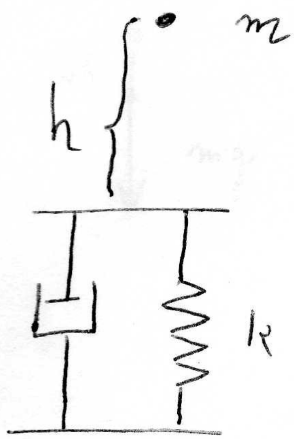
$$A\left[-\cancel{\frac{\cos(\phi)}{2}}^{0} - \frac{1}{\sqrt{21}} \cancel{sen(\phi)}^{1}\right] = 50$$

$$\frac{-A}{\sqrt{21}} = 50 \longrightarrow A = -50\sqrt{21}$$

$$X_{(t)} = -50\sqrt{21}\, e^{-\frac{t}{2}} \cos\left(\frac{t}{\sqrt{21}} + \frac{\pi}{2}\right)$$

P. 2] Se deja caer una masa m de una altura h sobre un sistema de suspensión compuesto por un resorte y un amortiguador. La masa queda adherida al sistema y comienza a oscilar. determine la solución para la oscilación.

R]



La velocidad a la que cae la masa es: $v_0 = \sqrt{2gh}$

y la ecuación de newton $\sum \vec{F} = m\ddot{x}$ queda:

$$m\ddot{x} = -b\dot{x} - kx \rightarrow \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$2\eta = \frac{b}{m} \;;\; \omega_0^2 = \frac{k}{m} \rightarrow \eta = \frac{b}{2m} \;;\; \omega_0 = \sqrt{\frac{k}{m}}$$

y la solución es: $x(t) = A e^{-\frac{b}{2m}t} \cos\left(\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \cdot t + \phi\right)$

con C.I. $x_0 = 0$ y $\dot{x}(0) = \sqrt{2gh}$

$$\dot{x}(t) = A\left[-\frac{b}{2m} e^{-\frac{b}{2m}t} \cos\left(\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} \cdot t + \phi\right) - e^{-\frac{b}{2m}t} \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}\, \text{sen}(\right.$$

$$= \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$$

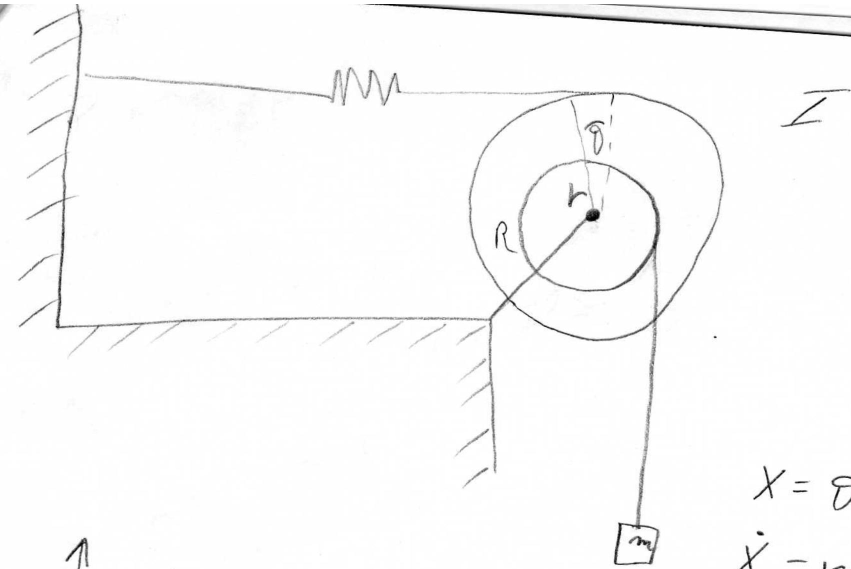
$$X(0) = A\cos(\phi) = 0 \rightarrow \phi = \frac{\pi}{2}$$

$$\dot{X}(0) = A\left[\frac{-b}{2m}\cancel{\cos(\phi)}^{0} - \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}\,\cancel{\operatorname{sen}(\phi)}^{1}\right] = \sqrt{2gh}$$

$$-A\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} = \sqrt{2gh}$$

$$A = -\frac{\sqrt{2gh}}{\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}}$$

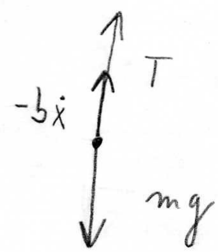
P.5



$x = \theta r$

$\dot{x} = r\dot{\theta}$

$\ddot{x} = r\ddot{\theta}$



$$T - mg - b\dot{x} = m\ddot{x}$$

$$T - mg - br\dot{\theta} = mr\ddot{\theta}$$

$$T = mr\ddot{\theta} + br\dot{\theta} + mg$$

$$-kx'R - T\cdot r = I\ddot{\theta} \qquad x' = \theta R$$

$$I\ddot{\theta} + kR^2\theta + T\cdot r = 0$$

$$I\ddot{\theta} + kR^2\theta + mr^2\ddot{\theta} + br^2\dot{\theta} + mgr = 0$$

$$(I + mr^2)\ddot{\theta} + br^2\dot{\theta} + kR^2\theta + mgr = 0$$

$$\ddot{\theta} + \frac{br^2}{I+mr^2}\dot{\theta} + \underbrace{\frac{kR^2}{I+mr^2}\theta + \frac{mgr}{I+mr^2}}_{\frac{kR^2}{I+mr^2}\left(\theta + \frac{mgr}{kR^2}\right)} = 0$$

$$(0) = A\cos(\phi) = 0 \longrightarrow \boxed{\phi = \frac{\pi}{2}}$$

$$0) = A\left[\frac{-b}{2m}\cancelto{0}{\cos(\phi)} - \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}\,\cancelto{1}{\text{sen}(\phi)}\right] = \sqrt{2gh}$$

$$-A\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} = \sqrt{2gh}$$

$$\boxed{A = \frac{-\sqrt{2gh}}{\sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}}}$$