se puede expresar el problema como $\min\limits_{\vec{x}\in\mathbb{R}^m} \|Q\vec{x}-\vec{d}\|_2^2$ (0.2)

(ii) Haciendo $\nabla_{\vec{x}} \underbrace{\|Q\vec{x}-\vec{d}\|_2^2}_{=\sum_{i=1}^{n} e_i^2} = \vec{0}$ queda que:

$$\left(\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} \left(y_i - \left(a_0 + \sum_{j=1}^{m-1} a_j x_i^j\right)\right)^2\right)$$

Para cada $k \in \{1, \dots, m-1\}$,

$$\sum_{i=1}^{n} y_i x_i^k = \sum_{i=1}^{n} a_0 x_i^k + \sum_{i=1}^{n}\left(\sum_{j=1}^{m-1} a_j x_i^j\right) x_i^k$$

$(k=0)$

$$\sum_{i=1}^{n} y_i = n \cdot a_0 + \sum_{i=1}^{n}\left(\sum_{j=1}^{m-1} a_j x_i^j\right)$$

Lo que equivale a:

Para cada $k \in \{1, \dots, m-1\}$

$$\frac{1}{n}\sum_{i=1}^{n} y_i x_i^k = a_0 \frac{1}{n}\left(\sum_{i=1}^{n} x_i^k\right) + \sum_{j=1}^{m-1}\left(a_j \frac{1}{n}\sum_{i=1}^{n} x_i^{j+k}\right)$$

$$\frac{1}{n}\sum_{i=1}^{n} y_i = a_0 + \sum_{j=1}^{m-1}\left(a_j \frac{1}{n}\sum_{i=1}^{n} x_i^j\right)$$

Es decir,

$$\begin{pmatrix} 1 & \frac{1}{n}\sum_{i=1}^{n} x_i & \frac{1}{n}\sum_{i=1}^{n} x_i^2 & \dots & \frac{1}{n}\sum_{i=1}^{n} x_i^{m-1} \\ \vdots & \vdots & \vdots & & \vdots \\ \frac{1}{n}\sum_{i=1}^{n} x_i^{m-1} & \frac{1}{n}\sum_{i=1}^{n} x_i^{1+m-1} & \frac{1}{n}\sum_{i=1}^{n} x_i^{2+m-1} & \dots & \frac{1}{n}\sum_{i=1}^{n} x_i^{m-1+m-1} \end{pmatrix} \begin{pmatrix} a_0 \\ \vdots \\ a_{m-1} \end{pmatrix} = \begin{pmatrix} \frac{1}{n}\sum_{i=1}^{n} y_i \\ \vdots \end{pmatrix}$$ (0.5)

Se requiere A invertible y entonces: (0.4)

$(a_0 \dots a_{m-1})^t = A^{-1}\vec{r}$