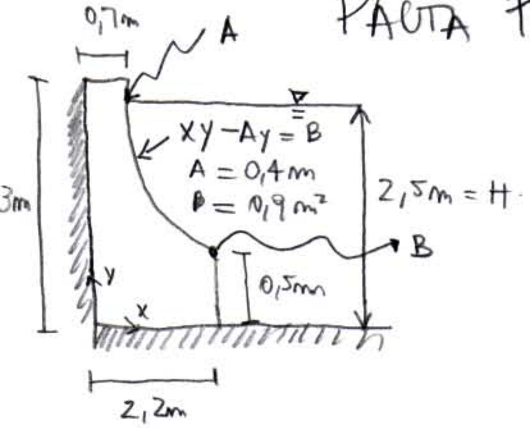
PAUTA Problema 1, Control 1



* Fuerza Vertical

$$F_v = \int \rho \, dA_y$$

$$F_v = \int_{x_A}^{x_B} \rho g h b \, dx = \rho g b \int_{x_A}^{x_B} (H - y) \, dx$$

$$y(x - A) = B \rightarrow y = \frac{B}{x - A}$$

$$\Rightarrow F_v = \rho g b \int_{x_A}^{x_B} \left(H - \frac{B}{x - A}\right) dx$$

$$F_v = \rho g b \left[Hx - B \ln(x - A) \right] \Big|_{x_A}^{x_B}$$

$$F_v = \rho g b \left[H(x_B - x_A) - B \ln\left(\frac{x_B - A}{x_A - A}\right) \right]$$

Reemplazando $\rho = 1000 \, kg/m^3$; $g = 9{,}8 \, m/s^2$; $b = 50m$, $H = 2{,}5m$

$A = 0{,}4m$; $B = 0{,}9 \, m^2$; $x_A = 0{,}7$, $x_B = 2{,}2$

$$\Rightarrow F_v = 1{,}05 \times 10^6 \, N$$

Para obtener el punto de aplicación:

$$x' F_v = \int x \, dF_v = \int_{x_A}^{x_B} x \rho g b \left(H - \frac{B}{x - A}\right) dx = \rho g b \int_{x_A}^{x_B} \left[Hx - \frac{Bx}{x - A} \right] dx$$

$$x' F_v = \rho g b \left[H \frac{x^2}{2} - Bx - BA \ln(x - A) \right] \Big|_{x_A}^{x_B}$$

$$x' F_v = \rho g b \left[\frac{H}{2}(x_B^2 - x_A^2) - B(x_B - x_A) - BA \ln\left(\frac{x_B - A}{x_A - A}\right) \right]$$

Reemplazando los valores:

$$x' = 1{,}61 \, m$$

Para la fuerza horizontal:

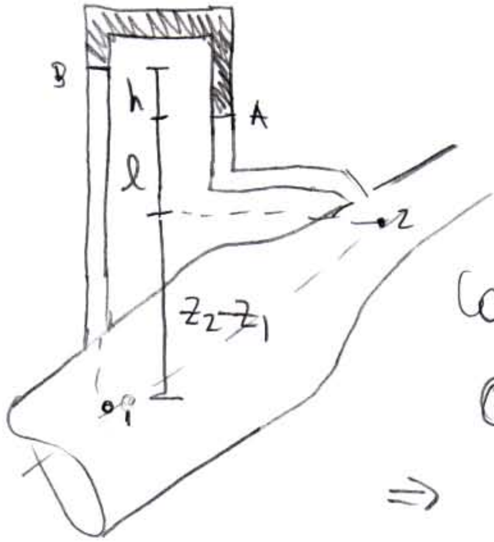
$$F_H = P_c \overset{\text{Area}}{A} = \rho g h_c b H = \rho g \frac{H}{2} b \cdot H = \rho g b \frac{H^2}{2}.$$

Para el punto de aplicación:

$$h' = h_c + \frac{I_{\hat{x}\hat{x}}}{h_c \, A_{\text{Area}}} = \frac{H}{2} + \frac{bH^3}{12} \cdot \frac{1}{\frac{H}{2} \cdot bH} = \frac{H}{2} + \frac{H}{6} = \frac{2}{3} H.$$

$$h' = \frac{2}{3} \cdot 2{,}5\,m =$$

PAUTA Problema 2, control 1



Por Bernoulli:

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$$

Como el caudal se conserva.

$$Q = A_1 V_1 = A_2 V_2$$

$$\Rightarrow \quad V_1 = \frac{Q}{A_1} = \frac{Q}{\left(\frac{\pi D_1^2}{4}\right)} \qquad (2)$$

$$V_2 = \frac{Q}{A_2} = \frac{Q}{\left(\frac{\pi D_2^2}{4}\right)} \qquad (3)$$

Para obtener $P_2 - P_1$:

$$P_2 - P_A = \rho g l$$

$$P_1 - P_B = \rho g (h + l + z_2 - z_1)$$

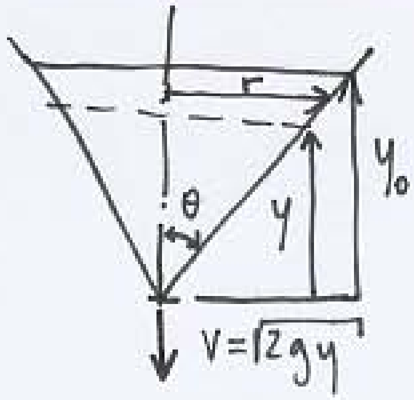
$$P_B - P_A = -SG\rho g h$$

$$\Rightarrow \quad P_2 - P_1 = SG\rho g h + \rho g (z_1 - z_2 - h) \qquad (4)$$

Ahora reemplazando (2), (3) y (4) en (1) y despejando Q:

$$Q = \left(\frac{h\pi^2 g (1 - SG) D_1^4 D_2^4}{8 (D_2^4 - D_1^4)}\right)^{1/2}$$

PAUTA Pregunta 3 Control 1



$$Q_0 = A\sqrt{2gy_0}$$

$$\boxed{r = y\,tg\,\theta}$$

Conservación de masa: $0 = \frac{\partial}{\partial t}\int_{cv}\rho\, d\forall + \int_{cs}\rho \vec{V} d\vec{A}$

Volumen: $\pi r^2 \cdot y = \frac{\pi y^3 tg^2\theta}{3}$

$$\Rightarrow 0 = \rho \frac{\partial}{\partial t}\left(\frac{\pi\, tg^2\theta\, y^3}{3}\right) + \rho A\sqrt{2gy}$$

$$0 = \cancel{\rho}\, \pi\, tg^2\theta\, y^2 \frac{dy}{dt} + \cancel{\rho} A\sqrt{2gy}$$

$$y^{3/2} dy = \frac{-\sqrt{2g}\,A}{\pi\, tg^2\theta}\, dt \qquad \Big/ \int_0^{y_0} \rightarrow \int_t^0$$

$$\frac{2}{5} y_0^{5/2} = \frac{\sqrt{2g}\,A}{\pi\, tg^2\theta}\, t$$

$$\Rightarrow \boxed{t = \frac{2}{5}\frac{tg^2\theta \cdot \pi\, y_0^{5/2}}{\sqrt{2g}\,A}}$$

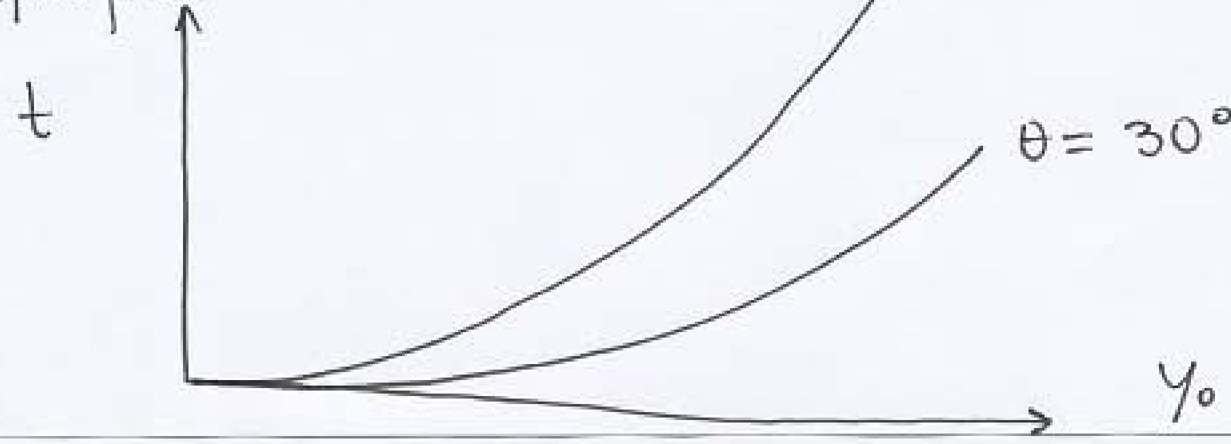
→ 4 puntos hasta acá.

$$t = \frac{2\cdot 3}{5} \cdot \underbrace{\frac{1}{3}\pi y_0^3\, tg^2\theta}_{\forall_0} \cdot \underbrace{\frac{1}{A y_0^{1/2}\sqrt{2g}}}_{1/Q_0}$$

$$\Rightarrow \boxed{t = \frac{6}{5}\frac{\forall_0}{Q_0}}$$

→ 1 punto.

Gráfico:



→ 1 punto