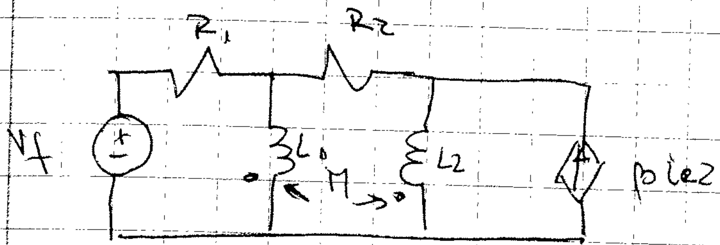
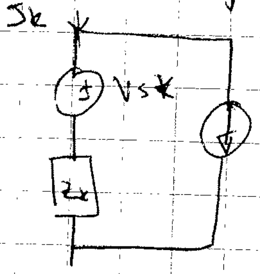
Plantee las ecuaciones de los circuitos regionales



Solución:

① Rama típica (1^er paso



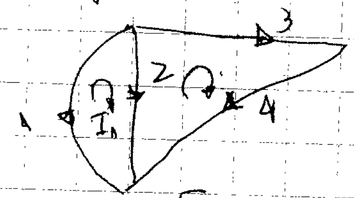
$$V_k = V_{sk} + Z_k(J_k - J_{sk})$$

$$= Z_k J_k + V_{sk} - Z_k J_{sk}$$

$$MV = 0$$

$$J = M^T I$$

② Grafo de la red



matriz $M = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$

las dos corrientes salen por el punto

$$Z_b = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & j\omega L_1 & 0 & j\omega M \\ 0 & 0 & R_2 & 0 \\ 0 & j\omega M & 0 & j\omega L_2 \end{bmatrix}$$

$$V_s = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad J_s = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\beta i_{R2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\beta J_3 \end{bmatrix}$$

$$\Rightarrow Z_n = M Z_b M^T = \begin{bmatrix} R_1 + j\omega L_1 & -j\omega L_1 + j\omega M \\ -j\omega L_1 + j\omega M & j\omega L_1 + j\omega L_2 - 2j\omega M + R_2 \end{bmatrix}$$

ahora:

$M(Z_b J_s - V_s)$

primero $Z_b J_s - V_s = \begin{bmatrix} -V_f \\ -jM\beta J_3 \\ 0 \\ -j\omega L_2 \beta J_3 \end{bmatrix}$

$$\Rightarrow M(Z_b J_s - V_s) = \begin{bmatrix} V_f - j\omega M \beta J_3 \\ jM\beta J_3 - j\omega L_2 J_3 \end{bmatrix}_{2\times 1}$$

$$Mv = M Z_b J + M(V_s - Z_b J_s) = 0$$

$$0 = M Z_b M^T I + M(V_s - Z_b J_s)$$

$$\underbrace{M Z_b M^T}_{Z_n} I = M(Z_b J_s - V_s)$$

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega L_1 + j\omega M \\ -j\omega L_1 + j\omega M & j\omega L_1 + j\omega L_2 - 2j\omega M + R_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \begin{pmatrix} V_f - j\omega M \beta I_2 \\ j\omega M \beta I_2 - j\omega L_2 \beta I_2 \end{pmatrix}$$

$$\Rightarrow \begin{bmatrix} R_1 + j\omega L_1 & -j\omega L_1 + j\omega M + j\omega M\beta \\ -j\omega L_1 + j\omega M & j\omega L_1 + j\omega L_2 - 2j\omega M - j\omega M\beta + j\omega L_2\beta + R_2 \end{bmatrix}$$

$$\cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{pmatrix} V_f \\ 0 \end{pmatrix}$$