$a)\ f(x,y) = 3x^2y^2 + 2x^3 + 2y^3$

$\operatorname{int} Q$

$$\nabla f(x,y) = \begin{pmatrix} 6xy^2 + 6x^2 \\ 6yx^2 + 6y^2 \end{pmatrix} = 0$$

$\Leftrightarrow\ 6x(y^2 + x) = 0$

$\quad 6y(x^2 + y) = 0$

puntos estacionarios : $(0,0)$ y $(-1,-1)$

$$\text{Hessiano} : H_f(x,y) = \begin{bmatrix} 6y^2 + 12x & 12xy \\ 12xy & 6x^2 + 12y \end{bmatrix}$$

$$H_f(0,0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; H_f(-1,-1) = \begin{bmatrix} -6 & 12 \\ 12 & -6 \end{bmatrix}$$

Frontera Q

$L(x,y,\lambda) = 3x^2y^2 + 2x^3 + 2y^3 + \lambda(x^2 + y^2 - 4)$

$$\frac{\partial L}{\partial x}(x,y,\lambda) = 6xy^2 + 6x^2 + 2\lambda x = 0$$

$$\frac{\partial L}{\partial y}(x,y,\lambda) = 6yx^2 + 6y^2 + 2\lambda y = 0$$

$$\frac{\partial L}{\partial \lambda}(x,y,\lambda) = x^2 + y^2 - 4 = 0$$

$\Leftrightarrow$

$$2x(3y^2 + 3x + \lambda) = 0$$
$$2y(3x^2 + 3y + \lambda) = 0$$
$$x^2 + y^2 = 4$$

puntos estacionarios : $(0,2), (0,-2), (2,0), (-2,0)$

si $x, y \neq 0$

$$3y^2 + 3x + \lambda = 0$$
$$3x^2 + 3y + \lambda = 0$$
$$x^2 + y^2 = 4$$

si $x = y$ se obtienen los puntos $(\sqrt{2},\sqrt{2}), (-\sqrt{2},\sqrt{2}), (\sqrt{2},-\sqrt{2})$
$(-\sqrt{2},-\sqrt{2})$.

si $x \neq y$ *se obtiene* $(\frac{1}{2} + \frac{\sqrt{7}}{2}, \frac{1}{2} - \frac{\sqrt{7}}{2}), (\frac{1}{2} - \frac{\sqrt{7}}{2}, \frac{1}{2} + \frac{\sqrt{7}}{2})$.

evaluando se obtienen los maximos y minimos:

$f(\sqrt{2},\sqrt{2}) \approx 23.3 \qquad f(0,-2) = f(-2,0) = -16$

$b)\, i)$ Plano tangente: $z = 3 + \dfrac{\partial z}{\partial x}(1,2)(x-1) + \dfrac{\partial z}{\partial y}(1,2)(y-2)$

$$z = z(x,y) = \frac{3xy}{2x + y - 3xy}$$

$$\frac{\partial z}{\partial x}(x,y) = \frac{3y^2}{(2x+y-3xy)^2}$$

$$\frac{\partial z}{\partial y}(x,y) = \frac{6y^2}{(2x+y-3xy)^2}$$

El plano tangente queda

$$z = 3 + 3(x-1) + \frac{3}{2}(y-2)$$

(ii)

$$L(x,y,z,\lambda) = x + y + z + \lambda\left(\frac{1}{x} + \frac{2}{y} + \frac{3}{z} - 3\right)$$

$$\frac{\partial L}{\partial x}(x,y,z,\lambda) = 1 - \frac{\lambda}{x^2} = 0$$

$$\frac{\partial L}{\partial y}(x,y,z,\lambda) = 1 - \frac{2\lambda}{y^2} = 0$$

$$\frac{\partial L}{\partial z}(x,y,z,\lambda) = 1 - \frac{3\lambda}{z^2} = 0$$

$$\frac{\partial L}{\partial \lambda}(x,y,z,\lambda) = \frac{1}{x} + \frac{2}{y} + \frac{3}{z} - 3 = 0$$

$\Rightarrow$

$$x^2 = \lambda;\ y^2 = 2\lambda;\ z^2 = 3\lambda$$

en la ultima ecuación queda $\lambda = 1$ (se omiten raices <0 pues x,y,z>0)

$\Rightarrow x = 1;\ y = \sqrt{2};\ z = \sqrt{3}$

$c)$

$$\frac{\partial z}{\partial x}(x,y)=\frac{\partial z}{\partial u}(u,v)+\frac{\partial z}{\partial v}(u,v)$$

$$\frac{\partial z}{\partial y}(x,y)=2\frac{\partial z}{\partial u}(u,v)-\frac{\partial z}{\partial v}(u,v)$$

$$\frac{\partial^2 z}{\partial y^2}(x,y)=4\frac{\partial^2 z}{\partial u^2}(u,v)-4\frac{\partial^2 z}{\partial u\partial v}(u,v)+\frac{\partial^2 z}{\partial v^2}(u,v)$$

$$\frac{\partial^2 z}{\partial x^2}(x,y)=\frac{\partial^2 z}{\partial u^2}(u,v)+2\frac{\partial^2 z}{\partial u\partial v}(u,v)+\frac{\partial^2 z}{\partial v^2}(u,v)$$

$$\frac{\partial^2 z}{\partial x\partial y}(x,y)=2\frac{\partial^2 z}{\partial u^2}(u,v)+\frac{\partial^2 z}{\partial u\partial v}(u,v)-\frac{\partial^2 z}{\partial v^2}(u,v)$$

Reemplazando se obtiene lo pedido

si z no depende de v:

$$\frac{\partial z}{\partial u}(u,v)=0\Rightarrow z=k\in\mathbb{R}$$