PAUTA EXAMEN

El Profesor corrigió de manera que la pauta está más resumida...

Problema 1.

Un trompo simétrico de masa m cuyos momentos principales de Inercia en su púa son $C = 3mh^2$, $A = 2mh^2$se coloca en movimiento de modo que inicialmente .

$$\theta(0) = \pi/2, \quad \dot{\theta}(0) = 0, \quad \dot{\phi}(0) = 2\sqrt{\frac{g}{h}}, \quad s = \sqrt{\frac{g}{h}}$$

Calculamos

$$\begin{aligned}
\alpha &= A\dot{\phi}\sin^2\theta + Cs\cos\theta = 2A\sqrt{\frac{g}{h}} = 4mh^2\sqrt{\frac{g}{h}} \\
2E - Cs^2 &= A\dot{\phi}^2\sin^2\theta + A\dot{\theta}^2 + 2mgh\cos\theta = 8mh^2\frac{g}{h} = 8mgh
\end{aligned}$$

la energía será

$$E = \frac{1}{2}Cs^2 + 2A\frac{g}{h} = \frac{1}{2}3mh^2\frac{g}{h} + 4mh^2\frac{g}{h} = \frac{11}{2}mgh \qquad \text{(a) 2 p)}$$

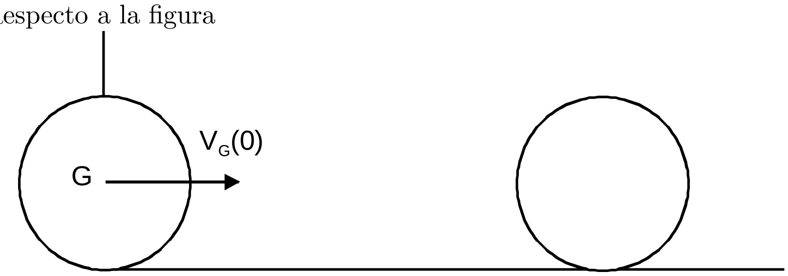
$$\begin{aligned}
\dot{u}^2 &= f(u) = (8mgh - 2mghu)\frac{1-u^2}{2mh^2} - \left(\frac{4mh^2\sqrt{\frac{g}{h}} - 3mh^2\sqrt{\frac{g}{h}}u}{2mh^2}\right)^2 \\
&= \frac{g}{h}\left((-4+u)(u-1)(u+1) - \frac{1}{4}(-4+3u)^2\right) = \frac{g}{h}\frac{1}{4}u\left(-25u + 20 + 4u^2\right) \\
\theta_1 &= \frac{\pi}{2}, \ \cos\theta_2 = \frac{25}{8} - \frac{1}{8}\sqrt{305} = 0.942\,0, \quad \theta_2 = 19.61^o \qquad \text{(b) 2 p)}
\end{aligned}$$

$$\dot{\phi} = \sqrt{\frac{g}{h}}\frac{4 - 3\cos\theta}{2\sin^2\theta}$$

$$\begin{aligned}
\dot{\phi}(u) &= \sqrt{\frac{g}{h}}\frac{4-3u}{2(1-u^2)} \\
\dot{\phi}_{\max}(0.942\,0) &= 5.\,211\sqrt{\frac{g}{h}} \qquad \text{(c) 2 p)}
\end{aligned}$$

Problema 2.

Respecto a la figura



tenemos que

$$\begin{aligned} Ma_G &= -f = -\mu_k Mg \quad G \text{ desacelera} \\ Rf &= I_G \alpha \Rightarrow \alpha = \frac{R\mu_k Mg}{I_G}, \quad \dot{\theta} \text{ aumenta} \end{aligned}$$

ambas pueden integrarse

$$\begin{aligned} v_G &= v_G(0) - \mu_k g t \\ \omega &= \frac{R\mu_k Mg}{I_G} t = \frac{2\mu_k g}{R} t \end{aligned}$$

la condición de rodar sin resbalar se satisface cuando

$$v_G = R\omega$$

de donde

$$\begin{aligned} v_G(0) - \mu_k g t &= 2\mu_k g t \Rightarrow t = \frac{v_G(0)}{3\mu_k g} \\ x_G &= v_G(0)t - \frac{1}{2}\mu_k g t^2 = \frac{5}{18}\frac{v_G^2(0)}{\mu_k g} \end{aligned}$$

Problema 3.

Esto si que es trivial.....Tenemos que

$$\begin{aligned} x_G &= a \sin \Omega t + x, \quad \dot{x}_G = a\Omega \cos \Omega t + \dot{x}, \quad \omega = \frac{\dot{x}}{R} \\ L &= \frac{1}{2} M (a\Omega \cos \Omega t + \dot{x})^2 + \frac{1}{2}\frac{1}{2} M R^2 \frac{\dot{x}^2}{R^2} \\ p_x &= \frac{\partial L}{\partial \dot{x}} = M(a\Omega \cos \Omega t + \dot{x}) + \frac{1}{2} M \dot{x} \\ \frac{3}{2}\ddot{x} - a\Omega^2 \sin \Omega t &= 0 \qquad \text{(6 p)} \end{aligned}$$

Problema 4.

Lo que cae

$$\begin{aligned} s &= \frac{1}{2} g t^2 \\ m(t) &= \frac{M}{L}\frac{1}{2} g t^2, \\ u &= -gt, \quad v = 0 \end{aligned}$$

luego

$$
\begin{aligned}
R - mg &= m\frac{dv}{dt} - (u-v)\frac{dm}{dt} \\
R &= mg - u\frac{dm}{dt} = \frac{M}{L}\frac{1}{2}g^2t^2 + gt\frac{M}{L}gt \\
&= \frac{3}{2}\frac{M}{L}g^2t^2. \qquad \text{(6 p)}
\end{aligned}
$$

Problema 5.

Solución: tenemos $\mu \simeq m$. Como siempre se caluclan E, l_0

$$
\begin{aligned}
v_0 &= \frac{1}{2}\sqrt{\frac{GM}{R}}, \\
l_0 &= mR\frac{1}{2}\sqrt{\frac{GM}{R}}\frac{1}{2}\sqrt{2} \\
E &= \frac{1}{2}m\frac{1}{4}\frac{GM}{R} - \frac{GMm}{R} = -\frac{7}{8}mG\frac{M}{R} \qquad \text{(2 p)}
\end{aligned}
$$

luego

$$
e^2 = 1 + \frac{2(-\frac{7}{8}mG\frac{M}{R})(mR\frac{1}{2}\sqrt{\frac{GM}{R}}\frac{1}{2}\sqrt{2})^2}{mG^2M^2m^2} = \frac{25}{32} \qquad \text{(1 p)}
$$

$$
\begin{aligned}
r &= \frac{(mR\frac{1}{2}\sqrt{\frac{GM}{R}}\frac{1}{2}\sqrt{2})^2}{mGMm}\frac{1}{1-\sqrt{\frac{25}{32}}\cos(\theta-\alpha)}, \\
r &= \frac{1}{8}R\frac{1}{1-\sqrt{\frac{25}{32}}\cos(\theta-\alpha)} \qquad \text{(1 p)}
\end{aligned}
$$

Sea $r = R$ en $\theta = 0$

$$
R = \frac{1}{8}R\frac{1}{1-\sqrt{\frac{25}{32}}\cos(\alpha)} \qquad \text{(1 p)}
$$

el otro ángulo θ donde $r = R$ es $\theta = 2\alpha$.

$$
\cos\alpha = \frac{7}{10}\sqrt{2} \Rightarrow \alpha = 0.141\,9 = (8.\,13^o)
$$

el arco será

$$
s = 0.283\,8R \qquad \text{(1 p)}
$$