PAUTA P1, C3.

a) $$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy = \delta \int_0^1 \left(1 - \frac{u}{U}\right) d\eta \quad ; \quad \eta = \frac{y}{\delta}$$

$$\Rightarrow \delta^* = \int_0^1 (1 - \eta^{1/7}) \cdot d\eta = \delta \cdot \left(\eta - \frac{7}{8}\eta^{8/7}\right)\Big|_0^1 = \frac{\delta}{8}$$

$$\Rightarrow \boxed{\delta^* = \frac{\delta}{8}} \qquad (1\ \text{pto}).$$

b) Por continuidad $V_1 A_1 = V_2 \cdot A_2$

$$\Rightarrow V_1 \cdot w \cdot H = V_2 \cdot w (H - 2 \cdot \delta_2^*) \longrightarrow (1\ \text{pto}).$$

$$\Rightarrow V_2 = \frac{V_1 \cdot H}{H - 2 \cdot \delta_2^*} = 10\ \frac{m}{s} \cdot \frac{300\ mm}{(300 - 25)\ mm} = 10{,}9\ m/s.$$

TOMANDO UN PUNTO DETRÁS DE LAS PLACAS DONDE, $P = P_0$ Y $V = 0$.

$$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 = P_0 + \underbrace{V_0}_{0}.$$

$$\Rightarrow P_1 = P_0 - \frac{1}{2}\rho V_1^2$$

$$P_1 = 10^5 [Pa] - \frac{1}{2} \cdot 1{,}23\ \frac{kg}{m^3} \cdot (10\ m/s)^2$$

$$P_1 = 99938{,}5\ [Pa] \longrightarrow (0{,}5\ \text{ptos})$$

Ahora con P_2:

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + V_2^2 \cdot \frac{1}{2} \cdot \rho$$

$$\Rightarrow P_2 = P_1 + \frac{1}{2}\rho (V_1^2 - V_2^2)$$

$$P_2 = 99938{,}5\ [Pa] + \frac{1}{2} \cdot 1{,}23\ \frac{kg}{m^3} \cdot (10^2 - 10{,}9^2)\ \frac{m^2}{s^2}$$

$$\boxed{P_2 = 99926{,}81\ [Pa]} \longrightarrow (0{,}5\ \text{ptos}).$$

i) ESFUERZO DE CORTE PROMEDIO. (3 ptos).

$$\sum F_x = \int_{SC} u\cdot\rho\,\vec{V}\cdot d\vec{A}$$

$$\underbrace{(P_1-P_2)\cdot w\cdot\frac{H}{2}}_{\text{FZAS EN LA ENTRADA Y SALIDA}} - \underbrace{\bar{\tau}\cdot w\cdot L}_{\text{FZA POR ESFUERZO DE CORTE.}} = \overbrace{\bar{V}_1\cdot\left\{-\rho\bar{V}_1\frac{H}{2}w\right\}}^{\text{A LA ENTRADA}} + \overbrace{\int_0^{\delta_2} u\rho u w\,dy}^{\text{EN LA SALIDA EN LA C.L}} + \underbrace{V_2\left\{+\rho V_2\left(\frac{H}{2}-\delta_2\right)w\right\}}_{\text{A LA SALIDA FUERA DE LA C.L.}}$$

La integral en el segundo término de la ecuación anterior se puede resolver como:

$$\int_0^{\delta_2} u\rho u w\,dy = \rho V_2^2\delta_2 w\int_0^1 \eta^{2/7}\cdot d\eta = \rho V_2^2\cdot\left(\frac{7}{9}\eta^{9/7}\right)\Big|_0^1\cdot\delta_2 = \rho V_2^2\cdot\frac{7}{9}\delta_2\cdot w$$

$$\Rightarrow \bar{\tau}\cdot\cancel{w}\,L = (P_1-P_2)\cdot\cancel{w}\frac{H}{2} + \rho V_1^2\frac{H}{2}\cancel{w} - \rho V_2^2\left(\frac{H}{2}-\frac{2}{9}\delta_2\right)\cdot\cancel{w}$$

$$\Rightarrow \bar{\tau} = \frac{1}{L}\left[(P_1-P_2)\cdot\frac{H}{2} + \rho V_1^2\cdot\frac{H}{2} - \rho V_2^2\left(\frac{H}{2}-\frac{2}{9}\delta_2\right)\right)$$

$$\Rightarrow \bar{\tau} = \frac{1}{5}\left[11{,}6\cdot 0{,}15 + 1{,}23\cdot 10^2\cdot 0{,}15 - 1{,}23\cdot 10{,}9^2\cdot(0{,}15-0{,}022)\right]$$

$$\boxed{\bar{\tau} = 0{,}3\ \frac{N}{m^2}.}$$

Pauta P.2, Control 3.

- Se asumen las siguientes variables para definir F_A, F_S

$$F_A = F_A(D, V, \omega, d, \rho, \mu) \; ; \; F_S = F_S(D, V, \omega, d, \rho, \mu)$$

$$D \Rightarrow L \qquad V \Rightarrow \frac{L}{T} \qquad ; \qquad \mu \Rightarrow \frac{M}{LT} \; ; \; F = M \cdot L T^{-2}$$

$$d \Rightarrow L \qquad \rho \Rightarrow \frac{M}{L^3}$$

$$\omega \Rightarrow \frac{1}{T}$$

(1 pto).

- Número de grupos adimensionales:

$$7 - 3 = 4.$$

- Variables a repetir: V, ρ, D.

- $\Pi_1 = \omega \cdot V^a \rho^b D^c \Rightarrow M^0 L^0 T^0$

$$\Rightarrow T^{-1} (LT^{-1})^a (ML^{-3})^b (L)^c = M^0 L^0 T^0$$

$$-1 - a = 0 \rightarrow a = -1$$

$$a - 3b + c = 0 \longrightarrow c = 1$$

$$b = 0$$

$$\Rightarrow \boxed{\Pi_1 = \frac{\omega D}{V}}$$

1 pto

- $\Pi_2 = d V^a \rho^b D^c \Rightarrow M^0 L^0 T^0$

$$L (LT^{-1})^a (ML^{-3})^b (L)^c = M^0 L^0 T^0$$

(1 pto)

$$\left.\begin{array}{l} -a = 0 \rightarrow a = 0 \\ 1 + a - 3b + c = 0 \longrightarrow c = -1 \\ b = 0 \end{array}\right\} \Rightarrow \boxed{\Pi_2 = \frac{d}{D}}$$

$$\Pi_3 = \mu V^a \rho^b D^c \Rightarrow M^0 L^0 T^0$$

$$ML^{-1}T^{-1} \cdot (LT^{-1})^a (ML^{-3})^b (L)^c = M^0 L^0 T^0$$

$$1 + b = 0 \rightarrow b = -1$$

$$-1 + a - 3b + c = 0 \rightarrow c = -1$$

$$-1 - a = 0 \rightarrow a = -1$$

$$\Rightarrow \Pi_3 = \frac{\mu}{V \cdot D \rho} = \frac{1}{Re} \quad (1 \text{ pto})$$

$$\Pi_4 = F \cdot V^a \rho^b D^c \Rightarrow M^0 L^0 T^0$$

$$(MLT^{-2}) \cdot (LT^{-1})^a (ML^{-3})^b (L)^c = M^0 L^0 T^0$$

$$1 + b = 0 \rightarrow \boxed{b = -1}$$

$$1 + a - 3b + c = 0 \longrightarrow \boxed{c = -2}$$

$$-2 - a = 0 \rightarrow \boxed{a = -2}$$

$$\Pi_4 = \frac{F}{V^2 \rho D^2} \quad (1 \text{ pto})$$

- Diámetro del modelo. (0,5 ptos)

$$\left.\frac{\rho V D}{\mu}\right|_{modelo} = \left.\frac{\rho V D}{\mu}\right|_{prototipo} \therefore D_M = \frac{\rho_P}{\rho_M} \cdot \frac{V_P}{V_m} \frac{\mu_m}{\mu_P} D_P = 1 * \frac{240}{80} * 1 * D$$

$$\Rightarrow D_m = 3 D_p = 3 * 1{,}68 \text{ in} = 5{,}04 \text{ in}$$

$$D_m = 12{,}8016$$

- Velocidad de rotación del modelo: (0,5 ptos)

$$\left.\frac{WD}{V}\right|_M = \left.\frac{WD}{V}\right|_P \Rightarrow W_M = W_P \frac{D_P}{D_m} \cdot \frac{V_m}{V_P} = W_P \cdot \frac{1}{3} \cdot \frac{80}{240} = \frac{1}{9} W_P$$

$$\Rightarrow W_m = \frac{1}{9} W_P = \frac{1}{9} \cdot 9000 \text{ rpm} = 1000 \text{ rpm}$$