## •••ALGEBRAIC OPERATIONS FOR VECTORS AND TENSORS IN CARTESIAN COORDINATES

($s$ is a scalar; **v** and **w** are vectors; $\boldsymbol{\tau}$ is a tensor; dot or cross operations enclosed within parentheses are scalars, those enclosed in brackets are vectors)

$$(\mathbf{v}\cdot\mathbf{w}) = v_x w_x + v_y w_y + v_z w_z = (\mathbf{w}\cdot\mathbf{v})$$

$$[\mathbf{v}\times\mathbf{w}]_x = v_y w_z - v_z w_y = -[\mathbf{w}\times\mathbf{v}]_x$$

$$[\mathbf{v}\times\mathbf{w}]_y = v_z w_x - v_x w_z = -[\mathbf{w}\times\mathbf{v}]_y$$

$$[\mathbf{v}\times\mathbf{w}]_z = v_x w_y - v_y w_x = -[\mathbf{w}\times\mathbf{v}]_z$$

$$[\boldsymbol{\tau}\cdot\mathbf{v}]_x = \tau_{xx}v_x + \tau_{xy}v_y + \tau_{xz}v_z \qquad [\mathbf{v}\cdot\boldsymbol{\tau}]_x = v_x\tau_{xx} + v_y\tau_{yx} + v_z\tau_{zx}$$

$$[\boldsymbol{\tau}\cdot\mathbf{v}]_y = \tau_{yx}v_x + \tau_{yy}v_y + \tau_{yz}v_z \qquad [\mathbf{v}\cdot\boldsymbol{\tau}]_y = v_x\tau_{xy} + v_y\tau_{yy} + v_z\tau_{zy}$$

$$[\boldsymbol{\tau}\cdot\mathbf{v}]_z = \tau_{zx}v_x + \tau_{zy}v_y + \tau_{zz}v_z \qquad [\mathbf{v}\cdot\boldsymbol{\tau}]_z = v_x\tau_{xz} + v_y\tau_{yz} + v_z\tau_{zz}$$

*Note:* The above operations may be generalized to cylindrical coordinates by replacing $(x, y, z)$ by $(r, \theta, z)$, and to spherical coordinates by replacing $(x, y, z)$ by $(r, \theta, \phi)$. Descriptions of curvilinear coordinates are given in Figures 1.2-2, A.6-1, A.8-1, and A.8-2.

## •••DIFFERENTIAL OPERATIONS FOR SCALARS, VECTORS, AND TENSORS IN CARTESIAN COORDINATES

$$[\nabla s]_x = \frac{\partial s}{\partial x} \qquad [\nabla s]_y = \frac{\partial s}{\partial y} \qquad [\nabla s]_z = \frac{\partial s}{\partial z}$$

$$[\nabla\times\mathbf{v}]_x = \frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \qquad [\nabla\times\mathbf{v}]_y = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \qquad [\nabla\times\mathbf{v}]_z = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}$$

$$(\nabla\cdot\mathbf{v}) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \qquad (\mathbf{v}\cdot\nabla s) = v_x\frac{\partial s}{\partial x} + v_y\frac{\partial s}{\partial y} + v_z\frac{\partial s}{\partial z}$$

$$\nabla^2 s \equiv (\nabla\cdot\nabla s) = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$[\nabla^2 \mathbf{v}]_x \equiv [\nabla \cdot \nabla \mathbf{v}]_x = \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2}$$

$$[\nabla^2 \mathbf{v}]_y \equiv [\nabla \cdot \nabla \mathbf{v}]_y = \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}$$

$$[\nabla^2 \mathbf{v}]_z \equiv [\nabla \cdot \nabla \mathbf{v}]_z = \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}$$

$$[\mathbf{v} \cdot \nabla \mathbf{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \mathbf{v}\mathbf{v}]_x = \frac{\partial (v_x v_x)}{\partial x} + \frac{\partial (v_y v_x)}{\partial y} + \frac{\partial (v_z v_x)}{\partial z}$$

$$[\nabla \cdot \mathbf{v}\mathbf{v}]_y = \frac{\partial (v_x v_y)}{\partial x} + \frac{\partial (v_y v_y)}{\partial y} + \frac{\partial (v_z v_y)}{\partial z}$$

$$[\nabla \cdot \mathbf{v}\mathbf{v}]_z = \frac{\partial (v_x v_z)}{\partial x} + \frac{\partial (v_y v_z)}{\partial y} + \frac{\partial (v_z v_z)}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\begin{aligned}(\boldsymbol{\tau} : \nabla \mathbf{v}) = {} & \tau_{xx} \frac{\partial v_x}{\partial x} + \tau_{xy} \frac{\partial v_x}{\partial y} + \tau_{xz} \frac{\partial v_x}{\partial z} \\ & + \tau_{yx} \frac{\partial v_y}{\partial x} + \tau_{yy} \frac{\partial v_y}{\partial y} + \tau_{yz} \frac{\partial v_y}{\partial z} \\ & + \tau_{zx} \frac{\partial v_z}{\partial x} + \tau_{zy} \frac{\partial v_z}{\partial y} + \tau_{zz} \frac{\partial v_z}{\partial z}\end{aligned}$$

*Note:* the differential operations may *not* be simply generalized to curvilinear coordinates; see Tables A.7-2 and A.7-3.