Fecha / Date:

P3

1

(a) (i) Incógnitas: "m" y "b", pues con ellas se determina la recta. (0.2)

· Para pasar a un problema sin restricciones $e_i = y_i - mx_i - b$. Con lo cual (PMC) equivale a $\text{Min} \sum_{i=1}^{n} (y_i - (mx_i+b))^2$

$(m,b) \in \mathbb{R}^2$ (0.5)

(ii) Hacemos $\nabla f(m,b) = \vec{0}$.

Así $2\sum_{i=1}^{n} (y_i - (mx_i+b)) \cdot -x_i = 0$

$2\sum_{i=1}^{n} (y_i - (mx_i+b)) \cdot 1 = 0$ (0.4)

Que equivale a:

$$\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right) m + \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) b = \frac{1}{n}\sum_{i=1}^{n} x_i y_i$$

$$\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) m + 1 \cdot b = \frac{1}{n}\sum_{i=1}^{n} y_i$$

Que equivale a
$$\begin{pmatrix} \frac{1}{n}\sum_{i=1}^{n} x_i^2 & \frac{1}{n}\sum_{i=1}^{n} x_i \\ \frac{1}{n}\sum_{i=1}^{n} x_i & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{n}\sum_{i=1}^{n} x_i y_i \\ \frac{1}{n}\sum_{i=1}^{n} y_i \end{pmatrix}$$

(0.1) De donde
$$m = \frac{\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

y
$$b = \frac{\frac{1}{n}\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i\right) - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} x_i y_i\right)}{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$$

(ii) (0.6) siempre que
$$\begin{pmatrix} \frac{1}{n}\sum_{i=1}^{n} x_i^2 & \frac{1}{n}\sum_{i=1}^{n} x_i \\ \frac{1}{n}\sum_{i=1}^{n} x_i & 1 \end{pmatrix}$$
sea invertible, i.e. si $\sum_{i=1}^{n} x_i^2 \neq \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2$