P1)

a)

i) Notemos que $\lambda_k = \arg\min_{\lambda \geq 0} f(x_k - \lambda \nabla f(x_k))$

Sea $F(\lambda) = f(x_k - \lambda \nabla f(x_k))$

$\Rightarrow F'(\lambda) = \nabla f(x_k - \lambda \nabla f(x_k)) \cdot - \nabla f(x_k) = 0$

donde λ_k es solución de la ecuación anterior

$\Rightarrow$

$$\nabla f(\underbrace{x_k - \lambda_k \nabla f(x_k)}_{x_{k+1}}) \cdot - \nabla f(x_k) = 0$$

$\Rightarrow \nabla f(x_{k+1}) \perp \nabla f(x_k)$. (0,5)

ii)

1) $P(x) = f(x_k) + \langle \nabla f(x_k), x - x_k \rangle + \frac{1}{2}(x - x_k)^t H_f(x_k)(x - x_k)$ (0,2)

2) $P(x_k - \lambda \nabla f(x_k)) = \underbrace{f(x_k) - \lambda \langle \nabla f(x_k), \nabla f(x_k) \rangle + \frac{1}{2} \lambda^2 \nabla f(x_k)^t H_f(x_k) \nabla f(x_k)}_{\text{función de } \lambda.\ F(\lambda)}$ (0,2)

3) $F'(\lambda) = -\|\nabla f(x_k)\|_2^2 + \lambda \nabla f(x_k)^t H_f(x_k) \nabla f(x_k) = 0$

$$\Rightarrow \lambda^* = \frac{\|\nabla f(x_k)\|_2^2}{\nabla f(x_k)^t H_f(x_k) \nabla f(x_k)} \quad (0,5)$$

Ademas, $F''(\lambda^*) = \nabla f(x_k)^t H_f(x_k) \nabla f(x_k) > 0$ pues $x_k \in B(\bar{x}, \varepsilon)$ $\Rightarrow H_f(x_k)$ def. positivo.

$\Rightarrow \lambda^*$ es en mínimo de $F(\lambda)$. (0,4)

4) Por todo lo anterior, $\lambda_k = \dfrac{\|\nabla f(x_k)\|_2^2}{\nabla f(x_k)^t H_f(x_k) \nabla f(x_k)}$ (0,2)

b) i) Sea $F(x(s), y(s)) = c \quad / \frac{d}{ds}$

$$\Rightarrow \frac{\partial F}{\partial x}(\sigma(s)) \cdot x'(s) + \frac{\partial F}{\partial y}(\sigma(s))\, y'(s) = 0$$

$\Leftrightarrow \nabla F \cdot \hat{T} = 0 \quad \Rightarrow \nabla F \perp \hat{T}$. (0,2)

Ademas $\hat{T} = \left(\frac{-\partial F}{\partial y}, \frac{\partial F}{\partial x} \right) \cdot \frac{1}{\|\nabla F\|}$ (0,3)