

$$1) \mu(x, y) = \mu(x) \quad \mu(\text{fancy}(x_1))$$

$$\Rightarrow \frac{\partial \mu}{\partial y} = 0 \quad \text{and} \quad \frac{\partial \mu}{\partial x} = \frac{d\mu}{dx}$$

$$N \cdot \frac{\partial \mu}{\partial x} = N \cdot \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right)$$

$$\frac{1}{e^{\mu}} \frac{d\mu}{dx} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = g(x)$$

$$N(x) = C \cdot e^{\int g(x) dx}$$

$$\mu(x, y) = \mu(y)$$

$$\Rightarrow \frac{\partial \mu}{\partial x} = 0, \quad \frac{\partial \mu}{\partial y} = \frac{d\mu}{dy}$$

$$-M \frac{\partial \mu}{\partial y} = \mu \cdot \left(\frac{\partial M}{\partial y} - \frac{\partial \mu}{\partial x} \right)$$

$$\left(\frac{1}{\mu} \cdot \frac{\partial \mu}{\partial y} \right) = \left(\frac{\frac{\partial M}{\partial y} - \frac{\partial \mu}{\partial x}}{-M} \right) = h(y)$$

$$\Rightarrow \mu(y) = \phi \cdot e^{\int h(y) dy}$$

$$(x+y)dx + x \log x dy = 0$$

$$1) \frac{\partial M}{\partial y} = 1 \quad \frac{\partial N}{\partial x} = \log x + 1$$

$\Rightarrow$ it is not exact.

$$2) N = N(x)$$

$$\frac{1}{N} \frac{\partial N}{\partial x} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{-\log x}{x \log x} = -\frac{1}{x} = -\frac{1}{x}$$

$$N = 1 \cdot e^{\int -\frac{1}{x} dx} = e^{-\ln x} = e^{\ln x^{-1}} = \frac{1}{x} = f(x)$$

$$\overbrace{\left(1 + \frac{y}{x}\right)}^M dx + \overbrace{\log x}^N dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{1}{x} \quad \frac{\partial N}{\partial x} = \frac{1}{x} \Rightarrow \text{exact!}$$

$$\Rightarrow \frac{\partial E}{\partial x} = \textcircled{1} M, \quad \frac{\partial E}{\partial y} = \textcircled{2} N, \quad E(x, y) = \textcircled{3} C$$

$$\Rightarrow E(x, y) = y \cdot \log x + g(x)$$

$$\textcircled{1} \quad \cancel{\frac{y}{x}} + g'(x) = 1 + \cancel{\frac{y}{x}} \Rightarrow g(x) = x$$

$$\Rightarrow \textcircled{3} \quad E(x, y) = \underline{y \log x + x = C}$$

$$\left[2x - y \sin xy + k y^4 \right] dx + \left[-20xy^3 - x \sin xy \right] dy$$

Exact?

$$\frac{\partial}{\partial y} (2x - y \sin xy + k y^4) = -\sin xy - y \cos xy + 4k y^3$$

$$\frac{\partial}{\partial x} (-20xy^3 - x \sin xy) = -20y^3 - \sin xy - xy \cos xy$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow k = -5$$

$$C = -5xy^4 + \cos xy + (-y)$$

$$\mu(x, y) = \mu(x \cdot y) \quad z = x \cdot y$$

$$\mu(x, y) = \mu(z)$$

$$\frac{\partial \mu}{\partial x} = \frac{\partial \mu}{\partial z} \cdot \frac{\partial z}{\partial x} = \frac{\partial \mu}{\partial z} \cdot y$$

$$\frac{\partial \mu}{\partial y} = \frac{\partial \mu}{\partial z} \cdot \frac{\partial z}{\partial y} = \frac{\partial \mu}{\partial z} \cdot x$$

$$\Rightarrow \text{em EDP} \quad \mu \cdot \left(\frac{\partial \mu}{\partial y} - \frac{\partial \mu}{\partial x} \right) = \\ = (\mu y - \mu x) \frac{\partial \mu}{\partial z}$$

$$\frac{1}{N} \cdot \frac{\partial N}{\partial z} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{Ny - Mx} = g(z)$$

$$\Rightarrow N = c \cdot e^{\int g(z) dz} = N(x, y)$$

$$N(x^m y^m)$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{y^{m-1} \cdot x^{m-1} (m y N - m x M)} = g(x^m y^m)$$

$z = x^m y^m$

$$\frac{\partial N}{\partial x} = \frac{\partial N}{\partial z} \cdot m x^{m-1} y^m$$

$$\frac{\partial N}{\partial y} = \frac{\partial N}{\partial z} \cdot m x^m y^{m-1}$$

$$\frac{1}{N} \cdot \frac{\partial N}{\partial z} = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N \cdot m x^{m-1} y^m - M m x^m y^{m-1}}$$

$$\underbrace{(5x^2y + 6x^3y^2 + 4xy^2)}_M dx + \underbrace{(2x^3 + 3x^4y + 3x^2y)}_N dy = 0$$

$$\frac{\partial M}{\partial y} = 5x^2 + 12x^3y + 8xy$$

$$\frac{\partial N}{\partial x} = 6x^2 + 12x^3y + 6xy$$

$$g(z) = \frac{-x^2 + 2xy}{y^{m-1}x^{m-1}(my)}$$

$$(5x^2y + 6x^3y^2 + 4xy^2)dx + (2x^3 + 3x^4y + 3x^2y)dy = 0 \quad / \cdot x^m y^m$$

$$\frac{\partial M}{\partial y} = \frac{\partial (5x^{m+2}y^{m+1} + 6x^{m+3}y^{m+2} + 4x^{m+1}y^{m+2})}{\partial y}$$

$$= (m+1)5x^{m+2}y^m + 6(m+2)x^{m+3}y^{m+1} + 4(m+2)x^{m+1}y^{m+1}$$

$$\frac{\partial N}{\partial x} = 2(m+3)x^{m+2}y^m + 3(m+4)x^{m+3}y^m + 3(m+2)x^{m+1}y^{m+1}$$

$$4(m+2) = 2(m+3)$$

$$\Rightarrow 5(m+1) = 2(m+3) \wedge 6(m+2) = 3(m+4)$$