(3) sea $x_n = (x_{1n}, x_{2n})$ tal que $\lim_{n\to\infty} x_n = 0$

(a) $$\lim_{x\to 0} \frac{\sin\sqrt{x_1^2+x_2^2}}{\sqrt{x_1^2+x_2^2}} = \lim_{n\to\infty} \frac{\sin\sqrt{x_{1n}^2+x_{2n}^2}}{\sqrt{x_{1n}^2+x_{2n}^2}} = \lim_{n\to\infty} \frac{\sin h_n}{h_n} = 1$$

donde $h_n = \sqrt{x_{1n}^2+x_{2n}^2} \to 0$ 1.0

(b) $$\frac{\partial}{\partial x_1} f(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{\sin|h|}{|h|}-1}{h}$$

Demostraremos que: $\lim_{t\to 0} \frac{\frac{\sin t}{t}-1}{t} = 0$ $\left(\Rightarrow \frac{\partial f}{\partial x_1}(0,0) = 0\right)$

uno puede ocupar desigualdad:

$$|\sin t - t| \le C|t|^3 \Rightarrow \left|\frac{\frac{\sin t}{t}-1}{t}\right| \le C|t| \to 0$$

o usar l'Hospital:

$$\lim_{t\to 0}\left(\frac{\frac{\sin t}{t}-1}{t}\right) = \lim_{t\to 0}\left(\frac{\sin t - t}{t^2}\right) = \lim_{t\to 0}\left(\frac{\cos t - 1}{2t}\right)$$

$$= \lim_{t\to 0} \frac{\sin t}{2} = 0$$

De mismo modo uno demuestre $\frac{\partial f}{\partial x_2}(0,0) = 0$. 2.0

(c) si $(x_1,x_2) \neq (0,0) \Rightarrow \frac{\partial f}{\partial x_1}(x) = \frac{x_1 \cos\sqrt{x_1^2+x_2^2}}{x_1^2+x_2^2}$

$$- x_1 \frac{\sin\sqrt{x_1^2+x_2^2}}{(x_1^2+x_2^2)^{3/2}}$$

Calcularemos $\lim_{x\to 0} \frac{\partial f}{\partial x_1}(x) = \lim_{x\to 0} \left[\frac{x_1 \cos\sqrt{x_1^2+x_2^2}}{x_1^2+x_2^2} - x_1\frac{\sin\sqrt{x_1^2+x_2^2}}{(x_1^2+x_2^2)^{3/2}}\right.$

$$= \lim_{x \to 0} x_1 \left[\frac{\sqrt{x_1^2+x_2^2} \cos\sqrt{x_1^2+x_2^2} - \sin\sqrt{x_1^2+x_2^2}}{(x_1^2+x_2^2)^{3/2}} \right]$$

Sea $h = \sqrt{x_1^2+x_2^2}$, usando l'Hospital

$$\lim_{h\to 0} \left(\frac{h\cosh - \sinh}{h^{3/2}} \right) = \lim_{h \to 0} \frac{-h\sinh + \cosh - \cosh}{\frac{3}{2} h^{1/2}}$$

$$= \lim_{h\to 0} -\frac{2h^{1/2}}{3} \sinh = 0$$

$\Rightarrow \frac{\partial f}{\partial x_1}$ continua en $(0,0)$ y tambien en todos otros puntos (facil)

$\frac{\partial f}{\partial x_2}$ satisface lo mismo (2.0)

(d) $\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}$ son continuas $\forall x \in \mathbb{R}^2 \Rightarrow f$ es diferenciable $\forall x \in \mathbb{R}^2$. (1.0)