OVERVIEW OF ANALYTICAL AND EXPERIMENTAL MODAL MODEL CORRELATION TECHNIQUES

Peter Avitabile

MODAL MODEL CORRELATION TECHNIQUES

Peter Avitabile
11 March 1998
D R A F T

ABSTRACT

Correlation of analytical and experimental modal models for dynamic applications as well as the correction of analytical models to better reflect the actual structural dynamic system has become increasingly important in many engineering analyses for a wide variety of applications. Much work has been expended in the area of model reduction and model expansion to further this cause. Also, correlation tools have been developed to assist in the process. However, clear identification of where discrepancies exist is not always possible with existing tools.

The thrust of this work is to further define correlation tools that help better identify where discrepancies exist between the analytical and experimental data bases. In addition, some work is also presented to provide additional tools that assist in the evaluation of test measurement locations that may be critical to the success of the correlation process.

TABLE OF CONTENTS

CHAPTER 3

GENERAL MODEL REDUCTION TECHNIQUES

PREFACING REMARKS

This chapter presents some of the basic approaches to the reduction of finite element models from a general standpoint. Model reduction is typically performed to obtained a reduced model for efficiency purposes for other structural dynamic applications such as forced response analysis and component model synthesis techniques. However, as used for this work, model reduction is specifically used to form a mapping between the very large set of finite element degrees of freedom and the relatively small set of tested degrees of freedom.

GENERAL REDUCTION TECHNIQUES

Model reduction is generally performed to reduce the size of a large analytical model to develop a more efficient model for further analytical studies such as substructuring or forced response studies. Most reduction or condensation techniques affect the dynamic character of the resulting reduced model in the reduction process. Model reduction is performed for a number of reasons but here we are primarily interested in reduction as a mapping technique. A schematic of the reduction process is shown in Figure 1

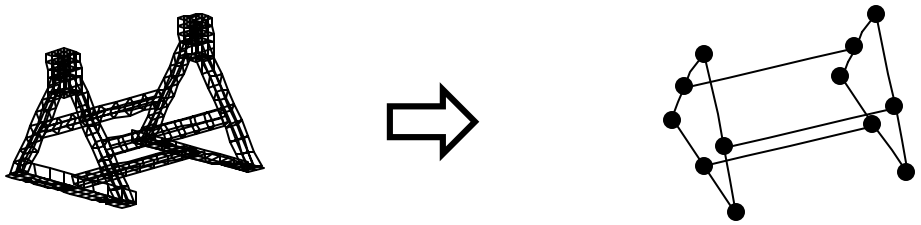


Figure 3-1 - Schematic of Reduction Process

In general we can write a relationship between the full set of analytical or finite element dof and the reduced set of active or condensed dof as

$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\} \qquad \text{or} \qquad \{x_1\} = [T_{12}]\{x_2\} \tag{3-1}$$

The 'n' subscript denotes the full set of analytical dofs, the 'a' subscript denotes the active set of dof (sometimes referred to as master dof and for correlation studies referred to as test dof) and the subscript 'd' denotes the deleted dof (sometimes referred to as embedded or omitted dof);

the [T] transformation relates the transformation between these two sets of dofs. (Note: that the subscript 1 and 2 are also sometimes ued to denote 'state 1' and 'state 2')

Since the energy of the system needs to be perserved then we can write a balance between the energy at state 1 and state 2 as

$$U = \frac{1}{2}\{x_1\}^T[K_1]\{x_1\} = \frac{1}{2}\{x_2\}^T[K_2]\{x_2\} \tag{3-2}$$

Substituting the transformation equation yields

$$U = \frac{1}{2}\{[T_{12}]\{x_2\}\}^T[K_1]\{[T_{12}]\{x_2\}\} = \frac{1}{2}\{x_2\}^T[K_2]\{x_2\} \tag{3-3}$$

and rearranging some terms gives

$$U = \frac{1}{2}\{x_2\}^T[T_{12}]^T[K_1][T_{12}]\{x_2\} = \frac{1}{2}\{x_2\}^T[K_2]\{x_2\} \tag{3-4}$$

Therefore, we can see that the reduced stiffness is related to the original stiffness as

$$[K_2] = [T_{12}]^T[K_1][T_{12}] \quad \text{or} \quad [K_a] = [T]^T[K_n][T] \tag{3-5}$$

Likewise for the system mass matrix we can write

$$[M_a] = [T]^T[M_n][T] \tag{3-6}$$

The [T] transformation can be a variety of different matrices depending on the transformation technique utilized.

Once these new mass and stiffness matrices are available in 'a' space then the equation of motion becomes

$$[M_a]\{\ddot{x}_a\} + [K_a]\{x_a\} = \{F_a(t)\} \tag{3-7}$$

with a corresponding eigensolution given as

$$[[K_a] - \lambda[M_a]]\{x_a\} = \{0\} \tag{3-8}$$

Depending on the reduction scheme utilized, the eigenvalues of the reduced system will generally

be greater than or at most equal to the eigenvalues of the full system.

Guyan Condensation

For a static system, the equation of motion can be written in partitioned form as

$$\begin{bmatrix} [K_{aa}] & [K_{ad}] \\ [K_{da}] & [K_{dd}] \end{bmatrix} \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = \begin{Bmatrix} F_a \\ F_d \end{Bmatrix} \tag{3-9}$$

again the 'a' subscript denotes the master or active set of dof and the 'd' subscript denotes the embedded or deleted dof.

Assuming that the forces on the deleted dof are zero, then the second equation can be expanded as

$$[K_{da}]\{x_a\} + [K_{dd}]\{x_d\} = \{0\} \tag{3-10}$$

which can be solved for the displacement at the deleted dof as

$$\{x_d\} = -[K_{dd}]^{-1}[K_{da}]\{x_a\} \tag{3-11}$$

The first equation of the partitioned set can be expanded as

$$[K_{aa}]\{x_a\} + [K_{ad}]\{x_d\} = \{F_a\} \tag{3-12}$$

Upon substitution of the displacement at the deleted dof

$$[K_{aa}]\{x_a\} + [K_{ad}][K_{dd}]^{-1}[K_{da}]\{x_a\} = \{F_a\} \tag{3-13}$$

Therefore, a relationship is available relating the the active dof to the full set of dof as

$$\{x_n\} = \begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix} \{x_a\} = [T_s]\{x_a\} \tag{3-14}$$

Using this transformation matrix, we can write the reduced system stiffness matrix as

$$\left[K_a^G\right] = [T_s]^T[K_n][T_s] \tag{3-15}$$

This transformation is exact in the static sense.

Both Guyan and Irons proposed that the same system transformation matrix used to modify the stiffness matrix be used to modify the system mass matrix

$$\left[M_a^G\right] = \left[T_s\right]^T \left[M_n\right] \left[T_s\right] \tag{3-16}$$

This transformation attempts to convert the system mass to the set of active dofs. However, since this technique is based soley on the static stiffness of the system, there is no guarantee that the reduced matrix will be accurate for dynamic applications.

The solution of the reduced problem will contain eigenvalues and eigenvectors that are similar to the eigenvalues and eigenvectors of the full system model. The degree of sililarity is heavily dependent on the selection of the set of adof - both the total number of dof as well as the distribution of the dof. In general the relative difference increases as the mode number increases with the lower order modes generally having less discrepancy than the higher order modes.

Improved Reduced System

As an extension of the Guyan reduction process, the Improved Reduced System (IRS) attempts to account for some of the effects of the deleted dofs that cause distortion in the Guyan reduction process. The development is based off of the fact that the static structural model containing distributed forces can be condensed producing a reduced system and solution. The displacements of the reduced system are then expanded and adjusted for the deleted forces producing an exact statical solution of the complete system. A first order approximation of the eigensystem is formed using a Guyan/Irons reduced model approach which is based on the static condensation process with no adjustment for the deleted distributed inertia forces. The modal vectors of the approximated solution can be adjusted in a similar fashion as in the static solution producing an improved set of eigenvectors. Finally an estimate of the transformation matrix from full space to reduced space can be formed for the IRS system. The resulting equations are summarized below but are not detailed herein

$$\left[T_i\right] = \begin{bmatrix} [I] \\ [t_s] \end{bmatrix} + \left[t_i\right] \tag{3-17}$$

where

$$\left[t_s\right] = -\left[K_{dd}\right]^{-1}\left[K_{da}\right] \quad ; \quad \left[t_i\right] = \begin{bmatrix} [0] & [0] \\ [0] & \left[K_{dd}^{-1}\right] \end{bmatrix} \left[M_n\right]\left[T_s\right]\left[M_a\right]^{-1}\left[K_a\right] \tag{3-18}$$

The IRS technique generally produces better approximations of the reduced eigensystem when compared to the Guyan/Irons approach since an estimate of the inertia associated with the deleted dof is developed as part of the reduction process. While IRS is useful as a model

reduction technique, expansion and correlation studies usually do not employ this method due to other much more accurate techniques.

Dynamic Condensation

A dynamic implementation of the Guyan reduction process is the Dynamic Condensation process which is often used in correlation studies, in particular for expansion of mode shapes.

Let's introduce a shift value, f, into the set of equation describing the dynamic system as

$$\left[\left[K_n\right]-(\lambda-f)\left[M_n\right]\right]\left\{x_n\right\}=\left\{0\right\} \tag{3-19}$$

and rearrange terms to group the constant term f times the mass matrix with the stiffness matrix to yield

$$\left[\left[\left[K_n\right]+f\left[M_n\right]\right]-\lambda\left[M_n\right]\right]\left\{x_n\right\}=\left\{0\right\} \tag{3-20}$$

and now let

$$\left[D_n\right]=\left[K_n\right]+f\left[M_n\right] \tag{3-21}$$

Using the same approach as done with Guyan condensation, these equations can be written in partitioned form as

$$\begin{bmatrix}\left[D_{aa}\right] & \left[D_{ad}\right]\\ \left[D_{da}\right] & \left[D_{dd}\right]\end{bmatrix}\begin{Bmatrix}x_a\\ x_d\end{Bmatrix}=\begin{Bmatrix}F_a\\ F_d\end{Bmatrix} \tag{3-22}$$

Assuming that the forces on the deleted dof are zero, then the second equation of the partitioned set can be expanded as

$$\left[D_{da}\right]\left\{x_a\right\}+\left[D_{dd}\right]\left\{x_d\right\}=\left\{0\right\} \tag{3-23}$$

which can be solved as

$$\left\{x_d\right\}=-\left[D_{dd}\right]^{-1}\left[D_{da}\right]\left\{x_a\right\} \tag{3-24}$$

The first equation of the partitioned set can be expanded as

$$\left[D_{aa}\right]\left\{x_a\right\}+\left[D_{ad}\right]\left\{x_d\right\}=\left\{F_a\right\} \tag{3-25}$$

Upon substitution

$$[D_{aa}]\{x_a\} + [D_{ad}][D_{dd}]^{-1}[D_{da}]\{x_a\} = \{F_a\} \tag{3-26}$$

Therefore, the relationship between the active dofs and the full set of analytical dofs can be written as

$$\{x_n\} = \begin{bmatrix} [I] \\ -[D_{dd}]^{-1}[D_{da}] \end{bmatrix} \{x_a\} = [T_f]\{x_a\} \tag{3-27}$$

So we can write the reduced mass and stiffness matrices as

$$\left[K_a^f\right] = [T_f]^T[K_n][T_f] \tag{3-28}$$

$$\left[M_a^f\right] = [T_f]^T[M_n][T_f] \tag{3-29}$$

Due to the formulation of the dydnamic condensation process, the eigensolution of the reduced matrices will result in on eigenvalue which will correspond to the shift value used for the reduction process. If the shift value happens to correspond exactly to one of the eigenvalues of the system, then this eigenvalue will be preserved accurately in the reduced mode and will also produce and expanded eigenvector which will be exactly the same as the corresponding eigenvector from the full finite element model relating to the shifted eigenvalue. None of the other eigenvalues will correspond to any of the eigenvalues of the full system.

System Equivalent Reduction Expansion Process (SEREP)

As done with the other reduction schemes, there is a relationship between the tested or active 'a' dof and the deleted 'd' dof which can be written in general form as

$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\} \tag{3-30}$$

The modal transformation can be rewritten using this notation as

$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = \begin{bmatrix} U_a \\ U_d \end{bmatrix} \{p\} \tag{3-31}$$

Notice that the modal matrix is also partitioned into the 'a' active and 'd' deleted set of degrees of freedom. Looking at just the relationship for the 'a' set of degrees of freedom, we can write

$$\{x_a\} = [U_a]\{p\} \tag{3-32}$$

The inverse specification of this equation involves a generalized inverse since the number of unkowns is not equal to the number of equations need to be solved. There are two possible solutions to this situation

- when the number of equations 'a' are greater than or equal to the number of solution variables 'm' (an overspecification or equivalence of the system)
- when the number of equations 'a' are less than the number of solution variables 'm' (an underspecification of the system)

Least Squares Solution - $a \geq m$

$$\{x_a\} = [U_a]\{p\}$$
$$[U_a]^T\{x_a\} = [U_a]^T[U_a]\{p\}$$
$$\left([U_a]^T[U_a]\right)^{-1}[U_a]^T\{x_a\} = \left([U_a]^T[U_a]\right)^{-1}[U_a]^T[U_a]\{p\}$$
$$\{p\} = \left([U_a]^T[U_a]\right)^{-1}[U_a]^T\{x_a\} = [U_a]^g\{x_a\}$$

Average Solution - $a \langle m$

$$\{p\} = [U_a]^T\left([U_a]^T[U_a]\right)^{-1}\{x_a\} = [U_a]^g\{x_a\}$$

For most structural dynamic applications in dynamic testing, the least sqaures solution is used since the number of master dof (or tested dof) is far greater than the number of modes in the system, then the generalized inverse is

$$\{p\} = \left([U_a]^T[U_a]\right)^{-1}[U_a]^T\{x_a\} = [U_a]^g\{x_a\} \tag{3-33}$$

This equation for the modal displacement can be substituted into the modal transformation eqaution to give

$$\{x_n\} = [U_n][U_a]^g\{x_a\} = [T_u]\{x_a\} \tag{3-34}$$

where

$$[T_U] = [U_n][U_a]^g \tag{3-35}$$

This is the SEREP transformation matrix that is used for either the reduction of the finite element mass and stiffness matrices or for the expansion of the measured experimental modal vectors.

The System Equivalent Reduction Expansion Process (SEREP) relies on a finite element model or analytical model from which an eigensolution is performed to develop the mapping between the full set of finite element dof and the reduced set of 'a' degress of freedom. The eigensolution of the full set of system matrices yields a set of modal vectors which can be partitioned into those degrees of freedom that correspond to the active set of 'a' dof and the inactive set of 'd' dof.

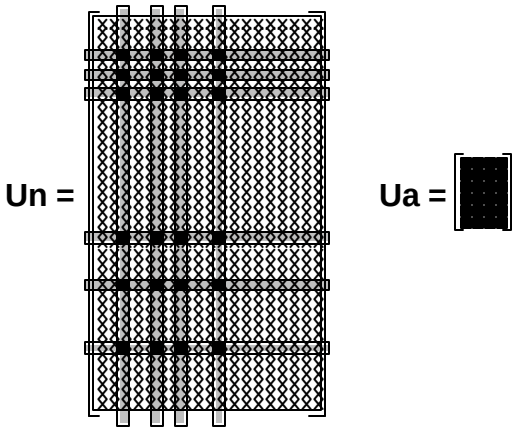


Figure 3-2 - Schematic of Ua Partion of Un

Reduction of System Matrices

Using this SEREP transformation matrix, the reduced mass and stiffness matrices can then be written as

$$\begin{aligned} \left[M_a^S\right] &= [T_U]^T [M_n][T_U] \\ \left[K_a^S\right] &= [T_U]^T [K_n][T_U] \end{aligned} \tag{3-36}$$

The equation of motion for the 'a' set of degrees of freedom can be written as

$$\left[M_a^S\right]\{\ddot{x}_a\} + \left[K_a^S\right]\{x_a\} = \{F_a(t)\} \tag{3-37}$$

Substituting in the SEREP transformation matrix in equation (3-35) into the reduced mass matrix in equation (17) gives

$$\left[M_a^S\right] = \left[U_a\right]^{gT}\left[U_n\right]^T\left[M_n\right]\left[U_n\right]\left[U_a\right]^g \tag{3-38}$$

From mass orthogonality in equation (2-10), equation (3-38) can be written for the reduced mass as

$$\left[M_a^S\right] = \left[U_a\right]^{gT}\left[U_a\right]^g \tag{3-39}$$

Note that the original system mass matrix is not needed in order to compute the reduced mass matrix.

Similarly for the reduced stiffness matrix, the SEREP transformation matrix can be substituted into the reduced stiffness matrix in equation (3-16) to give

$$\left[K_a^S\right] = \left[U_a\right]^{gT}\left[U_n\right]^T\left[K_n\right]\left[U_n\right]\left[U_a\right]^g \tag{3-40}$$

From stiffness orthogonality in equation (2-11), equation (3-40) can be written for the reduced stiffness matrix as

$$\left[K_a\right] = \left[U_a\right]^{gT}\left[\Omega^2\right]\left[U_a\right]^g \tag{3-41}$$

Note that the original system stiffness matrix is not needed in order to compute the reduced stiffness matrix.

While the size the these reduced mass and stiffness matrices is a by a, the rank of the reduced matrices is only m. Therefore, use of these matrices must be done so with caution. Due to this rank deficiency, an alternate form of the SEREP reduction process which invokes an exact solution can be obtained by using a=m for the reduction. This technique is referred to as SEREPa.

In order to better understand this rank deficiency problem, the following section reviews the reduced eigenproblem using the SEREP reduced matrices. Singular valued decomposition is used to illustrate some key features of the SEREP reduced matrices. (SVD is a procedure which allows for the inversion of coefficient matrices.)

Any matrix [A] can be decomposed into its orthogonal matrices and singular values as

$$\left[A\right] = \left[L\right]\left[S\right]\left[R\right]^T$$

where

$$[S] = \begin{bmatrix} [\sigma_r] & [0] \\ [0] & [0] \end{bmatrix}$$

Reduced Modal Matrices

Using SVD, the active set of dof in the modal matrix can be decomposed into

$$[U_a] = [L][S][R]^T$$

where

$$[S] = \begin{bmatrix} [\sigma_m] \\ [0] \end{bmatrix}$$

Then the generalized inverse can be formed as

$$[U_a]^g = [R][S]^g[L]^T$$

where

$$[S]^g = \begin{bmatrix} [\sigma_m]^{-1} & [0] \end{bmatrix}$$

Reduced Mass and Stiffness Matrices

The reduced mass matrix is

$$[M_a] = [L]\left([S]^{gT}[S]^g\right)[L]^T$$

where

$$[S]^{gT}[S]^g = \begin{bmatrix} [\sigma_m^{-2}] & [0] \\ [0] & [0] \end{bmatrix}$$

The reduced stiffness matrix is

$$[K_a] = [L]\left([S]^{gT}[R]^T[\Omega^2][R][S]^g\right)[L]^T$$

and the terms in [ ] are reduced using SVD to

$$\left([S]^{gT}[R]^{T}\left[\Omega^{2}\right][R][S]^{g}\right)=[L_1][S_1][L_1]^{T}$$

where

$$[S_1]=\begin{bmatrix}[\gamma_m] & [0]\\ [0] & [0]\end{bmatrix}$$

Reduced Mass and Stiffness Matrices

The SEREP condition occurs when a=m. It is for this that the system is truely equivalent and the rank of the system is equal to the order of the reduced matrices.

When a<m, the reduced system matrices are shown to be of proper rank but this condition is not normally useful since it involves an average of the solution variables.

When a>m, the reduced system matrices are rank deficient but produce the proper eigensolution for the 'm' variables retained in the reduced model.

Reduced Eigen System

The reduced eigen system is

$$\left[[K_a]-\lambda[M_a]\right]\{x_a\}=[0]$$

which can be expressed as

$$[L]\begin{bmatrix}\left(\left[\gamma_m^{-1}\right][R]^{T}\left[\Omega^{2}\right][R]\left[\gamma_m^{-1}\right]-\lambda\left[\sigma_m^{-2}\right]\right) & [0]\\ [0] & \left([0]-\lambda[0]\right)\end{bmatrix}[L]^{T}\{x_a\}=\{0\}$$

which is

$$\left[[U_a]^{gT}\quad[0]\right]\begin{bmatrix}\left(\left[\Omega^{2}\right]-\lambda[I]\right) & [0]\\ [0] & \left([0]-\lambda[0]\right)\end{bmatrix}\begin{bmatrix}[U_a]^{g}\\ [0]\end{bmatrix}\{x_a\}=\{0\}$$

and finally

$$[U_a]^{gT}\left(\left[\Omega^2\right]-\lambda[I]\right)[U_a]^g\{x_a\}=\{0\}$$

or

$$\left(\left[\Omega^2\right]-\lambda[I]\right)\{p\}=\{0\}$$

Hybrid Reduction

Another method for reduction of the system matrices utilizes the exactness of the SEREP process and overcomes the rank deficiency by incorporating the effects of Guyan condensation into the process and is referred to as the Hybrid Reduction [9] technique and is formulated as

$$[T_H]=[T_S]+\left[[T_U]-[T_S]\right]\left[[U_a][U_a]^T[T_U]^T[M_n][T_U]\right] \tag{3-42}$$

While this reduction technique overcomes some of the rank problems associated with SEREP when using the reduced model for forced response and other studies, there is no inherent advantage in using this technique as a model reduction scheme for correlation studies.

CONCLUDING REMARKS

Several of the more popular and commonly used reduction schemes were presented. Any of the transformations described above may be used for the development of the reduced mass and stiffness for processing such as needed for correlation studies. However, there will be inaccuracies introduced in the reduced matrices depending on the technique employed and the set of degrees of freedom chosen for the master set of degrees of freedom.

CHAPTER 4

MODE SHAPE EXPANSION

PREFACING REMARKS

This chapter presents some of the basic approaches for the expansion of measured experimental modal vectors. Expansion of experimental vectors is often needed for correlation studies and also used in the model updating procedures typically implemented utilizing current technology.

MODE SPAPE EXPANSION

Experimental mode shapes only exist at the DOF associated with the test points (ADOF). Since the mass and stiffness matrices are described at the full set of finite element DOFs (NDOF), the system mass and stiffness matrices need to be reduced to the set of experimental DOF for correlation studies. However, there is a need to also expand the measured experimental mode shape over the full set of finite element DOF for further correlation studies as well as for model updating and localization studies. Therefore, expansion techniques are necessary for further studies. A schematic of the expansion process is shown in Figure 4-1.

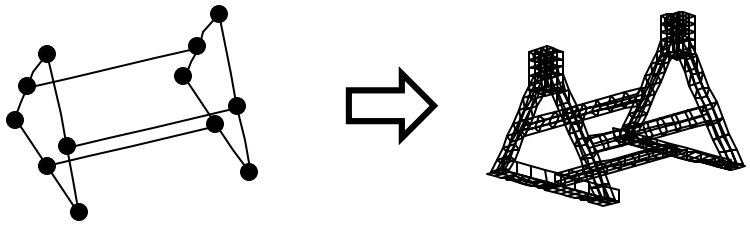


Figure 4-1 - Schematic of the Expansion Process

Early expansion techniques evolved around using spline fits and polynomial expansion based on geometry and measured data. While in concept they are useful, in practice, using these approaches for general structural systems is not feasible. Most expansion techniques utilized today involve the use of the finite element model as a mechanism to complete the unmeasured DOF from the experimental modal model. In essence, the finite element model is used as a high order polynomial curvefitter to estimate the experimental mode shapes at the deleted DOF. The majority of the expansion techniques use the model reduction transformation matrix as an expansion mechanism.

Recall that the basic relationship relating the ADOF to the NDOF is

$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\} \tag{4-1}$$

Using this expansion concept along with measured experimental modal data, we can write

$$[E_n] = \begin{bmatrix} E_a \\ E_d \end{bmatrix} = [T][E_a] \tag{4-2}$$

So we see that the measured experimental modal vectors at ADOF are expanded over all the finite element NDOF using the transformation matrix [T]. This transformation matrix will take on various forms depending on which technique is utilized.

Guyan Expansion

The Guyan Expansion technique [4] uses the static condensation transformation matrix to expand the measured DOF over all the finite element DOF. The transformation is given by

$$[T_s] = \begin{bmatrix} [I] \\ [t_s] \end{bmatrix} = \begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix} \tag{4-3}$$

and the expanded mode shapes are

$$[E_n] = \begin{bmatrix} E_a \\ E_d \end{bmatrix} = [T_s][E_a] = \begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix}[E_a] \tag{4-4}$$

Notice that the ADOF remain unchanged as seen by the upper partition of this equation

$$[E_a] = [I][E_a] \tag{4-5}$$

and that the deleted DOF are estimated by

$$[E_d] = \left[-[K_{dd}]^{-1}[K_{da}]\right][E_a] \tag{4-6}$$

Of course, the Guyan condensation process will not produce acceptable results unless there are sufficient DOF to describe the mass inertia of the system (as previously discussed in the reduction section). If sufficient DOF are available, then the Guyan process will produce reasonably good results but will never produce exact results since the inherent formulation of the reduction matrix is approximate. The Guyan reduction process is still widely used for model reduction applications due to it's long historical background but is not widely used for expansion of mode shapes due to other more accurate techniques that have been developed.

IRS Expansion

The IRS Expansion technique [6] uses the static condensation transformation matrix along with adjustment terms to compensate for the inertia associated with the deleted DOF to expand the measured DOF over all the finite element DOF. The transformation is given by

$$[T_i] = \begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix} + \begin{bmatrix} [0] & [0] \\ [0] & [K_{dd}^{-1}] \end{bmatrix} [M_n][T_s][M_a]^{-1}[K_a] \tag{4-7}$$

and the expanded mode shapes are

$$\begin{aligned} [E_n] &= \begin{bmatrix} E_a \\ E_d \end{bmatrix} = [T_i]\{E_a\} \\ &= \left(\begin{bmatrix} [I] \\ -[K_{dd}]^{-1}[K_{da}] \end{bmatrix} + \begin{bmatrix} [0] & [0] \\ [0] & [K_{dd}^{-1}] \end{bmatrix} [M_n][T_s][M_a]^{-1}[K_a] \right) [E_a] \end{aligned} \tag{4-8}$$

Notice that the ADOF remain unchanged as seen by the upper partition of this equation

$$[E_a] = [I][E_a] \tag{4-9}$$

and that the deleted DOF are estimated by

$$\{E_d\} = \left([-[K_{dd}]^{-1}[K_{da}]] + [[K_{dd}^{-1}][M_n][T_s][M_a]^{-1}[K_a]] \right) [E_a] \tag{4-10}$$

Of course, the IRS technique will improve on the Guyan expansion process but will not produce acceptable results unless there are sufficient DOF to describe the mass inertia of the system (as previously discussed in the reduction section). If sufficient DOF are available, then the IRS process will produce reasonably good results (which are improved over the Guyan results) but will never produce exact results since the inherent formulation of the reduction matrices is approximate. While the use of IRS is popular for reduction of matrices for reduced model processing such as for structural response studies, the technique is not widely used for expansion of mode shapes due to other techniques more appropriate for this type of process.

Dynamic Expansion

The Dynamic Expansion technique [7,22] is very similar in technique to the static expansion process except that the stiffness matrix is modified to include the effects of the mass of the system at a particular frequency; this is accomplished by adding an adjustment term of the reference frequency times the system mass to the stiffness matrix as shown in the development

of the model reduction equations. In essence, this matrix is exact for this one particular frequency and the transformation matrix will be exact in regards to expanding a mode shape at that particular frequency; of course, the shift frequency must correspond to one of the eigenvalues of the system. The transformation is given by

$$[T_f] = \begin{bmatrix} [I] \\ [t_f] \end{bmatrix} = \begin{bmatrix} [I] \\ -[D_{dd}]^{-1}[D_{da}] \end{bmatrix} \tag{4-11}$$

and the expanded mode shapes are

$$[E_n] = \begin{bmatrix} E_a \\ E_d \end{bmatrix} = [T_s][E_a] = \begin{bmatrix} [I] \\ -[D_{dd}]^{-1}[D_{da}] \end{bmatrix} [E_a] \tag{4-12}$$

Notice that the ADOF remain unchanged as seen by the upper partition of this equation

$$[E_a] = [I][E_a] \tag{4-13}$$

and that the deleted DOF are estimated by

$$[E_d] = \left[-[D_{dd}]^{-1}[D_{da}]\right][E_a] \tag{4-14}$$

The Dynamic Expansion process will produce exact results for one frequency and only one frequency. Providing that the shift frequency corresponds exactly to one of the eigenvalues of the system, then the expansion will produce an exact mode shape for this one eigenvalue. If additional eigenvectors need to be expanded, then separate shift values need to be processed. While many matrices need to be processed for each eigenvector that needs to be expanded, the exactness of the process warrants the additional processing.

SEREP Expansion

The SEREP Expansion technique [8] uses the SEREP transformation matrix to expand the measured DOF over all the finite element DOF. The transformation is given by

$$[T_u] = [U_n][U_a]^g = \begin{bmatrix} [U_a]\left([U_a]^T[U_a]\right)^{-1}[U_a]^T \\ [U_d]\left([U_a]^T[U_a]\right)^{-1}[U_a]^T \end{bmatrix} \tag{4-15}$$

and the expanded mode shapes are

$$[E_n] = \begin{bmatrix} E_a \\ E_d \end{bmatrix} = [T_u][E_a] = [U_n][U_a]^g[E_a] = \begin{bmatrix} [U_a] \\ [U_d] \end{bmatrix}[U_a]^g[E_a] \tag{4-16}$$

Notice that the ADOF may be changed as seen by the upper partition of this equation

$$[E_a]' = [U_a][U_a]^g[E_a] \tag{4-17}$$

and that the deleted DOF are estimated by

$$[E_d] = [U_d][U_a]^g[E_a] \tag{4-18}$$

When the ADOF are expanded, there is the possibility that the initial measured DOF may be modified by the expansion process; this is referred to as smoothing of the measured DOF. This occurs since the SEREP process is based on a generalized inverse using a least squares error minimization. Therefore, the measured data is smoothed as part of the process. While much controversy exists over where or not to smooth the actual measured data, this is the most proper way to process the data, from a mathematical standpoint.

The SEREP expansion technique is extremely accurate in regards to the expanded mode shapes - actually it is exact due to it's inherent formulation. However, if the experimental mode shapes are not correlated well with regards to the analytical mode shapes, then the results can produce very poor expanded mode shapes. The SEREP process is very unforgiving of small errors that exist in the measured experimental data base. While the SEREP process is often looked at as being too harsh in the evaluation of modal vectors, this is exactly what is needed in order to more clearly identify where errors exist in the measured and/or analytical model.

SEREPa Expansion

The SEREPa Expansion technique [23] extends the SEREP expansion technique such that the generalized inverse is formulated as a standard inverse. This is accomplished by assuring that a=m; that is, the number of measurement DOF is equal to the number of modes in the system. The transformation is given by

$$[T_u] = [U_n][U_a]^{-1} = \begin{bmatrix} [U_a][U_a]^{-1} \\ [U_d][U_a]^{-1} \end{bmatrix} \tag{4-119}$$

This assures that the measured DOF remain unchanged in the expansion process since

$$[E_a] = [U_a][U_a]^{-1}[E_a] \tag{4-20}$$

However, due to the large number of modes that are possibly needed, this technique is not widely used for most expansion applications.

Modal Expansion

Since the SEREP process generally smoothes the data on the measured DOF in the expansion process, another technique sometimes used is the Modal Expansion technique [24]. Actually, this technique is the same as the SEREP process except that the upper partition of the transformation matrix is forced to be identity which then forces the measured DOF to remain unchanged in the expansion process. The transformation is written as

$$[T_M] = \begin{bmatrix} [I] \\ [U_d][U_a]^g \end{bmatrix} \tag{4-21}$$

$$[E_n] = \begin{bmatrix} E_a \\ E_d \end{bmatrix} = [T_M][E_a] = \begin{bmatrix} [I] \\ [U_d][U_a]^g \end{bmatrix} [E_a] \tag{4-22}$$

The measured DOF remain unchanged in the expansion process and the deleted DOF are expanded in the same manner as in the typical SEREP expansion process. While this approach attempts to retain the measured DOF as seen from test, there is a mathematical mismatch between the ADOF and the deleted DOF which can cause some errors in any further processing using the expanded mode shapes.

Hybrid Expansion

The Hybrid Expansion technique [9] uses the Hybrid condensation transformation matrix to expand the measured DOF over all the finite element DOF. The transformation is given by

$$[T_H] = [T_S] + [[T_U] - [T_S]]\left[[U_a][U_a]^T[T_U]^T[M_n][T_U]\right] \quad (4\text{-}23)$$

This transformation was developed to address some of the rank deficiencies inherent in the SEREP reduction process when the number of DOF is greater than the number of modes in the system. While Hybrid is a very useful technique for model reduction applications, the extra computation necessary does not warrant it use as an expansion process.

CONCLUDING REMARKS

Several of the more popular and commonly used expansion schemes were presented. Any of the transformations described above may be used for the development of the expansion matrix for processing such as needed for correlation studies. However, there will be inaccuracies introduced in the expansion process depending on the technique employed and the set of degrees of freedom chosen for the master set of degrees of freedom.

CHAPTER 6

CORRELATION TECHNIQUES

PREFACING REMARKS

Many different correlation techniques exist for the comparison of analytical and experimental modal vectors. The correlation techniques can be broken down into techniques relating to the modal correlation in a vector sense and those relating to the correlation in a DOF sense; both of these categories can be further broken down into those techniques which do not use any mass scaling and those techniques that do use mass scaling.

In general, the techniques which do not employ any mass scaling are easier to implement and use form a practical standpoint but usually are not as discerning since they lack any scaling of the system mass properties. Techniques that do employ mass scaling are more difficult to use since the mass matrix needs to be condensed but usually offer more robust evaluation of the data evaluated.

Vector based correlation techniques are the MAC, POC and RVAC whereas DOF based techniques are the COMAC, ECOMAC, CORTHOG and FRAC. All of these techniques are described in the following sections with the exception of CORTHOG which is discussed in Chapter 7.

CORRELATION TECHNIQUES

Modal Assurance Criteria (MAC)

The Modal Assurance Criteria (MAC) [1] is an extremely useful technique which gives a first indication as to the level of correlation that exists between the analytical and experimental modal vectors and is given by

$$MAC_{ij} = \frac{\left[\{u_i\}^T\{e_j\}\right]^2}{\left[\{u_i\}^T\{u_i\}\right]\left[\{e_j\}^T\{e_j\}\right]} \tag{6-1}$$

In this formulation, the values of MAC range between 0 and 1, where zero indicates that there is little or no correlation between the vectors and one indicates that there is a high degree of similarity between the modal vectors. MAC is ideal for identify which analytical modes correspond to which experimental modes and is very useful when identifying mode switching.

MAC is very sensitive to the DOF that are largest in value and is very insensitive to very small DOF in the mode shape vector. Mass weighting is not used in this formulation which has an advantage in that mass reduction is not needed but the scaling effects of the mass matrix are useful in weighting various dofs for a correlation study.

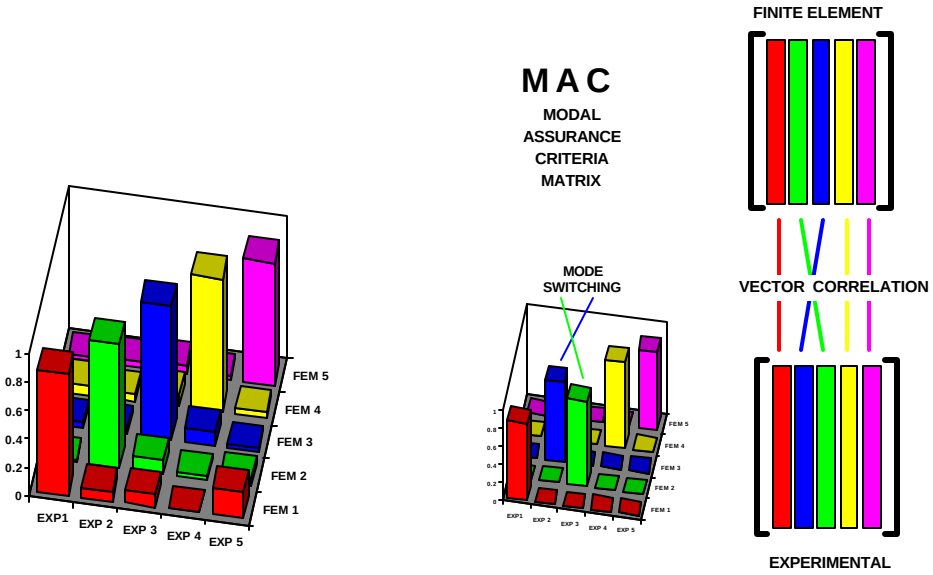


Figure 6-1 - Modal Assurance Criteria Matrix

Orthogonality Checks

Two orthogonality checks are often made in evaluating vector correlation. The Cross Orthogonality Check or the Pseudo Orthogonality Check. The obvious hurdle to overcome is whether to reduce the finite element mass and stiffness matrices to the set of tested DOF or to expand the measured experimental modal vectors to the full space of the finite element model; alternately, there could also be some combination of both reduction and expansion in order to compute the orthogonality.

The Cross Orthogonality Check is an orthogonality check where the modal vector matrices are obtained from the experimentally measured modal data.

$$\text{CROSS} = [E]^T [M][E] \stackrel{?}{=} [I] \tag{6-2}$$

The Pseudo Orthogonality Check [10] is essentially an orthogonality check where one of the modal vector matrices is replaced with the experimentally measured modal data.

$$\text{POC} = [E]^T [M][U] \stackrel{?}{=} [I] \tag{6-3}$$

As mentioned above, in order to accomplish this triple product, the size of the matrices must be consistent. Therefore, either the system mass matrix must be reduced to the set of tested DOF (and corresponding ADOF from the modal matrix) or the experimentally measured modal vectors must be expanded to the full space of the finite element model. Of course, the results of either of these checks will be dependent on the type of reduction or expansion utilized with the exception of the SEREP process which preserves the dynamics of the system in the reduced model.

In reviewing the results of the Cross Orthogonality Check and the Pseudo Orthogonality Check, there are similarities that exist since one check basically uses the experimentally measured vectors twice in the computation of orthogonality and therefore, the resulting terms are larger. However, there is no benefit of using these vectors twice in the Cross Orthogonality Check; no additional insight is gained in this process. Thus, there is no benefit in using the Cross Orthogonality Check rather than the Pseudo Orthogonality Check for general orthogonality using the usual model reduction and expansion processes such as Guyan and IRS. However, when using the SEREP process, there are other significant computational benefits that are obtained when using the Pseudo Orthogonality Check that are described next.

Pseudo Orthogonality Checks using the SEREP Process

In all of the reduction/expansion techniques, there is some numerical processing necessary to either reduce the mass matrix or expand the experimental modal vectors. Due to the inherent formulation of the SEREP Process, there are some very important items to note. The POC at the set of tested ADOF is exactly equal to the POC at the full set of NDOF and that the POC can be performed without the use of any system matrices [25]. The theoretical development to support this is presented next.

The Pseudo Orthogonality Check (POC) relating the correlation between the analytical and experimental modal vectors can be performed at the full space or reduced space by either a pre or post multiplication of the experimental modal vector with the analytical mass/mode shape using one of the following forms

Pre-multiplying by the experimental modal vectors

$$POC = [E_a]^T [M_a][U_a] \tag{6-4a}$$

$$POC = [E_n]^T [M_n][U_n] \tag{6-4b}$$

Post-multiplying by the experimental modal vectors

$$POC = [U_a]^T[M_a][E_a] \tag{6-4c}$$

$$POC = [U_n]^T[M_n][E_n] \tag{6-4d}$$

Now let's consider one of the forms above and shown that whether the POC is performed at the reduced model size or at the full model size produces exactly the same results.

$$POC = [E_a]^T[M_a][U_a] \tag{6-5}$$

Substituting the reduced mass matrix in equation (3-36) into equation (6-5) gives

$$[E_a]^T[M_a][U_a] = [E_a]^T\left([T_U]^T[M_n][T_U]\right)[U_a] \tag{6-6}$$

Substituting equation (3-35) into equation (6-6) gives

$$[E_a]^T[M_a][U_a] = [E_a]^T\left([U_a]^{gT}[U_n]^T[M_n][U_n][U_a]^g\right)[U_a] \tag{6-7}$$

The expanded experimental modal vector in equation (4-16) can be transposed to give

$$[E_n]^T = [E_a]^T[U_a]^{gT}[U_n]^T \tag{6-8}$$

and it can be seen that the first three terms of equation (6-7) are equation (6-8). Now also realize that the last two terms of equation (6-7) can be regrouped

$$[U_a]^g[U_a] = \left[\left([U_a]^T[U_a]\right)^{-1}[U_a]^T\right][U_a] \tag{6-9}$$

so that equation (6-9) is actually identity

$$[U_a]^g[U_a] = \left(\left([U_a]^T[U_a]\right)^{-1}\right)\left([U_a]^T[U_a]\right) = [I] \tag{6-10}$$

Substituting equation (6-8) and equation (6-10) into equation (6-7) it can easily be seen that

$$[E_a]^T[M_a][U_a] = [E_n]^T[M_n][U_n] \tag{6-11}$$

This implies that the POC will produce exactly the same results at either the full set of 'n' finite element degrees of freedom or at the reduced set of 'a' tested degrees of freedom. Obviously

there are significant computational benefits to be obtained if the computations are performed at the set of 'a' tested degrees of freedom rather than at the set of 'n' finite element degrees of freedom.

Now let's show that the POC can be written such that neither the full or reduced analytical mass matrix is needed. Recall the POC at the full set of finite element degrees of freedom in equation (6-4) and substitute the relationship for the expanded experimental modal vectors in equation (23)

$$[E_n]^T[M_n][U_n] = \left([U_n][U_a]^g[E_a]\right)^T[M_n][U_n] \tag{6-12}$$

Taking the transpose of the relationship for the expanded experimental modal vectors in equation (4-16) gives

$$[E_n]^T[M_n][U_n] = [E_a]^T[U_a]^{gT}[U_n]^T[M_n][U_n] \tag{6-13}$$

Recognizing the mass orthogonality relationship in equation (2-10) we see that

$$[E_n]^T[M_n][U_n] = [E_a]^T[U_a]^{gT} \tag{6-14}$$

Since the POC was shown to be identical at either the full set of finite element degrees of freedom or at the reduced set of tested degrees of freedom in equation (6-11), we see that the POC is no more than

$$[E_a]^T[M_a][U_a] = [E_n]^T[M_n][U_n] = \left([U_a]^g[E_a]\right)^T \tag{6-15}$$

Transposing this equation (to obtain an easier expression) we get

$$[U_a]^T[M_a][E_a] = [U_n]^T[M_n][E_n] = \left([U_a]^g[E_a]\right) \tag{6-16}$$

Similarly for the stiffness POC

$$[E_a]^T[K_a][U_a] = [E_n]^T[K_n][U_n] = \left([U_a]^g[E_a]\right)^T\left[\Omega^2\right] \tag{6-17}$$

and (to obtain an easier expression) its transpose

$$[U_a]^T[K_a][E_a] = [U_n]^T[K_n][E_n] = \left[\Omega^2\right]\left([U_a]^g[E_a]\right) \tag{6-18}$$

Two important items can be noted from the development presented above. First, the Pseudo Orthogonality Check will provide exactly the same results whether the check is done at the full space of the finite element model or at the reduced space of the tes model. Therefore, the computation is most efficiently performed in the reduced space of the test model. Second, tremendous computational and procedural benefits are obtained for the Pseudo Orthogonality Check using the SEREP process since the mass matrix is not needed for this computation.

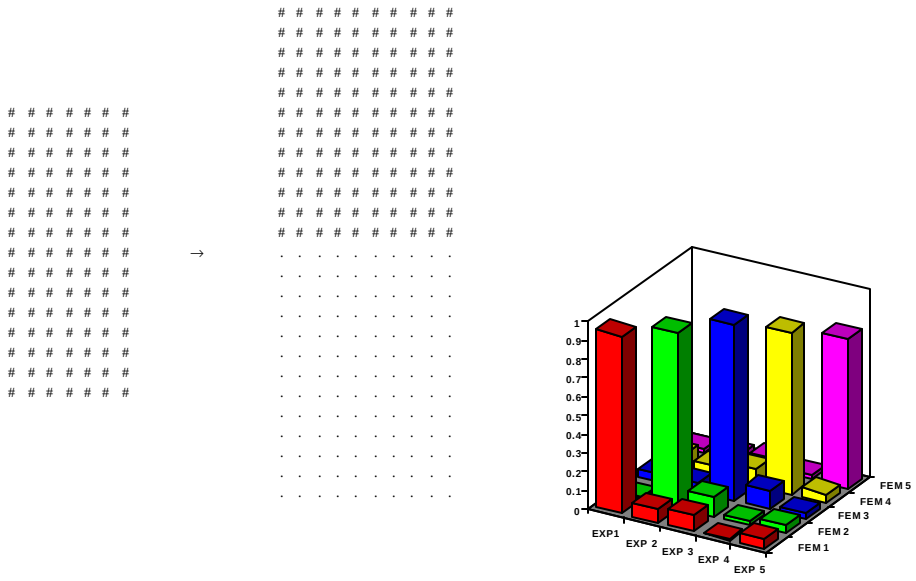


Figure 6-2 - Schematic of Expansion for Orthogonality Checks

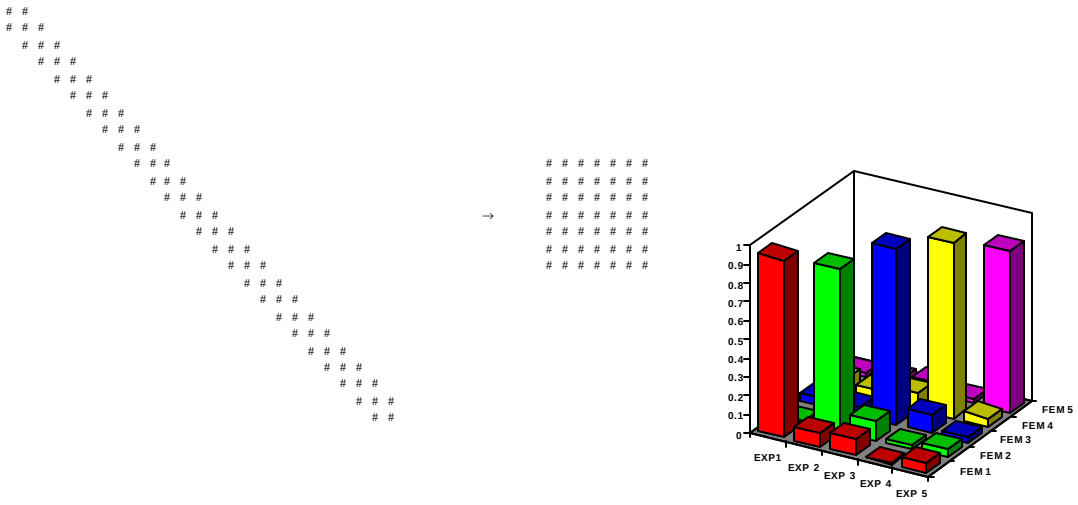


Figure 6-3 - Schematic of Reduction for Orthogonality Checks

Coordinate Modal Assurance Criteria (CoMAC)

The Coordinate Modal Assurance Criteria [2] follows the same formulation as MAC in that a correlation coefficient is developed to determine the degree of correspondence that exists for a particular DOF over a set of correlated mode pairs. COMAC is useful in determining how correlated each individual DOF may be over a set of modes and provides some insight into where some discrepancies may exist.

$$\mathrm{CoMAC}(k) = \frac{\left[\sum_{c=1}^{m} \left| u_k^{(c)} \cdot e_k^{(c)} \right|\right]^2}{\sum_{c=1}^{m} \left(u_k^{(c)}\right)^2 \cdot \sum_{c=1}^{m} \left(e_k^{(c)}\right)^2} \qquad (6\text{-}19)$$

However, without mass scaling to properly weight the dofs, at times it is difficult to determine the degree of correlation that exists. Another drawback of the CoMAC is that it can only be used for correlated mode pairs which implies that only the diagonal related terms of the MAC correlation matrix can be assessed (once the vectors are arranged in propoer correlated order if need be).

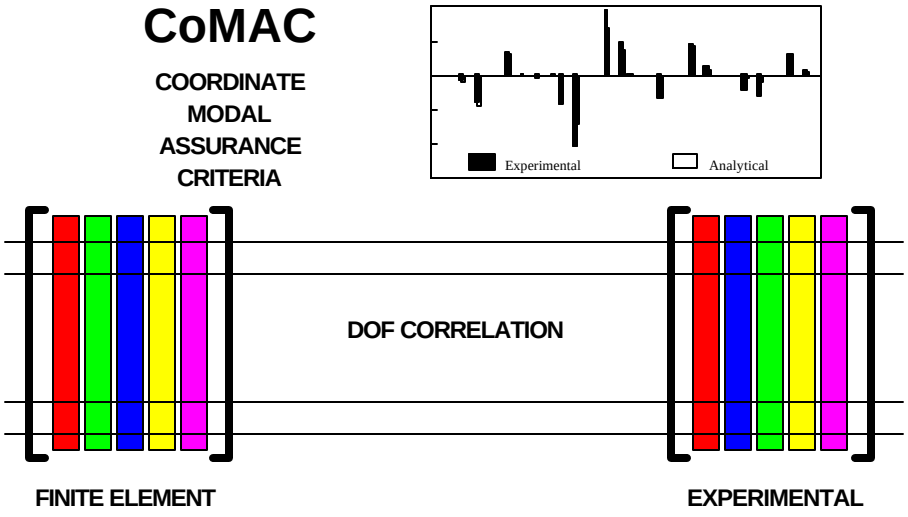


Figure 6-4 - Schematic of CoMAC for Vector Correlation

Modulus Difference

The modulus difference is another tool that helps to identify the discrepancy that a given mode pair may have.

$$\text{Modulus Difference}(k) = \left|u_k^{(c)}\right| - \left|e_k^{(c)}\right| \tag{6-20}$$

This is a subset of COMAC since it evaluates the difference between DOF for a given correlated mode pair rather that over a set of correlated modes.

Enhanced Coordinate Modal Assurance Criteria (ECoMAC)

The enhanced COMAC [3] was developed to address some of the scaling issued that exist with the original formulation of COMAC

$$\text{ECoMAC}(k) = \frac{\left[\sum_{c=1}^{m}\left|u_k^{(c)} - e_k^{(c)}\right|\right]}{2m} \tag{6-21}$$

The ECOMAC is very good for identifying gross problems in the measured modal vectors such as polarity of a transducer from a test. Again no system mass matrix is used to weight the dofs in this evaluation.

Frequency Response Assurance Criteria (FRAC)

The Frequency Response Assurance Criteria [15] evaluates each DOF based on the FRF comparison of the analytical and experimentally derived functions. The formulation is very similar to the MAC function and has similar interpretation. The FRAC is given by

$$\text{FRAC}(\beta) = \frac{\left(\{h_{test}\}\{h_{fem}(\beta)\}^*\right)^2}{\left(\{h_{test}\}\{h_{test}\}^*\right)\left(\{h_{fem}(\beta)\}\{h_{fem}(\beta)\}^*\right)} \tag{6-22}$$

The FRAC is a useful tool for evaluating FRFs but the main drawback is that the analytical model may have similar shape characteristics but differ slightly in frequency which can cause significanly low FRAC values - so the function is constructed with a shifting function to allow for some frequency adjustment due to global stiffness differences. The FRAC is mainly used for correlation that leads into frequency response based model updating studies. The majority of studies in this research are directed towards modal vector based correlation; FRAC is included here for completeness of the presentation of different correlation tools.

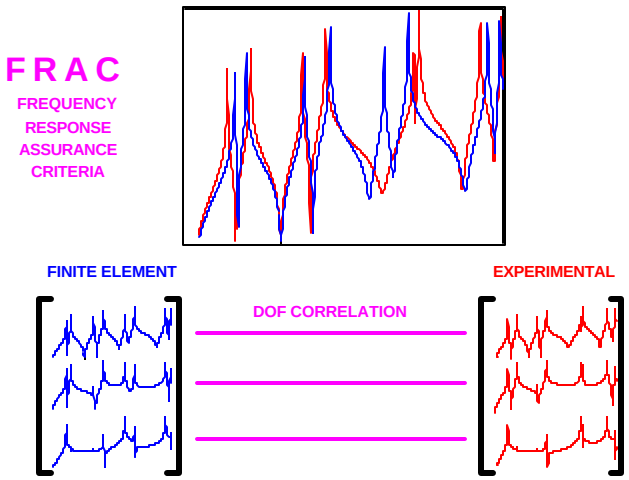


Figure 6-5 - Schematic of FRAC for Vector Correlation

Response Function Assurance Criteria (RVAC)

Companion to the FRAC is the Response Function Assurance Criteria (RVAC) [15] which compares a specific spectral line of the FRF ands evaluates it over all the FRFs of the analytical and experimental data base. Essentially RVAC is a MAC correlation technique for the analytical and experimental vectors (approximated using a peak pick technique).

$$RVAC(\omega) = MAC\left(\left\{E_{test}(\omega)\right\},\left\{U_{fem}(\omega\beta)\right\}\right) \tag{6-23}$$

Again this technique is more applicable to correlation for frequency response based model updating and is included here for completeness.

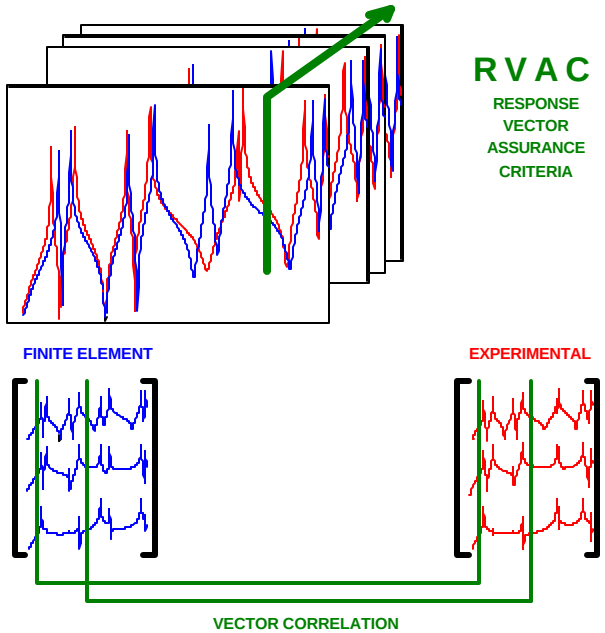


Figure 6-6 - Schematic of RVAC for Vector Correlation

CHAPTER 7

COORDINATE ORTHOGONALITY CHECK

PREFACING REMARKS

The correlation tools discussed thus far consisted of vector correlation techniques (both with and without mass scaling) and degree of freedom correlation techniques (that use no mass scaling). The vector correlation techniques assess the vector in a global sense and the correlation of the vector is stated in terms of a scalar quantity that provides a measure of the level of correlation achieved. The Modal Assurance Crieria was very easy to compute and provides a first level of correlation of the vector sets. However, mass scaling can hamper the technique. Orthogonality Checks such as the Cross Orthogonality Check and Pseudo Orthogonality Check provide a better indicator of the level of correlation achieved since mass scaling is included but again, the correlation of the vector is stated in terms of a scalar quantity that provides a measure of the level of correlation achieved. The degree of freedom correlation techniques such as CoMAC and ECoMAC provide a better assessment of the correlation on a degree of freedom basis but also are hampered by the fact that mass scaling is not included in the formulation. A new technique referred to as the Coordinate Orthogonality Check [12,14] has been developed to address the degree of freedom correlation of two modal vectors on a mass scaled basis. The Coordinate Orthogonality Check was developed to more clearly identify the discrepancy that exists between analytical and experimental modal vectors on a degree of freedom basis and is formulated with the mass as a weighting function on the DOF.

The simplest statement concerning the Coordniate Orthogonality Check is as follows:

The Coordinate Orthogonality Check (CORTHOG) is simply the comparison of what should have been obtained analytically for each degree of freedom in an orthogonality check to what was actually obtained for each degree of freedom in a POC from test.

The CORTHOG is shown schematically in Figure 7-1. The formulation of the CORTHOG is presented herein.

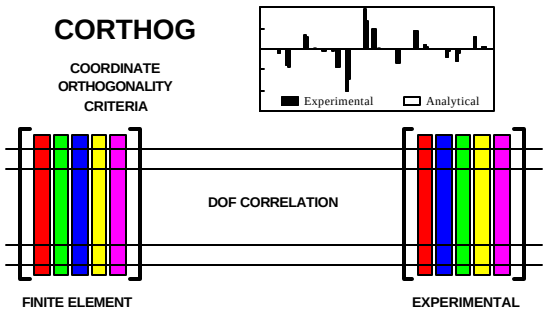


Figure 7-1 - Schematic of the Coordinate Orthogonality Check

Basis of the Coordinate Orthogonality Check

The basis of the Coordinate Orthogonality Check stems from the Pseudo Orthogonality Check. The terms that make up the Pseudo Orthogonality Check are evaluated and compared to the terms that would have been obtained from an orthogonality check for each individual degree of freedom.

The standard mass orthogonality can also be recalled to be

$$[U]^T [M][U] \equiv [I] \tag{7-1}$$

This is the condition for which the analytical vectors are orthogonal with respect to the system mass matrix.

The experimental vectors can be used as one of the set of vectors in equation (7-1) to obtain an indication as to how well the measured experimental modal vectors are related to (or are orthogonal to) the analytical mass matrix and the analytical modal vectors. The Pseudo Orthogonality Check relating the correlation between the analytical modal vectors, [U], and the experimental modal vectors, [E], with an analytical mass matrix, [M], can be performed using

$$POC = [E]^T [M][U] \stackrel{?}{=} [I] \tag{7-2}$$

and can be done at either the set of 'a' tested DOF or at the set of 'n' finite element DOF. (Note that unit modal mass scaling is assumed throughout this theoretical development.) At the reduced model size, the mass matrix can be obtained through the SEREP reduction scheme. At the full model size, the measured experimental vectors can be expanded using the SEREP expansion process.

Each term of the POC matrix can be described as

$$POC_{ij} = \sum_k \sum_p e_{ki}\ m_{kp}\ u_{pj} \tag{7-3}$$

To easily describe the concept of CORTHOG, let's consider a simple 3 DOF system with a lumped mass matrix. The POC can be written as

$$POC = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}^T \begin{bmatrix} m_{11} & & \\ & m_{22} & \\ & & m_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix} \tag{7-4}$$

Considering only the 'i'th experimental 'e' mode with the 'j'th analytical 'u' mode,

$$POC_{ij} = \begin{bmatrix} e_{1i} & e_{2i} & e_{3i} \end{bmatrix} \begin{bmatrix} m_{11} & & \\ & m_{22} & \\ & & m_{33} \end{bmatrix} \begin{Bmatrix} u_{1j} \\ u_{2j} \\ u_{3j} \end{Bmatrix} \tag{7-5}$$

Equation (7-5) can be written in expanded form for the 3 DOF system as

$$POC_{ij} = (e_{1i}m_{11}u_{1j} + e_{2i}m_{22}u_{2j} + e_{3i}m_{33}u_{3j}) \stackrel{?}{=} \begin{cases} 1 \,(\text{for } i = j) \\ 0 \,(\text{for } i \neq j) \end{cases} \tag{7-6}$$

Clearly, all DOFs have a contribution to one particular diagonal or off-diagonal term of the POC matrix. It is very important to note that for an off-diagonal term to become zero, the vectors need not be correlated; also that for off-diagonal terms, each of the individual multiplications are not zero themselves but rather the summation of all the multiplications should produce a value of zero. The individual multiplications that make up a POC term can be inspected, but there is no way to assess whether a given value is too high or too low if just the individual 'emu' terms are evaluated.

If we recall the statement of mass orthogonality for this simple 3 DOF system then Equation (7-1) can be written in expanded form as

$$ORTHOG_{ij} = (u_{1i}m_{11}u_{1j} + u_{2i}m_{22}u_{2j} + u_{3i}m_{33}u_{3j}) \equiv \begin{cases} 1 \,(\text{for } i = j) \\ 0 \,(\text{for } i \neq j) \end{cases} \tag{7-7}$$

By definition of orthogonality, this must be true.

In terms of the individual DOFs for the 3 DOF system, the contribution to each POC and orthogonality term, respectively is

	'emu' term	'umu' term
DOF 1	$e_{1i}m_{11}u_{1j}$	$u_{1i}m_{11}u_{1j}$
DOF 2	$e_{2i}m_{22}u_{2j}$	$u_{2i}m_{22}u_{2j}$
DOF 3	$e_{3i}m_{33}u_{3j}$	$u_{3i}m_{33}u_{3j}$
	$\overline{\sum POC_{ij}}$	$\overline{\sum ORTHOG_{ij}}$

(7-8)

Figure 7-2 shows a plot of the comparison of the actual individual 'emu' values from POC and

expected individual 'umu' values from one orthogonality term for each degree of freedom for a given 'ij' mode pair; this is a plot of the individual values shown in Equation (7-8). In Figure 7-2, it is clear that both the POC and orthogonality terms all sum to zero, but that the difference between them on a degree of freedom by degree of freedom basis is not the same. From this plot, this discrepancy becomes apparent and an assessment can be made as to the correlation that exists on a degree of freedom basis.

The Coordinate Orthogonality Check (CORTHOG) is simply the comparison of what should have been obtained analytically for each degree of freedom in an orthogonality check using Equation (7-7) to what was actually obtained experimentally for each degree of freedom in a POC using Equation (7-8).

Now let's extend this to the general case of a fully populated mass matrix for the 3 DOF system. Considering only the 'i'th experimental 'e' mode with the 'j'th analytical 'u' mode,

$$POC_{ij} = \begin{bmatrix} e_{1i} & e_{2i} & e_{3i} \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{Bmatrix} u_{1j} \\ u_{2j} \\ u_{3j} \end{Bmatrix} \tag{7-9}$$

This can be written in expanded form for the 3 DOF system as

$$\begin{aligned} POC_{ij} = (&e_{1i}m_{11}u_{1j} + e_{1i}m_{12}u_{2j} + e_{1i}m_{13}u_{3j} \\ &+ e_{2i}m_{21}u_{1j} + e_{2i}m_{22}u_{2j} + e_{2i}m_{23}u_{3j} \\ &+ e_{3i}m_{31}u_{1j} + e_{3i}m_{32}u_{2j} + e_{3i}m_{33}u_{3j}) \end{aligned} \tag{7-10}$$

As stated earlier, all DOFs have a contribution to one particular diagonal or off-diagonal term of the POC matrix. In terms of the individual DOFs, the contribution to the POC for the first, second and third DOF, respectively, is

$$\begin{aligned} POC_{ij}^{DOF1} &= e_{1i}m_{11}u_{1j} + e_{1i}m_{12}u_{2j} + e_{1i}m_{13}u_{3j} \\ &= e_{1i}(m_{11}u_{1j} + m_{12}u_{2j} + m_{13}u_{3j}) \end{aligned} \tag{7-11a}$$

$$\begin{aligned} POC_{ij}^{DOF2} &= e_{2i}m_{21}u_{1j} + e_{2i}m_{22}u_{2j} + e_{2i}m_{23}u_{3j} \\ &= e_{2i}(m_{21}u_{1j} + m_{22}u_{2j} + m_{23}u_{3j}) \end{aligned} \tag{7-11b}$$

$$\begin{aligned} POC_{ij}^{DOF3} &= e_{3i}m_{31}u_{1j} + e_{3i}m_{32}u_{2j} + e_{3i}m_{33}u_{3j} \\ &= e_{3i}(m_{31}u_{1j} + m_{32}u_{2j} + m_{33}u_{3j}) \end{aligned} \tag{7-11c}$$

and the contribution to the orthogonality for the first, second and third DOF, respectively, is

$$ORTHOG_{ij}^{DOF1} = u_{1i}m_{11}u_{1j} + u_{1i}m_{12}u_{2j} + u_{1i}m_{13}u_{3j} = u_{1i}(m_{11}u_{1j} + m_{12}u_{2j} + m_{13}u_{3j}) \quad (7\text{-}12a)$$

$$ORTHOG_{ij}^{DOF2} = u_{2i}m_{21}u_{1j} + u_{2i}m_{22}u_{2j} + u_{2i}m_{23}u_{3j} = u_{2i}(m_{21}u_{1j} + m_{22}u_{2j} + m_{23}u_{3j}) \quad (7\text{-}12b)$$

$$ORTHOG_{ij}^{DOF3} = u_{3i}m_{31}u_{1j} + u_{3i}m_{32}u_{2j} + u_{3i}m_{33}u_{3j} = u_{3i}(m_{31}u_{1j} + m_{32}u_{2j} + m_{33}u_{3j}) \quad (7\text{-}12c)$$

Using this notation, the subscript on the POC and ORTHOG refer to the analytical and experimental mode pair being evaluated whereas the superscript refers to the particular DOF that is being evaluated. For purposes of general notation used hereafter, equation (7-11) and (7-12) can be written in general terms as

$$POC_{ij}^{k} = \sum_{p} e_{ki}m_{kp}u_{pj} \qquad ORTHOG_{ij}^{k} = \sum_{p} u_{ki}m_{kp}u_{pj}$$

for an individual DOF. The sum over all DOF yields the usual 'ij' term of the POC or orthogonality.

Coordinate Orthogonality Check - CORTHOG

The individual contribution of the 'emu' and 'umu' terms for each *k* DOF can be inspected, compared, and evaluated for each '*ij'* mode pair. Several different formulations are presented below for a variety of different scaling and normalizing schemes that have currently been considered. Each case has its own advantages and disadvantages. In cases where the values are normalized, relative differences can be assessed, but there is no way to know if a given DOF difference is either too high or too low. In cases where absolute values are used, differences are easily compared, however, directional information is lost.

The most commonly used forms of the CORTHOG are the Comparison and Simple Difference techniques which tend to be used over the other techniques due their simplicity, ease of use and inclusion of specific, particular values relative to each degree of freedom; the other normalization techniques are useful at times but the relative relation of the values between various mode pairs is lost. Each technique is briefly described below.

Comparison

The simplest of techniques is performed by plotting the individual 'emu' and 'umu' values for each DOF in bar chart form, side by side, as shown in Figure 7-2. This provides very quick visual information as to the differences between the expected analytical values and the actual

experimental values for each individual DOF. All other techniques described below provide similar information but are presented in some sort of scaled fashion which may accentuate differences and allow easier interpretation of the data.

Simple Difference

A simple approach is to determine the difference between the summation of the individual multiplications for each DOF for a POC off-diagonal term and its expected value based on the analytical vectors given as

$$\text{SD} = \text{CORTHOG}_{ij}^{k} = \sum_{p} e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj} \tag{7-13}$$

An advantage for this formulation is that the magnitude and direction of the error is retained. By retaining the magnitude of the error, the value may be evaluated but there is no way to assess whether a given discrepancy is either acceptable or unacceptable. This magnitude may be considered in the same way as off-diagonal terms are evaluated; an arbitrary limit for a DOF, such as 0.01, may be used as a criteria for acceptance. By retaining directional information, differences relative to other degrees of freedom can be assessed. The summation of the plus values and minus values that make up the POC terms may be inspected lending further insight into the discrepancies.

Normalized SD Maximum

Another approach is to normalize the simple difference to a certain value such as the maximum difference given as

$$\text{NSD}_{M} = \text{CORTHOG}_{ij}^{k} = \frac{\sum_{p} e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj}}{\left(\sum_{p} e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj} \right)_{max}} \tag{7-14}$$

which has the advantage of setting the maximum difference equal to 1; all other differences would be a percentage of the maximum. However, since every mode pair evaluated may have widely varying levels of correlation, it is difficult to determine which mode pairs of vectors are the most and least correlated. That is to say that this is a tool to be used in conjunction with one of the other scaling methods.

Normalized SD Total

By normalizing to the summation of the differences given as

$$NSD_T = CORTHOG_{ij}^{k} = \frac{\sum_p e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj}}{\sum_k \sum_p e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj}} \tag{7-15}$$

each difference will be given as a percentage of the POC term. Thus, the maximum DOFs contributing to the given off-diagonal term can easily be evaluated. Note that if the POC term is exactly equal to zero then this normalizing scheme cannot be used.

Absolute SD

By taking the absolute difference between the summation of individual multiplications for each experimental DOF of a POC off-diagonal term and its expected value based on the analytical vectors, the absolute difference is given as

$$ASD = CORTHOG_{ij}^{k} = \sum_p \left| e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj} \right| \tag{7-16}$$

and can be easily compared. This again highlights which DOFs are most important to the given off-diagonal term. The advantage here is that the sum of all the terms will not arbitrarily become zero due to the addition of many different plus and minus contributions. When used in this fashion with an orthogonality check, this is referred to as an Absolute POC.

Normalized ASD Maximum

The absolute difference when normalized to the maximum difference is given as

$$NASD_M = CORTHOG_{ij}^{k} = \frac{\sum_p \left| e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj} \right|}{\left(\sum_p \left| e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj} \right| \right)_{max}} \tag{7-17}$$

and sets the maximum DOF difference to 1. All other differences are percentages of the maximum difference. This will provide a consistent scale but does not allow for direct comparison with other mode pairs evaluated.

Normalized ASD Total

Normalizing to the summation of absolute differences is given as

$$NASD_T = CORTHOG_{ij}^{k} = \frac{\sum_p \left| e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj} \right|}{\sum_k \sum_p \left| e_{ki} m_{kp} u_{pj} - u_{ki} m_{kp} u_{pj} \right|} \tag{7-18}$$

and will put the DOF discrepancy in terms of percent of the total difference.

The formulations discussed above are plotted in Figures 7-3 through 7-7 for the 3 DOF example presented in Figure 7-2. Note that the normalized absolute difference could not be plotted due to division by zero. Again, not all of the normalization schemes are routinely used but are included here for reference.

Coordinated Orthogonality Check without a Mass Matrix

The Coordinate Orthogonality Check can be performed with any reduced mass matrix discussed in Chapter 3. However, if the SEREP technique is utilized then the CORTHOG can be reformulated into a much more efficient form which does not require the use of system matrices similar to that shown in Chapter 6.

Basically, the equations can be written as follows. If the SEREP technique is utilized, then the standard mass orthogonality can be written as

$$POC = \left[U_n\right]^T \left[M_n\right] \left[U_n\right] = \left[U_a\right]^T \left[M_a\right] \left[U_a\right] = \left[U_a\right]^g \left[U_a\right] \tag{7-19}$$

The Pseudo Orthogonality Check can be written as

$$POC = \left[U_n\right]^T \left[M_n\right] \left[E_n\right] = \left[U_a\right]^T \left[M_a\right] \left[E_a\right] = \left[U_a\right]^g \left[E_a\right] \tag{7-20}$$

Now expanding the last term of this matrix gives

$$\left[U_a\right]^g \left[E_a\right] = \begin{bmatrix} u_{11}^g & u_{12}^g & u_{13}^g & \cdots & u_{1a}^g \\ u_{21}^g & u_{22}^g & u_{23}^g & \cdots & u_{2a}^g \\ \vdots & \vdots & \vdots & \ddots & \\ u_{m1}^g & u_{m2}^g & u_{m3}^g & & u_{ma}^g \end{bmatrix} \begin{bmatrix} e_{11} & e_{12} & \cdots & e_{1m} \\ e_{21} & e_{22} & \cdots & e_{2m} \\ e_{31} & e_{32} & \cdots & e_{3m} \\ \vdots & \vdots & \ddots & \\ e_{a1} & e_{a2} & & e_{am} \end{bmatrix} \tag{7-21}$$

If we now look at a particular 'ij'term then this can be written as

$$POC_{ij} = \begin{bmatrix} u_{i1}^g & u_{i2}^g & u_{i3}^g & \cdots & u_{ia}^g \end{bmatrix} \begin{bmatrix} e_{1j} \\ e_{2j} \\ e_{3j} \\ \vdots \\ e_{aj} \end{bmatrix} \tag{7-22}$$

Now if we multiply out this relationship for an 'ij' term, we get

$$POC_{ij} = u_{i1}^g e_{1j} + u_{i2}^g e_{2j} + u_{i3}^g e_{3j} + \cdots + u_{ia}^g e_{aj} \tag{7-23}$$

and if we multiply out the corresponding term from the orthogonality relationship, we get

$$ORT_{ij} = u_{i1}^g u_{1j} + u_{i2}^g u_{2j} + u_{i3}^g u_{3j} + \cdots + u_{ia}^g u_{aj} = 0 \tag{7-24}$$

The CORTHOG manipulation is the comparison of each of the terms of the relationships in Equation 7-23 and 7-24 for corresponding degrees of freedom.

$$\begin{array}{lll} POC_{ij} = u_{i1}^g e_{1j} + u_{i2}^g e_{2j} + u_{i3}^g e_{3j} + \cdots + u_{ia}^g e_{aj} & & \\ \qquad\quad \Updownarrow \qquad \Updownarrow \qquad \Updownarrow \qquad\qquad \Updownarrow & & CORTHOG_{ij} \\ ORT_{ij} = u_{i1}^g u_{1j} + u_{i2}^g u_{2j} + u_{i3}^g u_{3j} + \cdots + u_{ia}^g u_{aj} & & \\ \qquad\quad \vdots \qquad\ \vdots \qquad\ \vdots & & \\ \qquad dof\,1 \quad dof\,2 \quad dof\,3 & & \end{array} \tag{7-25}$$

Then any one of the comparison techniques (scaling, normalization, etc.) described previously can be computed. There are tremendous computation benefits to formulating the Coordinate Orthogonality Check using the SEREP process.

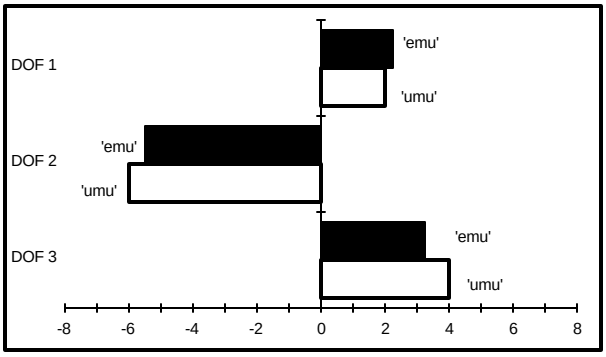


Figure 7-2 - Comparison of Analytical vs. Experimental Vectors

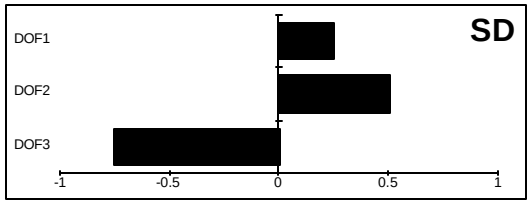


Figure 7-3 - Simple Difference Plot for 3 DOF Example

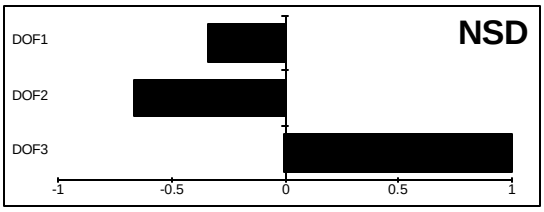


Figure 7-4 - Normalized SD Maximum Plot for 3 DOF Example

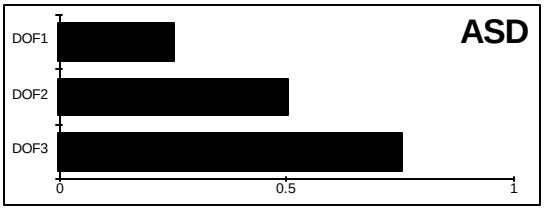


Figure 7-5 - Absolute Simple Difference Plot for 3 DOF Example

Figure 7-6 - Normalized ASD Maximum Plot for 3 DOF Example

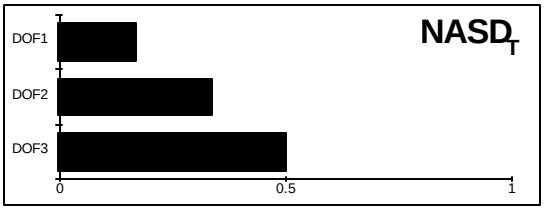


Figure 7-7 - Normalized ASD Total Plot for 3 DOF Example

CHAPTER 8

PRE-TEST EVALUATION TECHNIQUES

PREFACING REMARKS

Selection of DOF for measurement locations can be critical to the success of an experimental modal survey. It is very important to assure that an adequate number of proper points are identified for the collection of data. This chapter briefly reviews some of the existing techniques used for pre-test evaluations and then offers some additional tools based mainly on the Coordinate Orthoganlity technique.

PRE-TEST EVALUATION TECHNIQUES

One of the first techniques utilized for the determination of measurement locations, was the visualization of the finite element mode shape. Locations of large amplitude in the shape are likely to be very good measurement locations. This produced a reasonably good method to select measurement locations but was fairly tedious for larger more complicated models.

Drive Point Residues (DPR)

One approach was derived from the fact that the residues are directly related to the mode shapes of the system and therefore,

$$\left[A(s)\right]_k = Q_k \left\{u_k\right\}\left\{u_k\right\}^T \tag{8-1}$$

So that the drive point residues (DPR) [16], that is where the input and output are measured at the same location, can be evaluated for each DOF over a set of modes from

$$a_{iik} = q_k \, u_{ik} u_{ik} \tag{8-2}$$

Thus as different modes, k, are evaluated, different DPRs are computed. Typical summation methods include an assessment on the minimum value, the maximum value, the average value and a weighted average (which is the average times the minimum value) for each DOF over a set of modes. The problems with the summing schemes is that often effects are smeared into one value which does not adequately depict the proper selection of points for test DOFs.

Effective Independence

One technique that is used is to assess the rank of the modal matrix for a set of DOF that is chosen as candidate measurement locations. The rank of the equation can be determined through the singular value decomposition of

$$\left[[U_a][U_a]^T\right] \Rightarrow [\Psi][\Gamma][\Psi]^T \tag{8-3}$$

The Effective Independence [17] can be used to determine the contribution of each retained DOF using

$$EfI = \text{diag}_i\left([U_a][\Psi][\Gamma]^{-1}[\Psi]^T[U_a]^T\right) = \text{diag}_i\left([U_a]\left[[U_a]^T[U_a]\right]^{-1}[U_a]^T\right) \tag{8-4}$$

This process is continued until an acceptable set of DOF are identified as candidate modal vectors. Due to it's inherent formulation, the DOF that are selected are very good for identifying the independent vectors for possible measurement locations. Unfortunately, this method will routinely specify DOF which are close to or actually at nodes of the system. These DOF are highly suspect in terms of accuracy and are not considered good measurement locations for an experimental modal test.

MAC Contribution

One fairly simple technique is to use the MAC and evaluate the effect of different DOF to the values of the off-diagonal terms of the MAC matrix from

$$MAC_{ij} = \frac{\left[\{u_i\}^T\{u_j\}\right]^2}{\left[\{u_i\}^T\{u_i\}\right]\left[\{u_j\}^T\{u_j\}\right]} \tag{8-5}$$

Referred to as the MAC Contribution (MACCO) [18], the MAC calculation can be extended to include the effect of an additional DOF as

$$MAC_{ij}^{\,p} = \frac{\left[\{u_i\}^T\{u_j\} + u_{pi}u_{pj}\right]\left[\{u_i\}^T\{u_j\} + u_{pi}u_{pj}\right]}{\left[\{u_i\}^T\{u_i\} + u_{pi}u_{pi}\right]\left[\{u_j\}^T\{u_j\} + u_{pj}u_{pj}\right]} \tag{8-6}$$

This requires only a minor calculational effort. The addition of extra DOF tend to make the off diagonal terms of the MAC more acceptable thereby indicating a better set of points has been selected. An iterative procedure can be developed to determine the effects of adding additional candidate DOFs for measurement and to rank them based on the effect on the reduction of the

off-diagonal terms of the MAC matrix. This can provide some useful information, however, as in the case of MAC used for correlation, there is no mass matrix available for appropriate weighting of important DOFs.

Other Analytical Methods (Henshell Method, etc.)

Other analytical techniques [19] exist for automatic selection of active DOF for model reduction purposes, but generally these techniques also use averaging by one means or another to obtain some "best" location for the active DOFs. The combination of all the DOFs for all the modes of the system generally break down in applications where the structure contains directional modes which is often the case.

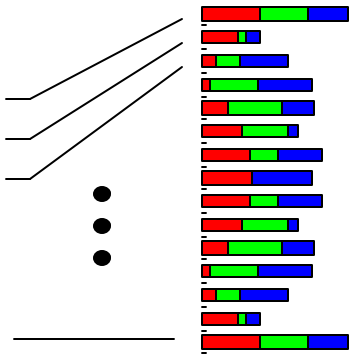
Mode Shape Evaluation Techniques

Since the CORTHOG was developed as a correlation tool which inherently contains mass scaling, then it would be reasonable to assume that it might also be useful for pre-test evaluations for selection of DOF especially when the resulting experimental modal data base will be used for correlation studies. Several new graphical tools have been developed, several of which are based off of the CORTHOG technique [20].

Mode Shape Summation Plot - MSSP

Due to the fact that information is averaged across many modes for the majority of the techniques, one possible alternative is to use a simple graphical approach to view the mode shapes. Recall that the modal matrix is nothing more than the mode shapes arranged in column form with the DOFs arranged in rows of the modal matrix. One technique is simply the graphical presentation of this data into the Mode Shape Summation Plot (MSSP).

$$[U] = \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1m} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2m} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3m} \\ u_{41} & u_{42} & u_{43} & \cdots & u_{4m} \\ u_{51} & u_{52} & u_{53} & \cdots & u_{5m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{a1} & u_{a2} & u_{a3} & \cdots & u_{am} \end{bmatrix}$$



(8-7)

Basically, the MSSP is a bar graph of the set of modes over the set of DOFs. Each (DOF) bar is created by stacking (summing) the corresponding mode shape components for all of the modes of interest. By creating the mode shape sum in this manner, the contribution of each mode to each DOF is preserved and more clearly identifies how the energy is distributed on a mode by mode basis for each DOF. Separate plots can be also be created for each mode. In

addition, the plots can be scaled to reflect displacement, velocity or acceleration, as desired.

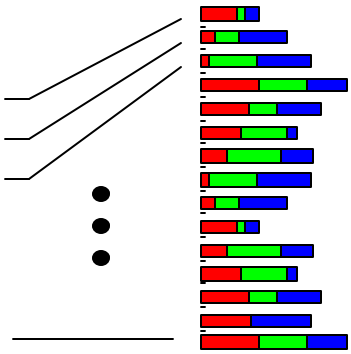
Generally, the "high amplitude" bars in the MSSP represent good candidates for test response or reference DOFs. However, typical, "high amplitude" DOFs for certain modes are at or near node DOFs for other modes. Therefore, it may be advantageous to use different sets of measurement DOFs in the correlation of each mode. The advantage of the MSSP over other approaches previously utilized is that the contribution of each mode is presented and the user can visually evaluate how well each mode will be represented by or excited at each DOF.

CORTHOG Pretest Plot - MSSP

A variation on the MSSP is the CORTHOG Pretest Plot (CPP). Recall that the Coordinate Orthogonality Check (CORTHOG) was developed as a tool to aid in determining the correlation of analytical and experimental modal data on a mass scaled DOF basis. Used as a Pre-Test Tool, CORTHOG helps to clearly show how important each individual DOF will be in an orthogonality check. This is a mass weighted preview of the mode shape, therefore, it will yield additional information that will not be evident when using other Pre-Test tools such as DPRs and MAC.

Like the MSSP, the CPP is a "summed" bar-chart of the "mass-scaled" mode shapes (columns of $[U_a]^g$). The importance of each individual DOF is mass scaled in the same way it is in an orthogonality check.

$$[U_n]^T[M_n][E_n] = [U_a]^T[M_a][E_a] = [U_a{}^g][E_a] \tag{8-8}$$



$$[U_a]^T[M_a] = [U_a^g] = \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1m} \\ u_{21} & u_{22} & u_{23} & \cdots & u_{2m} \\ u_{31} & u_{32} & u_{33} & \cdots & u_{3m} \\ u_{41} & u_{42} & u_{43} & \cdots & u_{4m} \\ u_{51} & u_{52} & u_{53} & \cdots & u_{5m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{a1} & u_{a2} & u_{a3} & \cdots & u_{am} \end{bmatrix}^g \tag{8-9}$$

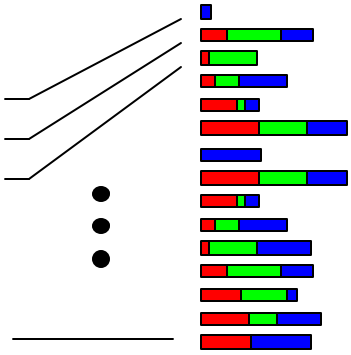
The CPP is useful in identifying and evaluating potential response and reference DOFs on the basis of modally active masses. The CPP plotted in conjunction with the MSSP helps to identify small mode shape components that may be difficult to measure but important to an orthogonality check.

Mode Power Plot - MPP

The Mode Power Plot (MPP) is a Pre-Test tool used to evaluate the effectiveness of the selected reference DOFs. Since the response DOF measurements will all be collected relative to a particular reference location(s), the MSSP can be scaled by the mode shape at the reference location as

$$[U]\begin{bmatrix} u_{r1} & & & & \\ & u_{r2} & & & \\ & & u_{r3} & & \\ & & & \ddots & \\ & & & & u_{rm} \end{bmatrix} = \begin{bmatrix} u_{11}u_{r1} & u_{12}u_{r2} & u_{13}u_{r3} & \cdots & u_{1m}u_{rm} \\ u_{21}u_{r1} & u_{22}u_{r2} & u_{23}u_{r3} & \cdots & u_{2m}u_{rm} \\ u_{31}u_{r1} & u_{32}u_{r2} & u_{33}u_{r3} & \cdots & u_{3m}u_{rm} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{r1}u_{r1} & u_{r2}u_{r2} & u_{r3}u_{r3} & \cdots & u_{rm}u_{rm} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{a1}u_{r1} & u_{a2}u_{r2} & u_{a3}u_{r3} & \cdots & u_{am}u_{rm} \end{bmatrix}$$

(79)



These "drive point scaled" modal vectors can be plotted in bar chart form and used to evaluate how well each mode will be represented in the set of measured frequency response functions using the chosen reference DOF.

While these new tools are not expected to replace previously developed techniques, they can provide additional insight into issues pertaining to correlation of analytical and experimental data.

REFERENCES

1) Allemang,R.J. and Brown,D.L., "A Correlation Coefficient for Modal Vector Analysis", First International Modal Analysis Conference, Orlando, Florida, November 1982
2) Lieven,N.A.J., Ewins,D.J., "Spatial Correlation of Mode Shapes, the Coordinate Modal Assurance Criteria (COMAC)", Sixth International Modal Analysis Conference, Orlando, Florida, February 1988
3) Hunt,D., "Application of an Enhanced Coordinate Modal Assurance Criterion", Tenth International Modal Analysis Conference, San Diego, California, February, 1992
4) Guyan, R.J., "Reduction of Stiffness and Mass Matrices", AIAA Journal, Vol. 3, No 2, 1965
5) Conti,P., "A Higher Order Approximation of the Guyan Reduction Method", Seventh International Modal Analysis Conference, Las Vegas, Nevada, February 1989
6) O'Callahan,J.C., "A Procedure for an Improved Reduced System (IRS)", Seventh International Modal Analysis Conference, Las Vegas, Nevada, February 1989
7) Kidder, RL, "Reduction of Structure Frequency Equations", AIAA, Vol 11, No 6, June 1973
8) O'Callahan,J.C., Avitabile,P., Riemer,R., "System Equivalent Reduction Expansion Process", Seventh International Modal Analysis Conference, Las Vegas, Nevada, February 1989
9) Kammer,D.C., "A Hybrid Approach to Test Analysis Model Development for Large Space Structures", submitted for publication in Journal of Spacecraft and Rockets, November 1989
10) Avitabile,P., O'Callahan,J.C., Milani,J. "Model Correlation and Orthogonality Criteria", Sixth International Modal Analysis Conference, Orlando, Florida, February 1988
11) Avitabile,P., Li,P., Foster,T., Effects of DOF Selection on Orthogonality using Reduced System Matrices", Eleventh International Modal Analysis Conference, Orlando, Florida, February 1992
12) Pechinsky,F., Avitabile,P., Coordinate Orthogonality Check (CORTHOG)", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1993
13) P.Avitabile, T.Foster, J.O'Callahan, "Comparison of Reduction vs. Expansion for Orthogonality Checks", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, January 1994.
14) Pechinsky,F., Avitabile,P., "Evaluation of DOF Correlation using the Coordinate Orthogonality Check", Thirteenth International Modal Analysis Conference, Nashville, Tennessee, February 1995.
15) Lammens,S., "Frequency Response Based Validation of Dynamic Structural Finite Element Methods", PhD Dissertation, Katholieke Universiteit, Leuven, Belgium, 1995
16) Kientzy,D., Richardson,M., Blakely,K., "Using Finite Element Data to Setup Modal Tests", Sound & Vibration, June 1989
17) Kammer, DC, "Effects of Noise on Sensor Placement for On-Orbit Modal Identification of Large Space Structures:, Journal of Dynamic System, Measurement and Control, Vol 114, September 1992
18) Carne,T.G., Dohrmann,C.R., "A Modal Test Design Strategy for Model Correlation, Thirteenth International Modal Analysis Conference, Nashville, Tennessee, February

1995.
19) O'Callahan, JC, Li, P, "An Automatic Selection of Reduced Degrees of Freedom", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1994.
20) DeClerck,J.P., Avitabile,P., Development of Several New Tools for Pre-Test Modal Evaluation", Fourteenth International Modal Analysis Conference, Dearborn, Michigan, February 1996.
21) Irons, B, "Structural Eigenvalue Problems: Elimination of Unwanted Variables", AIAA Journal, Vol 3, No 5, 1965
22) Flanigan, CC, Freymiller, JE, "Using Dynamic Expansion to Improve Test-Analysis Correlation", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1994.
23) O'Callahan, JC, Li, P, "The Effects of Modal Vector Expansion on Finite Element Model Updating", Thirteenth International Modal Analysis Conference, Nashville, Tennessee, February 1995.
24) Kammer,D.C., "Test-Analysis-Model Development Using an Exact Modal Reduction", International Journal of Analytical and Experimental Modal Analysis, October 1987
25) Avitabile,P., O'Callahan,J., "Mass and Stiffness Orthogonality Checks without a Mass or Stiffness Matrix", Thirteenth International Modal Analysis Conference, Nashville, Tenn, February 1995

BIBLIOGRAPHY

Model Reduction References

1) Kammer,D.C., "A Hybrid Approach to Test Analysis Model Development for Large Space Structures", submitted for publication in Journal of Spacecraft and Rockets, November 1989
2) Kammer,D.C., "Test-Analysis-Model Development Using an Exact Modal Reduction", International Journal of Analytical and Experimental Modal Analysis, October 1987
3) Freed,A.M., Flannigan,C.C., "A Comparison of Test Analysis Model Reduction Methods", Eighth International Modal Analysis Conference, Orlando, Florida, February 1990
4) Avitabile,P., Pechinsky,F., O'Callahan, J.C., "Study of Modal Vector Correlation using Various Techniques for Model Reduction", Tenth International Modal Analysis Conference, San Diego, California, February, 1992
5) Guyan, R.J., "Reduction of Stiffness and Mass Matrices", AIAA Journal, Vol. 3, No 2, 1965
6) O'Callahan,J.C., "A Procedure for an Improved Reduced System (IRS)", Seventh International Modal Analysis Conference, Las Vegas, Nevada, February 1989
7) O'Callahan,J.C., Avitabile,P., Riemer,R., "System Equivalent Reduction Expansion Process", Seventh International Modal Analysis Conference, Las Vegas, Nevada, February 1989
8) J.O'Callahan, "Comparison of Reduced Model Concepts", Eighth International Modal Analysis Conference, Orlando, Florida, January 1990.
9) P.Avitabile, J.O'Callahan, J.Milani, "Comparison of System Characteristics Using Various Model Reduction Techniques", Seventh International Modal Analysis

Conference, Las Vegas, Nevada, February 1989.
10) P.Avitabile, J.O'Callahan, E.Pan, "Effects of Various Model Reduction Techniques on Computed Response", Seventh International Modal Analysis Conference, Las Vegas, Nevada, February 1989.
11) Kammer,D.C., "A Hybrid Approach to Test Analysis Model Development for Large Space Structures", Journal of Vibrations and Acoustics, Vol 113, July 1991
12) Kammer,D.C., "Test-Analysis-Model Development Using an Exact Modal Reduction", International Journal of Analytical and Experimental Modal Analysis, October 1987
13) Gordis, J, "An Analysis of the Improved Reduced System (IRS) Model Reduction Procedure", Tenth International Modal Analysis Conference, San Diego, February 1992.
14) Urgueira, A, Lieven, NAJ, Ewins, D, "A Generalized Reduction Method for Modal Testing", Eighth International Modal Analysis Conference, Orlando, Florida, February 1990.
15) Irons, B, "Structural Eigenvalue Problems: Elimination of Unwanted Variables", AIAA Journal, Vol 3, No 5, 1965
16) Kidder, RL, "Reduction of Structure Frequency Equations", AIAA, Vol 11, No 6, June 1973
17) Paz, M, "Dynamic Condensation", AIAA Journal, Vol22, No 5, May 1984
18) Kammer, DC, "Test-Analysis Model Development Using the Exact Modal Reduction", The International Journal of Analytical and Experimental Modal Analysis, Oct 1987
19) Kammer, DC, "A Hybrid Approach to Test Analysis Model Development for Large Space Structures", Journal of Vibration and Acoustics, Vol 113, July 1991
20) O'Callahan, JC, "Comparison of Reduced Model Concepts", Eighth International Modal Analysis Conference, Orlando, Florida, February 1990
21) Friswell, MI, Garvey, SD, Penny, JET, "Model Reduction Using an Iterated IRS Technique", 5th International Conference on Recent Advances in Structural Dynamics, Southhampton, UK, July 1994
22) Goh, EL, Mottershead, JE, "On the Model Reduction Techniques for Finite Element Model Updating", NAFEMS/DTA International Conference on Structural Dynamics, Modeling, Test, Analysis and Correlation, Milton Keynes, July 1993

Vector Expansion References

1) P.Avitabile, J.O'Callahan, C.M.Chou, V.Kalkunte, "Expansion of Rotational Degrees of Freedom for Structural Dynamic Modification", Fifth International Modal Analysis Conference, London, England, April 1987.
2) J.O'Callahan, P.Avitabile, R.Madden, I.W.Lieu, "An Efficient Method of Determining Rotational Degrees of Freedom from Analytical and Experimental Modal Data", Fourth International Modal Analysis Conference, Los Angeles, California, February 1986.
3) J.O'Callahan, C.M.Chou, I.W.Lieu, "Determination of Rotational Degrees of Freedom for Moment Transfers in Structural Modifications", Third International Modal Analysis Conference, Orlando, Florida, January 1985.
4) O'Callahan, JC, Li, P, "The Effects of Modal Vector Expansion on Finite Element Model Updating", Thirteenth International Modal Analysis Conference, Nashville, Tennessee,

February 1995.
5) Flanigan, CC, Freymiller, JE, "Using Dynamic Expansion to Improve Test-Analysis Correlation", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1994.
6) Levine-West, MB, Kissil, A, Milman, M, "Evaluation of Mode Shape Expansion Techniques on the Micro-precision Interferometer Truss", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1994.
7) Hemez, FM, Farhat, C, "Comparing, Mode Shape Expansion Methods for Test-Analysis Correlation", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1994.
8) Roy, N, Girard, A, Bugeat, LP, "Expansion of Experimental Mode Shapes - An Improvement of the Projection Technique", Eleventh International Modal Analysis Conference, Orlando, Florida, February 1993.
9) Imregum, M, Ewins, D, "An Investigation into Mode Shape Expansion Techniques", Eleventh International Modal Analysis Conference, Orlando, Florida, February 1993.
10) Lieven, NAJ, Ewins, D, "Expansion of Modal Data for Correlation", Eighth International Modal Analysis Conference, Orlando, Florida, February 1990.
11) O'Callahan, JC, Lieu, IW, Chou, CM, "Determination of Rotational Degrees of Freedom for Moment Transfers in Structural Modifications", Third International Modal Analysis Conference, Orlando, Florida, February 1985.
12) Gysin, HP, "Comparison of Expansion Method for FE Modeling Error Localization", Eighth International Modal Analysis Conference, Orlando, Florida, February 1990
13) Gysin, HP, "Expansion: The Achilles Heel of Fe Model Updating", Proceedings of the 15th International Seminar on Modal Analysis, September 1990
14) O'Callahan, JC, Li, P, "A Non-Smoothing SEREP Process for Modal Vector Expansion", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1994
15) Lipkins, J., Vandeurzen, U., "The Use of Smoothing Techniques for Structural Modification Applications", Proceedings of the 12th International Seminar on Modal Analysis, 1987
16) Lang,G.F., "Including Rotational Terms in a Modal Model...An Experimental Prelude", Sound & Vibration, November 1989

Vector Correlation References

1) Allemang,R.J. and Brown,D.L., "A Correlation Coefficient for Modal Vector Analysis", First International Modal Analysis Conference, Orlando, Florida, November 1982
2) P.Avitabile, T.Foster, J.O'Callahan, "Comparison of Reduction vs. Expansion for Orthogonality Checks", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, January 1994.
3) Freed,A.M., Flannigan,C.C., "A Comparison of Test Analysis Model Reduction Methods", Eighth International Modal Analysis Conference, Orlando, Florida, February 1990
4) Avitabile,P., O'Callahan,J.C., Milani,J. "Model Correlation and Orthogonality Criteria", Sixth

International Modal Analysis Conference, Orlando, Florida, February 1988
5) Avitabile,P., O'Callahan,J., "Mass and Stiffness Orthogonality Checks without a Mass or Stiffness Matrix", Thirteenth International Modal Analysis Conference, Nashville, Tenn, February 1995
6) Blades, EL, Craig, RR, "A Craig-Bampton Test-Analysis Model", Fifteenth International Modal Analysis Conference, Orlando, Florida, February 1997.
7) Blaschke,PG, Bosch,R, Ewins, D, "The MAC Revisited and Updated", Fifteenth International Modal Analysis Conference, Orlando, Florida, February 1997.
8) O'Callahan, JC, "Development of a General Pseudo Orthogonality Correlation Procedure", Thirteenth International Modal Analysis Conference, Nashville, Tennessee, February 1995.
9) Flanigan, CC, Hunt, DL, "Integration of Pretest Analysis and Model Correlation Methods for Modal Surveys", Eleventh International Modal Analysis Conference, Orlando, Florida, February 1993.
10) Baker , M, "Review of Test/Analysis Correlation Methods and Criteria for Validation of Finite Element Models for Dynamic Analysis", Tenth International Modal Analysis Conference, San Diego, February 1992.
11) Blelloch, PA, "Direct Correlation of Test-Analysis Cross-Orthogonality", Tenth International Modal Analysis Conference, San Diego, February 1992.
12) Freed, A, Flanigan, CC, "A Comparison of Test-Analysis Model Reduction Methods", Eighth International Modal Analysis Conference, Orlando, Florida, February 1990.
13) Humbad, NG, Mallick, S, "Modal Comparison Criteria", Fifth International Modal Analysis Conference, London, England, February 1987.

DOF Correlation References

1) Lieven,N.A.J., Ewins,D.J., "Spatial Correlation of Mode Shapes, the Coordinate Modal Assurance Criteria (COMAC)", Sixth International Modal Analysis Conference, Orlando, Florida, February 1988
2) Pechinsky,F., Avitabile,P., Coordinate Orthogonality Check (CORTHOG)", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1993
3) Avitabile,P., Li,P., Foster,T., Effects of DOF Selection on Orthogonality using Reduced System Matrices", Eleventh International Modal Analysis Conference, Orlando, Florida, February 1992
4) Foster,T., "Evaluation of DOF Selection for Modal Vector Correlation and Expansion Studies", Masters Thesis, University of Massachusetts Lowell, Lowell, Massachusetts, July 1993
5) Pechinsky,F., Avitabile,P., "Evaluation of DOF Correlation using the Coordinate Orthogonality Check", Thirteenth International Modal Analysis Conference, Nashville, Tennessee, February 1995.
6) Mains, M, Vold, H, "Investigation of the Effects of Transducer Cross-Sensitivity and Misalignment Error on Modal Vector Correlation", Thirteenth International Modal Analysis Conference, Nashville, Tennessee, February 1995.

DOF Selection References

1) Imamovic, N, Ewins, D, "Optimization of Excitation DOF Selection for Modal Tests", Fifteenth International Modal Analysis Conference, Orlando, Florida, February 1997.
2) Schedlinski, C, Link, M, "An Approach to Optimal Pick-up and Exciter Placement", Fourteenth International Modal Analysis Conference, Dearborn, Michigan, February 1996.
3) Park, YS, Kim, HB, "Sensor Placement Guide for Model Comparison and Improvement", Fourteenth International Modal Analysis Conference, Dearborn, Michigan, February 1996.
4) O'Callahan, JC, Li, P, "An Automatic Selection of Reduced Degrees of Freedom", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1994.
5) Brillhart, RD, Kammer, DC, "Evaluation of Optimal Modal Sensor Placement Using a System Realization Method", Twelfth International Modal Analysis Conference, Honolulu, Hawaii, February 1994.
6) Penny, JET, Friswell, MI, Garvey, SD, "The Automatic Choice of Measurement Locations for Dynamic Testing", Tenth International Modal Analysis Conference, San Diego, February 1992
7) Kammer, DC, Triller, MJ, "Efficient Sensor Placement on On-Orbit Modal Identification of Sequentially Assembled Large Space Structures", Tenth International Modal Analysis Conference, San Diego, February 1992
8) Kammer, DC, "Effects of Noise on Sensor Placement for On-Orbit Modal Identification of Large Space Structures:, Journal of Dynamic System, Measurement and Control, Vol 114, September 1992
9) Hunt, DL, Peterson, EL, Vold, H, Williams, R, "Optimal Selection of Excitation Methods for Enhanced Modal Testing", AIAA paper, No. 84-1068, AIAA Dynamics Conference, Palm Springs, California, May 1984
10) Kientzy,D., Richardson,M., Blakely,K., "Using Finite Element Data to Setup Modal Tests", Sound & Vibration, June 1989

Experimental Modal Analysis References

1) Allemang,R.J. et al, "Experimental Modal Analysis and Dynamic Component Synthesis", Air Force Report # AFWAL-TR-87-3069, Vol I-VI, 1987
2) Avitabile,P., "Overview of Experimental Modal Analysis using the Frequency Response Method", Modal Seminar Lecture Notes, March 1994
3) Mitchell,L.D., "Signal Processing and the Fast Fourier Transform (FFT Analyzer - A Survey", International journal of Analytical and Experimental Modal Analysis, January 1986
4) Snoeys,R., Sas,P., Heylen,W., Van der Auweraer,H., "Trends in Experimental Modal Analysis", Mechanical Systems and Signal Processing, January 1987
5) Allemang,R.J., "Investigation of Some Multiple Input/Output Frequency Response Experimental Modal Analysis Techniques", PhD Dissertation, University of Cincinnati, Cincinnati, Ohio, 1980
6) Leuridan,J., "Some Direct Model Identification Methods Applicable for Multiple Input

Modal Analysis, PhD Dissertation, University of Cincinnati, Cincinnati, Ohio, 1984
7) Brown,D. et al, "Parameter Estimation Techniques for Modal Analysis" Society of Automotive Engineers Paper # 790221, 1979
8) "Fundamentals of Signal Analysis", Hewlett Packard Application Note 243
9) "Dynamic Signal Analyzer Applications", Hewlett Packard Application Note 243-1
10) "Fundamentals of Modal Testing", Hewlett Packard Application Note 243-3
11) Formenti,D., "Analytical and Experimental Modal Analysis", UC-SDRL-CN-20-263-662, University of Cincinnati Modal Analysis Notes, 1977, revised 1990-1992
12) Schmidtberg,R., Pal,T., "Solving Vibration Problems using Modal Analysis", Sound & Vibration, March 1986
13) "Techniques for Signal and System Analysis", Tektronix Signal Analysis Unit, PN 363-0100-01

Software References

1) Cada-X Collection of Modal Software Packages, Leuven Measurement Systems, Belgium
2) LINK Correlation/Updating Software Package, Leuven Measurement Systems, Belgium
3) Dynaview Correlation Software Package, MicroDyne, Inc
4) STAR Modal Analysis Software, Spectral Dynamics, San Jose, California
5) I-DEAS Collection of Software Packages- Structural Dynamics Research Corporation
6) NASTRAN Finite Element Software, McNeal-Schwendler Corp, Los Angeles, California
7) ANSYS Finite Element Software Package, Swanson Analysis Systems
8) FEMTools Correlation Software, Dynamic Systems Solutions, Belgium
9) FEMAP Pre/Post Processor Software, Enterprise Systems, Pennsylvania
10) MATSAP/MATRIX Toolbox, UMASS Lowell, J.C.O'Callahan

Miscellaneous References

1) Strang, G, "Linear Algebra and Its Applications", Academic Press, 2nd Edition, 1980
2) Cook, RD, Malkus, DS, Plashha, ME, "Concepts and Applications of Finite Element Analysis", John Wiley and Sons, Third Edition, 1989
3) Craig, RR, "Structural Dynamics - An Introduction to Computer Methods", John Wiley and Sons, 1981
4) Bathe, KJ, "Finite Element Procedures in Engineering", Prentice-Hall, Englewood Cliffs, NJ, 1982
5) Ewins, DJ, "Modal Testing: Theory and Practice", John Wiley and Sons, Inc, 1984