$$\gamma = \frac{T}{\bar{T}} \; ; \quad \bar{T} = \frac{2\pi}{\bar{\omega}} \Rightarrow \gamma = \frac{T \cdot \bar{\omega}}{2\pi} = \frac{0{,}5\ \text{seg} \cdot 15\ \text{rad/seg}}{2\pi} = 1{,}194$$

- Usando la expresión del problema anterior: $(D=1)$. $\left[1 = \frac{1}{\sqrt{(1-\gamma^2)^2 + (2\beta\gamma)^2}} \right]$

$$\Rightarrow \gamma^2 + (2\beta)^2 = 2$$

$$\Rightarrow \gamma = \sqrt{2 - 4\beta^2} \qquad \Rightarrow (2\beta)^2 = 2 - \gamma^2$$

$$\Rightarrow 2\beta = \sqrt{2 - \gamma^2}$$

$$\Rightarrow \beta = \frac{\sqrt{2 - \gamma^2}}{2}$$

$$\Rightarrow \beta = \frac{\sqrt{2 - (1{,}194)^2}}{2} = 0{,}379 \; ; \quad \beta = 37{,}9\ \%.$$

$$\beta = \frac{c^*}{2 I \omega_n} \Rightarrow \frac{c\ell^2}{2 I \omega_n} = 0{,}379 = \frac{c\ell^2}{2\sqrt{I \cdot k^*}}$$

$$\Rightarrow c = \frac{0{,}379 \cdot 2 \cdot \sqrt{17{,}96\ \text{kgf m s}^2 \cdot 2836{,}7\ \text{kgf} \cdot \text{m}}}{(0{,}5\ \text{m})^2} = 684{,}4\ \frac{\text{kgf s}}{\text{m}}$$

- Ecuación de movimiento:

$$\underbrace{\left(\frac{4}{3} \frac{w_n \ell^2}{g} + \frac{51}{8} \frac{w_2 \ell^2}{g} \right)}_{m^*} \ddot{\theta}(t) + \underbrace{c\ell^2}_{c^*} \dot{\theta}(t) + \underbrace{\left(\frac{6 + 4\sqrt{2}}{12\ell} \right)}_{k^*} EI\, \theta(t) = \pm Z m_0 \ell \operatorname{sen}(\bar{\omega} t)$$

↑ Depende de la convención de $\theta(t)$

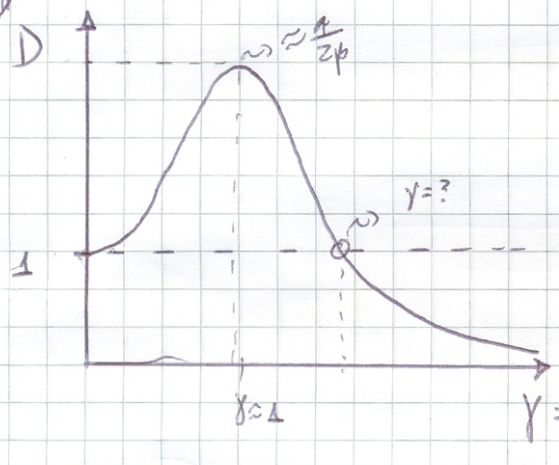
Ec. de movimiento

como $D = 1 \Rightarrow \theta_{max} = \theta_{est} = \frac{Z m_0 \ell}{k^*} = \frac{2 \cdot 100\ \text{kgf m/m} \cdot 0{,}5\ \text{m}}{2836{,}7\ \text{kgf} \cdot \text{m}} = 0{,}035\ \text{rad}$

$= 2^\circ$

Ejercicio Nº 2 CI 426

P1



Factor de Amplificación Dinámico.

$$\Rightarrow x(t) = \frac{P_0}{K} \cdot D \cdot \operatorname{sen}(\bar{\omega} t - \phi)$$

$$D = \frac{1}{\sqrt{(1-\gamma^2)^2 + (2\beta\gamma)^2}}$$

Se busca la región Tq $D = 1$. (Relación de frecuencia γ tq no existe Amplificación Dinámica).

$$\Rightarrow D = 1.$$

$$\Rightarrow 1 = \frac{1}{\sqrt{(1-\gamma^2)^2 + (2\beta\gamma)^2}} \Rightarrow (1-\gamma^2)^2 + (2\beta\gamma)^2 = 1$$

$$\Rightarrow 1 - 2\gamma^2 + \gamma^4 + 4\beta^2\gamma^2 = 1$$

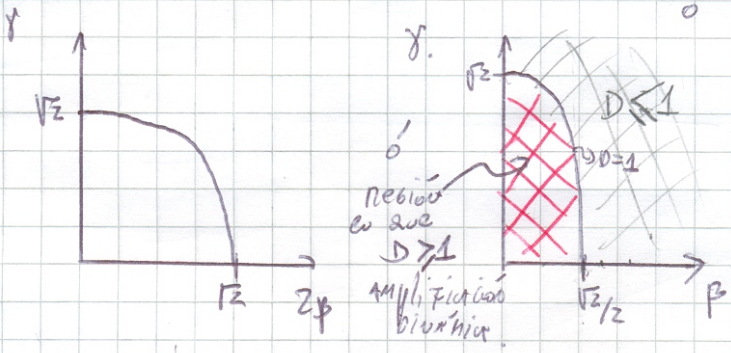
No tiene que una solución trivial es $\gamma = 0$. $\forall \beta$.

$$\Rightarrow \gamma^4 + (4\beta^2 - 2)\gamma^2 = 0 \quad \gamma \text{ v/s } 2\beta$$

$$\Rightarrow \gamma^2 + (2\beta)^2 = 2 \Rightarrow \text{círculo de radio } \sqrt{2}.$$

ó elipse con γ y β.

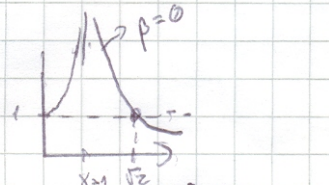
$\Rightarrow$



Cuando $D = 1$.

ojo: $\gamma = \sqrt{2 - 4\beta^2}$

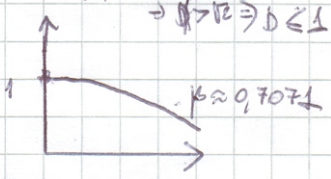
Luego: $\gamma \geq \sqrt{2} \Rightarrow D \leq 1 \quad \forall \beta$. ej.



$\Rightarrow \gamma > \sqrt{2} \Rightarrow D < 1$

$$\beta \geq \sqrt{2}/2 \Rightarrow D \leq 1 \quad \forall \gamma \Rightarrow$$

$\approx 0{,}7071$



Ejemplos:

$\beta = 0{,}05 \Rightarrow \gamma = 1{,}4107 < 1{,}4142$

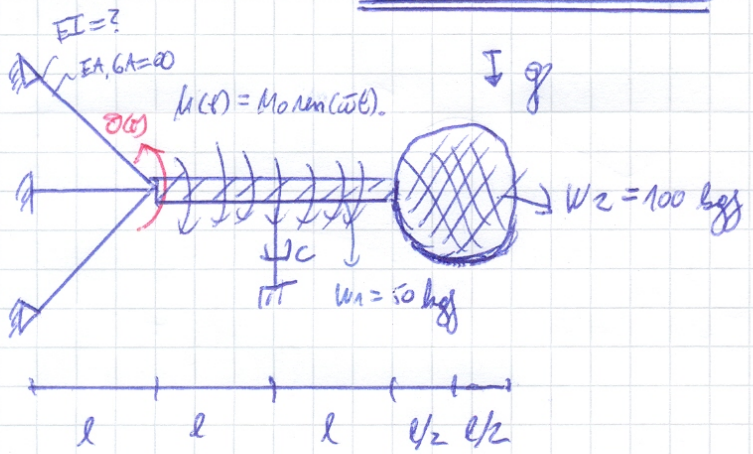
$\beta = 0{,}1 \Rightarrow \gamma = 1{,}40$

$\beta = 0{,}2 \Rightarrow \gamma = 1{,}3565$

$\beta = 0{,}4 \Rightarrow \gamma = 1{,}1662$

$\beta = 0{,}6 \Rightarrow \gamma = 0{,}7483$

Ejercicio 2 CI426



$g = 9{,}8\ m/s^2$

$W_1 = 50\ kgf$

$W_2 = 100\ kgf$

$m_0 = 100\ kgf\,m/m$

$l = 50\ cm$

$\omega = 15\ rad/seg.$

$$I = \frac{1}{3}\bar{m}(2l)^3 + \frac{\pi}{2}\left(\frac{l}{2}\right)^4 \gamma + \gamma\pi\left(\frac{l}{2}\right)^2 \cdot \left(\frac{5}{2}l\right)^2$$

$$\bar{m}\,2l = \frac{W_1}{g} \qquad \gamma\pi\left(\frac{l}{2}\right)^2 = \frac{W_2}{g}$$

$$\Rightarrow I = \frac{4}{3}\frac{W_1 l^2}{g} + \frac{W_2 l^2}{8g} + \frac{25}{4}\frac{W_2 l^2}{g} = \frac{4}{3}\frac{W_1 l^2}{g} + \frac{51}{8}\frac{W_2 l^2}{g}$$

$$I = \frac{4}{3}\cdot\frac{50\ kgf\cdot(0{,}5\ m)^2}{9{,}8\ m/s^2} + \frac{51}{8}\cdot\frac{100\ kgf\cdot(0{,}5\ m)^2}{9{,}8\ m/s^2} = 17{,}96\ kgf\,m\,s^2$$

$$k^* = \frac{3EI}{\sqrt{2}l}\times 2 + \frac{4EI}{l} = EI\cdot\left(\frac{6+4\sqrt{2}}{\sqrt{2}\,l}\right)$$

- $T = 0{,}5\ seg \Rightarrow 0{,}5\ seg = 2\pi\cdot\sqrt{\frac{I}{k^*}} \Rightarrow k^* = \left(\frac{2\pi}{0{,}5\ seg}\right)^2 \cdot I = 2836{,}7\ kgf\cdot m$

$$\Rightarrow EI = \left(\frac{\sqrt{2}\,l}{6+4\sqrt{2}}\right)\cdot\left(\frac{2\pi}{T}\right)^2 I$$

$$EI = \left(\frac{\sqrt{2}\,(0{,}5\ m)}{6+4\sqrt{2}}\right)\cdot\left(\frac{2\pi}{0{,}5\ seg}\right)^2 \cdot 17{,}96\ kgf\cdot m\cdot s^2 = 172{,}07\ kgf\,m^2$$

$$\Rightarrow K = 2836{,}7\ kgf\cdot m \quad \text{OK } \mathcal{N}.$$