Auxiliar #1 26/03/2007

- Grado de Indeterminación Estática (GIE)

$$GIE = \#\text{ incognitas} - \#\text{ ecuaciones}$$

- Marcos planos sin rótulas

$$3 \cdot m + r - 3j$$

- Marcos planos con rótulas

$$3m + r - 3j - R$$

- Marcos espacial sin rótulas

$$6m + r - 6j$$

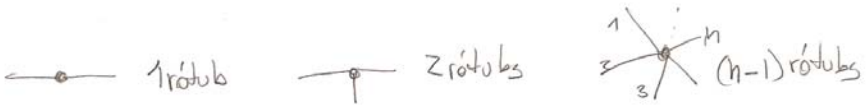
- Enrejados

$$m + r - 2j$$

m: # de barras
r: restricciones de apoyo
j: nodos.

Apoyos:

Rótulas



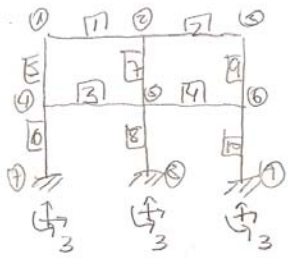
Ecuaciones de Equilibrio:

$\sum$ Fuerzas externas $(X_0 Y) = 0$

$\sum$ Momentos en rótula por todas las fuerzas externas $= 0$

Ejercicios GIE:

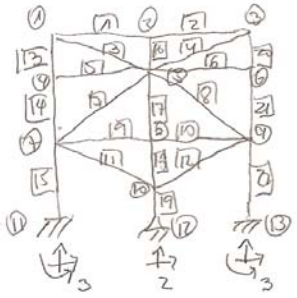
P1)



$m = 10$
$r = 9$
$j = 9$

$GIE = 3 \cdot (10) + (9) - 3(9)$
$= 30 + 9 - 27 = 12$
$\rightarrow GIE = 12$

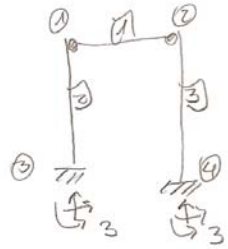
P2)



$m = 22$
$r = 8$
$j = 13$

$GIE = 3 \cdot (22) + (8) - 3(13)$
$= 66 + 8 - 39$
$\rightarrow GIE = 35$

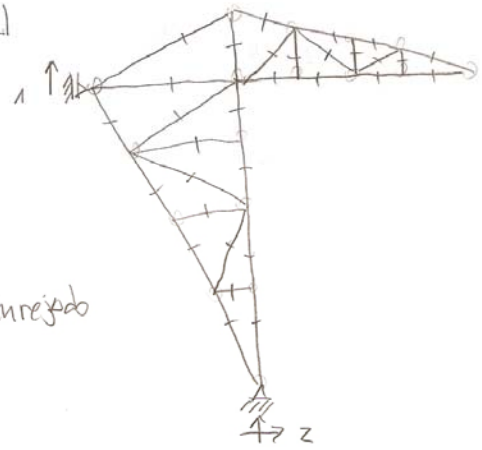
P3)



$m = 3$
$r = 6$
$j = 4$
$R = 2$

$GIE = 3 \cdot (3) + (6) - 3 \cdot (4) - (2)$
$= 9 + 6 - 12 - 2$
$\Rightarrow GIE = 1$

P4)

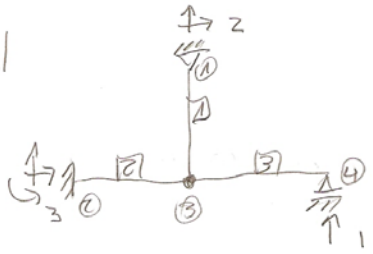


Empujado

$m = 31$
$r = 3$
$j = 17$

$GIE = (31) + (3) - 2(17)$
$= 31 + 3 - 34$
$\rightarrow GIE = 0$

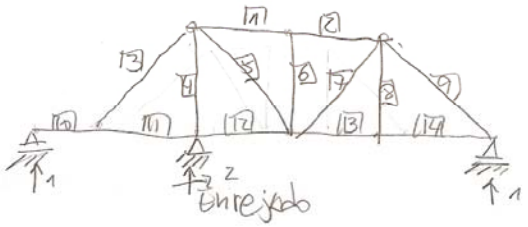
P5)



$m = 3$
$r = 6$
$j = 4$
$R = 2$

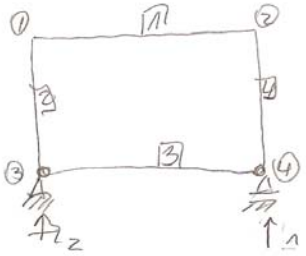
$GIE = 3 \cdot (3) + 6 - 3 \cdot (4) - 2$
$= 9 + 6 - 12 - 2$
$GIE = 1$

P6)



$m = 14$
$r = 4$
$j = 9$
$GIE = (14) + (4) - 2 \cdot (9)$
$= 14 + 4 - 18$
$GIE = 0$

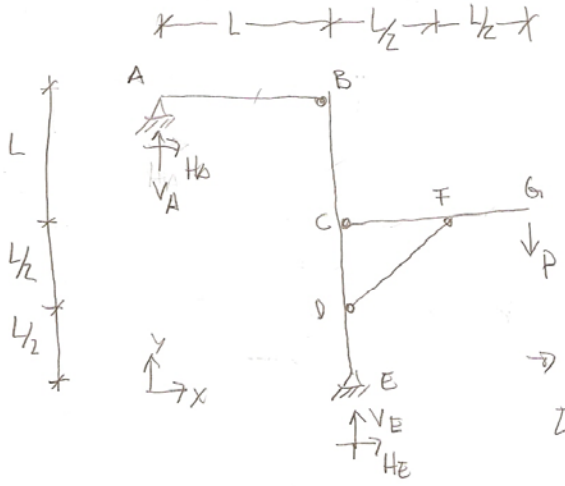
P7)



$m = 4$
$r = 3$
$j = 4$
$R = 2$

$GIE = 3 \cdot (4) + 3 - 3 \cdot (4) - (2)$
$= 12 + 3 - 12 - 2$
$GIE = 1$

Cálculo de Reacciones



$m = 7$ $\quad GIE = 3(j) + (4) - 3(7) - (4)$

$r = 4$

$j = 7$

$R = 4$ $\quad \rightarrow GIE = 0.$

$\sum F_x = 0$

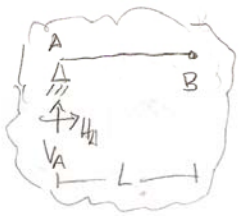
$\rightarrow H_D + H_E = 0 \quad (1)$

$\sum F_y = 0$

$\rightarrow V_D + V_E - P = 0 \quad (2)$

$\sum \eta_A = 0 \rightarrow V_E \cdot L + H_E \cdot 2L - P \cdot 2L = 0 \quad (3)$

$\sum \eta_B = 0$



$\Rightarrow V_A \cdot L = 0 \quad (4) \rightarrow \boxed{V_A = 0}$

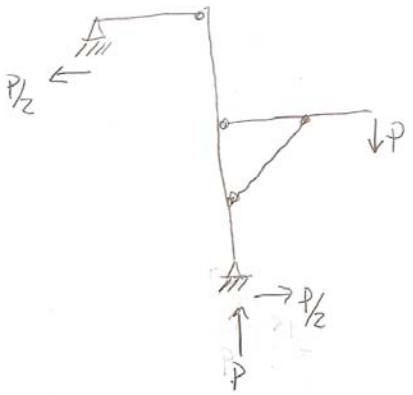
$(2) \rightarrow \boxed{V_E = P}$

$(3) \Rightarrow PL + H_E \cdot 2L - 2PL = 0$

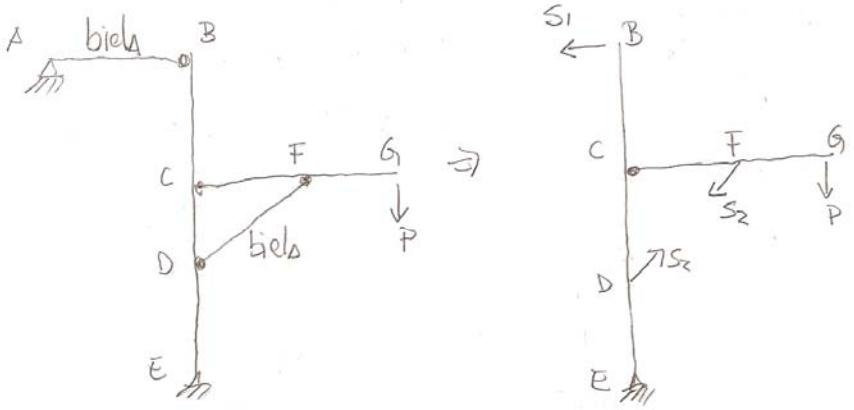
$\Rightarrow H_E = \frac{PL}{2L} = \frac{P}{2} \rightarrow \boxed{H_E = \frac{P}{2}}$

$(1) \Rightarrow \boxed{H_A = \frac{P}{2}}$

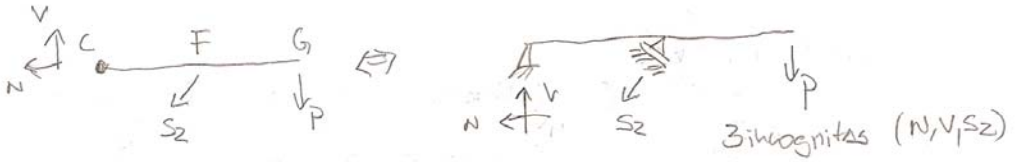
Reacciones:



Otra forma (separando la estructura en 2 fáciles)

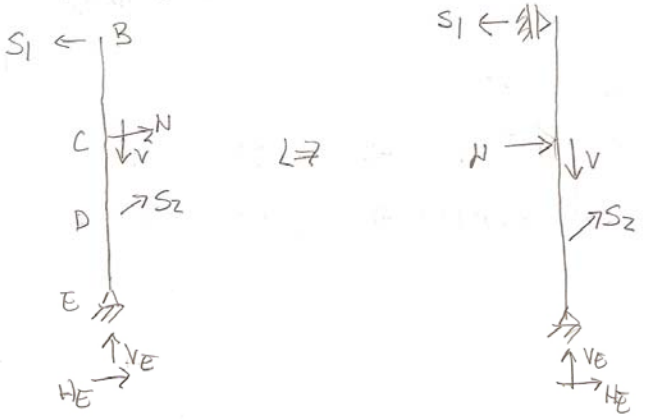


Estructura 1 (CFG):



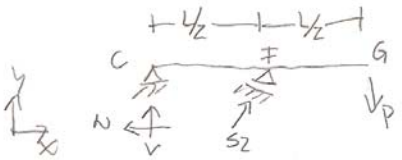
3 incognitas (N, V, S2)
3 ecuaciones "OK!

Estructura 2 (BCDE)



3 incognitas (V_E, H_E, S_1)
3 ecuaciones
OK!

- Estructura 1 (ITG)



$\sum F_x = 0 \Rightarrow -N + S_2 \cdot \cos(45) = 0$ (1)

$\sum F_y = 0 \Rightarrow V + S_2 \cdot \operatorname{sen}(45°) - P = 0$ (2)

$\sum \Pi_C = 0 \Rightarrow S_2 \cdot \operatorname{sen}(45°) \cdot \frac{L}{2} - P \cdot L = 0$ (3)

(1) $\Rightarrow N = S_2 \cdot \cos(45°)$

$= 2\sqrt{2}P \cdot \frac{\sqrt{2}}{2} = 2P$

$\Rightarrow \boxed{N = 2P}$

(3) $\Rightarrow S_2 \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - P = 0$

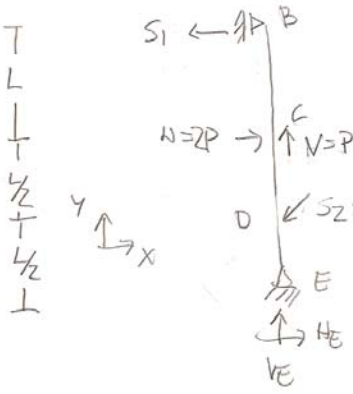
$\Rightarrow S_2 = \frac{P \cdot 2 \cdot 2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{2} = 2\sqrt{2}P$

$\boxed{S_2 = 2\sqrt{2}P}$

(2) $\Rightarrow V = P - S_2 \cdot \operatorname{sen}(45°)$

$= P - 2\sqrt{2}P \cdot \frac{\sqrt{2}}{2} = -P \Rightarrow \boxed{V = -P}$

- Estructura 2 (BCDE)



$\sum F_x = 0 \Rightarrow -S_1 + H_E + 2P - 2\sqrt{2}P \cdot \cos(45°) = 0$ (1)

$\sum F_y = 0 \Rightarrow P - 2\sqrt{2}P \cdot \operatorname{sen}(45) + V_E = 0$ (2)

$\Rightarrow V_E = 2\sqrt{2} \cdot \frac{\sqrt{2}}{2} \cdot P - P$

$\boxed{V_E = P}$

$\sum \Pi_E = 0$

$\Rightarrow 2\sqrt{2}P \cdot \cos(45) \cdot \frac{L}{2} - 2P \cdot L + S_1 \cdot 2L = 0$ (3)

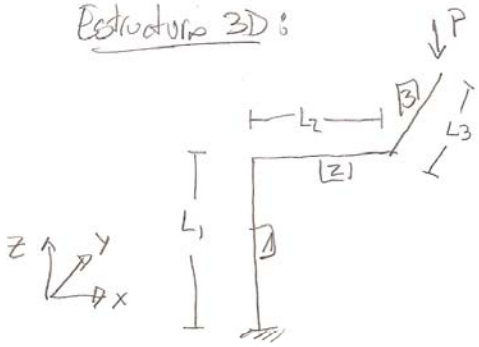
$\Rightarrow 2\sqrt{2}P \cdot \frac{\sqrt{2}}{2} \cdot \frac{L}{2} - 2PL + S_1 \cdot 2L = 0$

$\Rightarrow 2S_1 = P \Rightarrow \boxed{S_1 = P/2}$

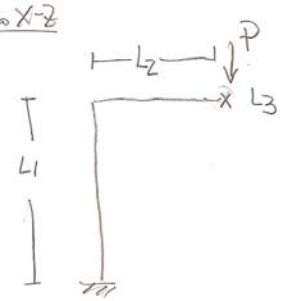
(1) $\Rightarrow -P/2 + H_E + 2P - 2\sqrt{2}P \cdot \frac{\sqrt{2}}{2} = 0$

$\Rightarrow \boxed{H_E = P/2}$

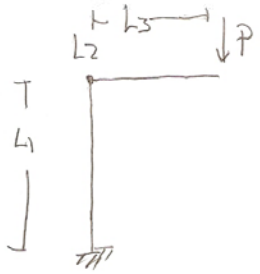
Estructura 3D:



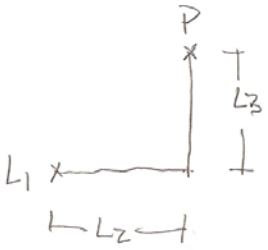
Plano X-Z



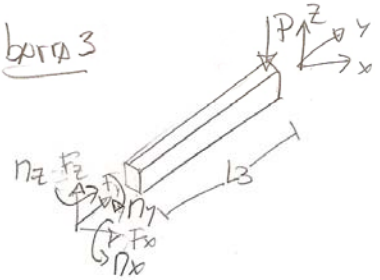
Plano Y-Z



Plano X-Y



barra 3



$\Rightarrow \sum M_x = 0 \Rightarrow M_x - PL_3 = 0$

$\Rightarrow \boxed{M_x = PL_3}$ Flexión

$\sum M_y = 0 \Rightarrow \boxed{M_y = 0}$ Torsión

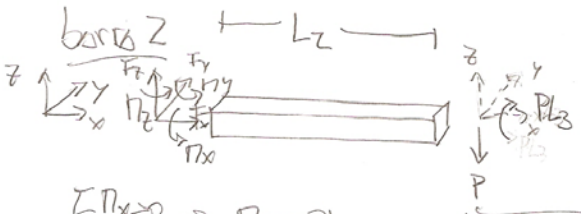
$\sum M_z = 0 \Rightarrow \boxed{M_z = 0}$ Flexión

$\sum F_x = 0 \Rightarrow \boxed{F_x = 0}$ corte

$\sum F_y = 0 \Rightarrow \boxed{F_y = 0}$ Axial

$\sum F_z = 0 \Rightarrow F_z - P = 0 \Rightarrow \boxed{F_z = P}$ corte

barra 2



$\sum n_x = 0 \Rightarrow n_x - PL_3 = 0 \Rightarrow \boxed{n_x = PL_3}$ Torsión

$\sum n_y = 0 \Rightarrow n_y + PL_2 = 0 \Rightarrow \boxed{n_y = -PL_2}$ Flexión

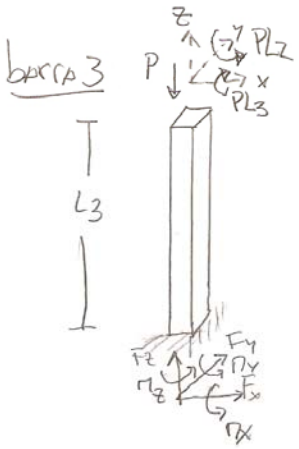
$\sum n_z = 0 \Rightarrow \boxed{n_z = 0}$ Flexión

$\sum F_x = 0 \Rightarrow \boxed{F_x = 0}$ Axial

$\sum F_y = 0 \Rightarrow \boxed{F_y = 0}$ corte

$\sum F_z = 0 \Rightarrow F_z - P = 0 \Rightarrow \boxed{F_z = P}$ Corte

barra 3



$\sum n_x = 0 \Rightarrow n_x - PL_3 = 0 \Rightarrow \boxed{n_x = PL_3}$ Flexión

$\sum n_y = 0 \Rightarrow n_y + PL_2 = 0 \Rightarrow \boxed{n_y = -PL_2}$ Flexión

$\sum n_z = 0 \Rightarrow \boxed{n_z = 0}$ Torsión

$\sum F_x = 0 \Rightarrow \boxed{F_x = 0}$ corte

$\sum F_y = 0 \Rightarrow \boxed{F_y = 0}$ corte

$\sum F_z = 0 \Rightarrow F_z - P = 0 \Rightarrow \boxed{F_z = P}$ Axial.