Universidad de Chile
Facultad de Ciencias Físicas y Matemáticas
Departamento de Ingeniería Matemática

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Problemas Resueltos

Problemas

1. Interpole la función $sen(\pi x)$ dentro del intervalo $[-1,1]$ utilizando Lagrange en los puntos $-1, -\frac{1}{2}, 0, \frac{1}{2}, 1$. Acote el error de lagrange. Grafique $sen(\pi x), L_4(x)$ y $T_5(x)$ (polinomio de Taylor)

2. Encuentre el polinomio de newton con los siguientes datos, y vea a que función se parece

i	x_i	$f(x_i)$
0	-1	$\frac{1}{2}$
1	0	1
2	1	2
3	2	4
4	4	16

3. Dada la función $f(x) = \frac{1}{x+1}$, encuentre el polinomio de newton que interpole a f y a su derivada f' en los puntos $0, 1, 3$.

4. Por las fuertes lluvias que se pronostican para el mes de Mayo, se desean hacer arreglos en la carretera Autopista Central para evitar inundaciones, pero el Ingeniera que diseño la carretera era de la Universidad de las Américas, y perdió los planos. El gerente, desesperado, llama a un Ingeniero amigo suyo de Beauchef, y le plantea el problema. El Beauchefiano, que pasó Calculo Numérico, sabe muy bien que hacer. Le pide al gerente una tabla de datos tomados en los Portales de Peaje que indiquen tiempo y posición de un vehículo. La tabla que el gerente le entrega es la siguiente:

t	0	3	5	8	11
x	0	225	383	623	1001
y	0	9	25	64	121
y'	0				22

Haga lo que un Ingeniero de Beauchef haría.

Hint: Use Spline Cúbica y Polinomio de Newton

5. Determine el sistema para aproximar una función $f(x)$ en un intervalo $[\alpha, \beta]$ por un exponencial de la forma $y(x) = be^{ax}$, donde a y b son las constantes a determinar.

6. Use los ceros de $\tilde{T}_3$ y las transformaciones del intervalo dado y construya un polinomio interpolante de segundo grado para $f(x) = x \ln x$,$[1,3]$

7. Obtenga el polinomio trigonométrico general de mínimos cuadrados continuos para $f(x) = \begin{matrix} 0 & si & -\pi < x \leq 0 \\ 1 & si & 0 < x < \pi \end{matrix}$

8. Detemine el polinomio trigonométrico $S_2(x)$ en $[-\pi, \pi]$ para $f(x) = x(\pi - x)$

Soluciones

1. Lagrange

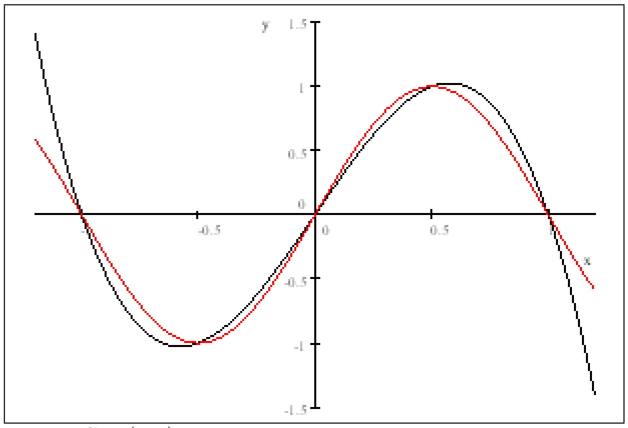
i	x_i	$f(x_i)$	$L_{4,i}(x)$		$L_{4,i}(x)$
0	-1	0	$\frac{(x+\frac{1}{2})x(x-\frac{1}{2})(x-1)}{(-1+\frac{1}{2})(-1)(-1-\frac{1}{2})(-1-1)}$		$\frac{1}{6}x - \frac{1}{6}x^2 - \frac{2}{3}x^3 + \frac{2}{3}x^4$
1	$-\frac{1}{2}$	-1	$\frac{(x+1)x(x-\frac{1}{2})(x-1)}{(-\frac{1}{2}+1)(-\frac{1}{2})(-\frac{1}{2}-\frac{1}{2})(-\frac{1}{2}-1)}$		$-\frac{4}{3}x + \frac{8}{3}x^2 + \frac{4}{3}x^3 - \frac{8}{3}x^4$
2	0	0	$\frac{(x+1)(x+\frac{1}{2})(x-\frac{1}{2})(x-1)}{(1)(\frac{1}{2})(\frac{1}{2})(-1)}$	$=$	$5x^2 - 4x^4 - 1$
3	$\frac{1}{2}$	1	$\frac{(x+1)(x+\frac{1}{2})x(x-1)}{(\frac{1}{2}+1)(\frac{1}{2}+\frac{1}{2})\frac{1}{2}(\frac{1}{2}-1)}$		$\frac{4}{3}x + \frac{8}{3}x^2 - \frac{4}{3}x^3 - \frac{8}{3}x^4$
4	1	0	$\frac{(x+1)(x+\frac{1}{2})x(x-\frac{1}{2})}{(1+1)(1+\frac{1}{2})1(1-\frac{1}{2})}$		$-\frac{1}{6}x - \frac{1}{6}x^2 + \frac{2}{3}x^3 + \frac{2}{3}x^4$

$$P_4(x) = 0 \times (L_{4,0}(x)) + (-1) \times \left(-\frac{4}{3}x + \frac{8}{3}x^2 + \frac{4}{3}x^3 - \frac{8}{3}x^4\right) + 0 \times (L_{4,2}(x)) + 1 \times \left(\frac{4}{3}x + \frac{8}{3}x^2 - \frac{4}{3}x^3 - \frac{8}{3}x^4\right) + 0 \times (L_{4,4}(x))$$

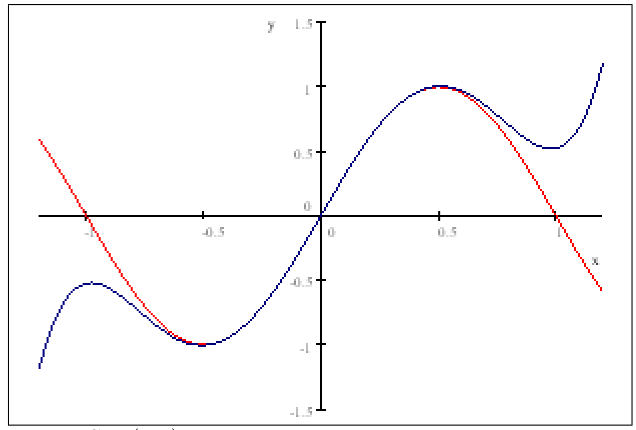
$$P_4(x) = \frac{4}{3}x + \frac{8}{3}x^2 - \frac{4}{3}x^3 - \frac{8}{3}x^4 + \frac{4}{3}x - \frac{8}{3}x^2 - \frac{4}{3}x^3 + \frac{8}{3}x^4 = \frac{8}{3}x - \frac{8}{3}x^3$$

b. $E_4(x) = \Pi_{i=0}^{4}(x-x_i)\frac{f^{(5)}(\xi)}{(5)!} = (x+1)\left(x+\frac{1}{2}\right)x\left(x-\frac{1}{2}\right)(x-1)\,\frac{\pi^5\cos(\pi\xi)}{120} \leq \pi^5\left(\frac{1}{2}\right)^5 = \frac{1}{32}\pi^5 \simeq 9.563$

c. $T_5(x) = \pi x - \frac{1}{6}\pi^3 x^3 + \frac{1}{120}\pi^5 x^5$



Rojo: $Sen(\pi x)$
Negro: Lagrange $\frac{8}{3}x - \frac{8}{3}x^3$



Rojo: $Sen(\pi x)$
Azul: Taylor $\pi x - \frac{1}{6}\pi^3 x^3 + \frac{1}{120}\pi^5 x^5$

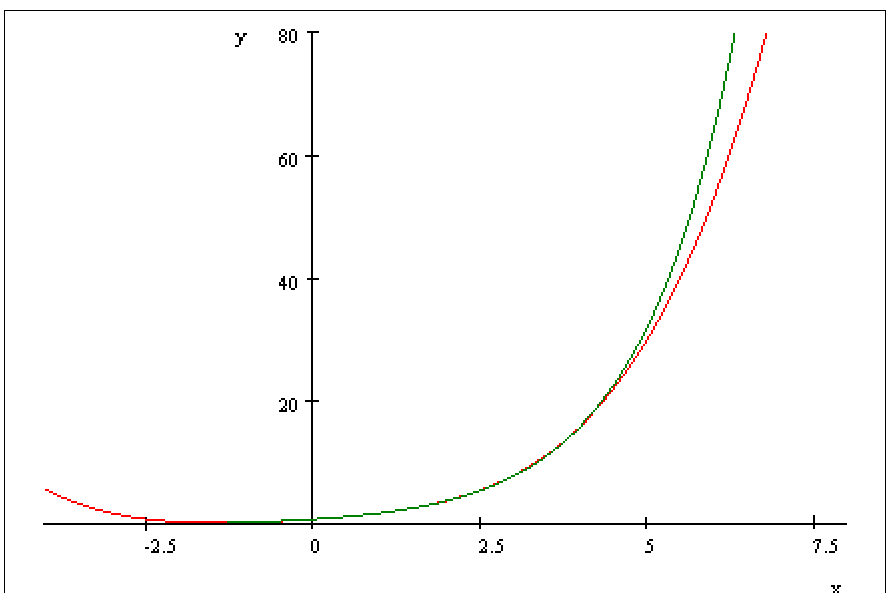
2. Newton

x_i	$f(x_i)$	$f[x_i,x_{i+1}]$	$f[x_i,x_{i+1},x_{i+2}]$	$f[x_i,x_{i+1},x_{i+2},x_{i+3}]$	$f[x_i,x_{i+1},x_{i+2},x_{i+3},x_{i+4}]$
-1	$\frac{1}{2}$	$\frac{1-\frac{1}{2}}{0+1} = 0.5$	$\frac{1-0.5}{1+1} = 0.25$	$\frac{0.5-0.25}{2+1} = \frac{1}{12}$	$\frac{\frac{5}{24}-\frac{1}{12}}{4+1} = \frac{1}{40}$
0	1	$\frac{2-1}{1-0} = 1$	$\frac{2-1}{2-0} = 0.5$	$\frac{\frac{4}{3}-0.5}{4-0} = \frac{5}{24}$	
1	2	$\frac{4-2}{2-1} = 2$	$\frac{6-2}{4-1} = \frac{4}{3}$		
2	4	$\frac{16-4}{4-2} = 6$			
4	16				

$P_4(x) = f[x_0] + (x-x_0)\,f[x_0,x_1] + (x-x_0)(x-x_1)\,f[x_0,x_1,x_2] + ... + \Pi_{j=0}^{n}(x-x_j)f[x_0...x_n]$

$P_4(x) = \frac{1}{2} + (x+1)\,0.5 + (x+1)\,x0.25 + (x+1)\,x(x-1)\frac{1}{12} + (x+1)\,x(x-1)(x-2)\frac{1}{40}$

$P_4(x) = 1 + \frac{43}{60}x + \frac{9}{40}\,x^2 + \frac{1}{30}x^3 + \frac{1}{40}x^4$



Rojo:Newton $1 + \frac{43}{60}x + \frac{9}{40}\,x^2 + \frac{1}{30}x^3 + \frac{1}{40}x^4$ Verde:2^x

3. Newton con derivadas

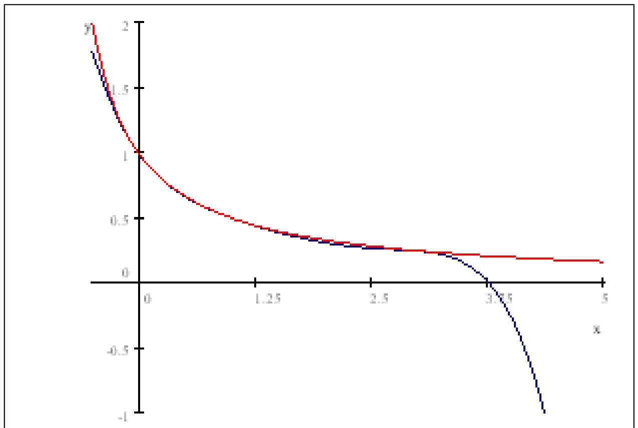
$f'(x) = -\frac{1}{(x+1)^2}$

x_i	$f(x_i)$	$f[x_i,x_{i+1}]$	$f[x_i,x_{i+1},x_{i+2}]$	$f[x_i,x_{i+1},x_{i+2},x_{i+3}]$	$f[x_i,x_{i+1},x_{i+2},x_{i+3},x_{i+4}]$	$f[x_i,x_{i+1},x_{i+2},x_{i+3},x_{i+4},x_{i+}$
0	1	$f'(0) = -1$	$\frac{-\frac{1}{2}+1}{1-0} = \frac{1}{2}$	$\frac{\frac{1}{4}-\frac{1}{2}}{1-0} = -\frac{1}{4}$	$\frac{-\frac{1}{16}+\frac{1}{4}}{3-0} = \frac{1}{16}$	$\frac{\frac{1}{64}-\frac{1}{16}}{3-0} = -\frac{1}{64}$
0	1	$\frac{\frac{1}{2}-1}{1-0} = -\frac{1}{2}$	$\frac{-\frac{1}{4}+\frac{1}{2}}{1-0} = \frac{1}{4}$	$\frac{\frac{1}{16}-\frac{1}{4}}{3-0} = -\frac{1}{16}$	$\frac{-\frac{1}{64}+\frac{1}{16}}{3-0} = \frac{1}{64}$	
1	$\frac{1}{2}$	$f'(1) = -\frac{1}{4}$	$\frac{-\frac{1}{8}+\frac{1}{4}}{3-1} = \frac{1}{16}$	$\frac{\frac{1}{32}-\frac{1}{16}}{3-1} = -\frac{1}{64}$		
1	$\frac{1}{2}$	$\frac{\frac{1}{4}-\frac{1}{2}}{3-1} = -\frac{1}{8}$	$\frac{-\frac{1}{16}+\frac{1}{8}}{3-1} = \frac{1}{32}$			
3	$\frac{1}{4}$	$f'(3) = -\frac{1}{16}$				
3	$\frac{1}{4}$					

$P_5(x) = 1 + x\,(-1) + x^2\frac{1}{2} + x^2(x-1)\left(-\frac{1}{4}\right) + x^2(x-1)^2\frac{1}{16} + x^2(x-1)^2(x-3)\left(-\frac{1}{64}\right) : \frac{55}{64}x^2 - \frac{31}{64}x^3 + \frac{9}{64}x^4 - \frac{1}{64}x^5$

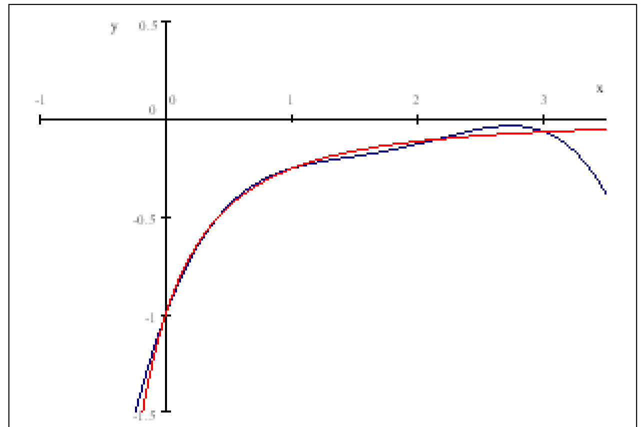
$P_5(x) = 1 - x + \frac{55}{64}x^2 - \frac{31}{64}x^3 + \frac{9}{64}x^4 - \frac{1}{64}x^5$

$\frac{\partial P_5(x)}{\partial x} = \frac{55}{32}x - \frac{93}{64}x^2 + \frac{9}{16}x^3 - \frac{5}{64}x^4 - 1$



Rojo: $\frac{1}{x+1}$

Azul: $1 - x + \frac{55}{64}x^2 - \frac{31}{64}x^3 + \frac{9}{64}x^4 - \frac{1}{64}x^5$



Rojo: $-\frac{1}{(x+1)^2}$

Azul: derivada $\frac{55}{32}x - \frac{93}{64}x^2 + \frac{9}{16}x^3 - \frac{5}{64}x^4 - 1$

4. Primero se construiye un Spline cúbico $S_x(t)$ el cual depende de $S_i(t), i = 0, 1, 2, 3$, donde

$$S_x(t) = \begin{cases} S_0(t) = S_{0,0} + S_{0,1}(t-0) + S_{0,2}(t-0)^2 + S_{0,3}(t-0)^3 & t \in [0,3] \\ S_1(t) = S_{1,0} + S_{1,1}(t-3) + S_{1,2}(t-3)^2 + S_{1,3}(t-3)^3 & t \in [3,5] \\ S_2(t) = S_{2,0} + S_{2,1}(t-5) + S_{2,2}(t-5)^2 + S_{2,3}(t-5)^3 & t \in [5,8] \\ S_3(t) = S_{3,0} + S_{3,1}(t-8) + S_{3,2}(t-8)^2 + S_{3,3}(t-8)^3 & t \in [8,10] \end{cases}$$

La matriz es

$$\begin{bmatrix} 1^\circ & \text{Condicion} & \text{de} & \text{borde} & \\ h_0 & 2\,(h_0+h_1) & h_1 & 0 & 0 \\ 0 & h_1 & 2\,(h_1+h_2) & h_2 & 0 \\ 0 & 0 & h_2 & 2\,(h_2+h_3) & h_3 \\ 2^\circ & \text{Condicion} & \text{de} & \text{borde} & \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} \mu_0 \\ \mu_1 = 3\,(d_1 - d_0) \\ \mu_2 = 3\,(d_2 - d_1) \\ \mu_3 = 3\,(d_3 - d_2) \\ \mu_4 \end{bmatrix}$$

Los coeficientes son

$$\begin{array}{ll} h_0 = 3 - 0 = 3 & d_0 = \frac{225-0}{3} = 75 \\ h_1 = 5 - 3 = 2 & d_1 = \frac{383-225}{2} = 79 \\ h_2 = 8 - 5 = 3 & d_2 = \frac{623-383}{3} = 80 \\ h_3 = 11 - 8 = 3 & d_3 = \frac{1001-623}{3} = 126 \end{array}$$

Reemplazando en la matriz es

$$\begin{bmatrix} & & & & \\ 3 & 10 & 2 & 0 & 0 \\ 0 & 2 & 10 & 3 & 0 \\ 0 & 0 & 3 & 12 & 3 \\ & & & & \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} \mu_0 \\ 12 \\ 3 \\ 138 \\ \mu_4 \end{bmatrix}$$

C.B: Spline Natural

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 3 & 10 & 2 & 0 & 0 \\ 0 & 2 & 10 & 3 & 0 \\ 0 & 0 & 3 & 12 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 3 \\ 138 \\ 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{116}{59} \\ -\frac{226}{59} \\ \frac{735}{59} \\ 0 \end{bmatrix}$$

$S_{k,0} = y_k \qquad S_{k,1} = d_k - \frac{h_k(2m_k + m_{k+1})}{3} \qquad S_{k,2} = m_k \qquad S_{k,3} = \frac{m_{k+1} - m_k}{3h_k}$

$$S_x(t) = \begin{cases} S_0(t) = 0 + \left(75 - \frac{3(2\times 0 + \frac{116}{59})}{3}\right)(t-0) + 0 \times (t-0)^2 + \frac{\frac{116}{59} - 0}{3\times 3}(t-0)^3 & t \in [0,3] \\ S_1(t) = 225 + \left(79 - \frac{2(2\times\frac{116}{59} - \frac{226}{59})}{3}\right)(t-3) + \frac{116}{59}(t-3)^2 + \frac{-\frac{226}{59} - \frac{116}{59}}{3\times 2}(t-3)^3 & t \in [3,5] \\ S_2(t) = 383 + \left(80 - \frac{3(2\times\left(-\frac{226}{59}\right) + \frac{735}{59})}{3}\right)(t-5) + \left(-\frac{226}{59}\right)(t-5)^2 + \frac{\frac{735}{59} + \frac{226}{59}}{3\times 3}(t-5)^3 & t \in [5,8] \\ S_3(t) = 623 + \left(126 - \frac{3(2\times\frac{735}{59} + 0)}{3}\right)(t-8) + \frac{735}{59} \times (t-8)^2 + \frac{0 - \frac{735}{59}}{3\times 3}(t-8)^3 & t \in [8,11] \end{cases}$$

$$S_x(t)=\begin{cases}\frac{4309}{59}t+\frac{116}{531}t^3 & \text{if} \quad 0\le t\wedge t\le 3\\ \frac{2422}{59}t+\frac{629}{59}t^2-\frac{57}{59}t^3+\frac{1887}{59} & \text{if} \quad 3\le t\wedge t\le 5\\ \frac{44\,116}{177}t-\frac{5483}{177}t^2+\frac{961}{531}t^3-\frac{167\,267}{531} & \text{if} \quad 5\le t\wedge t\le 8\\ \frac{2695}{59}t^2-364t-\frac{245}{177}t^3+\frac{233\,695}{177} & \text{if} \quad 8\le t\wedge t\le 11\end{cases}$$

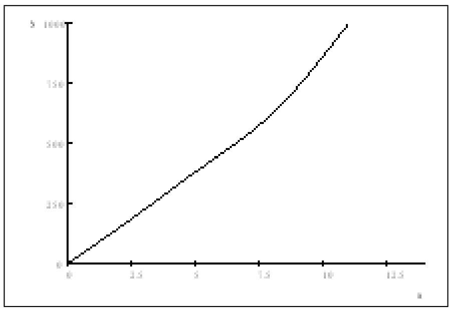
$$\begin{gathered}S(0)=0\\ S(3)=225\\ S(5)=383\\ S(8)=623\\ S(11)=1001\end{gathered}$$

$$D_x(t)=\begin{cases}\frac{116}{177}t^2+\frac{4309}{59} & \text{if} \quad 0\le t\wedge t\le 3\\ \frac{1258}{59}t-\frac{171}{59}t^2+\frac{2422}{59} & \text{if} \quad 3\le t\wedge t\le 5\\ -\frac{10\,966}{177}t+\frac{961}{177}t^2+\frac{44\,116}{177} & \text{if} \quad 5\le t\wedge t\le 8\\ \frac{5390}{59}t-\frac{245}{59}t^2-364 & \text{if} \quad 8\le t\wedge t\le 11\end{cases}$$

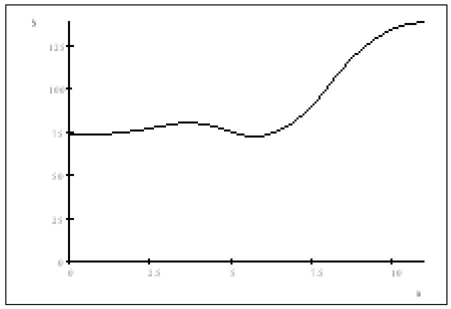
$$\begin{aligned}D(0)&=\tfrac{17\,807}{241}=73.887\,966\,804\,979\,253\,112\\ D(3)&=\tfrac{18\,611}{241}=77.224\,066\,390\,041\,493\,776\\ D(5)&=\tfrac{19\,359}{241}=80.327\,800\,829\,875\,518\,672\\ D(8)&=\tfrac{18\,804}{241}=78.024\,896\,265\,560\,165\,975\\ D(11)&=\tfrac{87\,909}{1205}=72.953\,526\,970\,954\,356\,846\end{aligned}$$

$$F_x(t)=\begin{cases}\frac{232}{177}t & \text{if} \quad 0\le t\wedge t\le 3\\ -\frac{342}{59}t+\frac{1258}{59} & \text{if} \quad 3\le t\wedge t\le 5\\ \frac{1922}{177}t-\frac{10\,966}{177} & \text{if} \quad 5\le t\wedge t\le 8\\ -\frac{490}{59}t+\frac{5390}{59} & \text{if} \quad 8\le t\wedge t\le 11\end{cases}$$

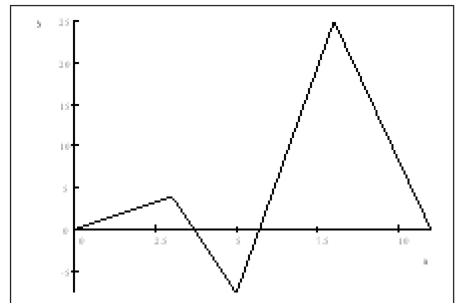
$$\begin{aligned}F(0)&=0\\ F(3)&=\tfrac{232}{59}=3.932\,203\,389\,830\,508\,474\,6\\ F(5)&=-\tfrac{452}{59}=-7.661\,016\,949\,152\,542\,372\,9\\ F(8)&=\tfrac{1470}{59}=24.915\,254\,237\,288\,135\,593\\ F(11)&=0\end{aligned}$$



$S_x(t)$



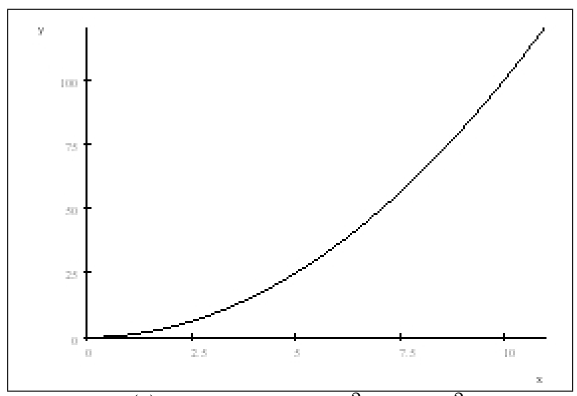
$D_x(t)=\frac{dS_x(t)}{dt}$



$F_x(t)=\frac{d^2S_x(t)}{dt^2}$

Luego, para la coordenada y hacemos una interpolación de Newton

0	0	$y'(0)=0$	$\frac{3-0}{3-0}=1$	0	0	0	0
0	0	$\frac{9-0}{3-0}=3$	$\frac{8-3}{5-0}=1$	0	0	0	
3	9	$\frac{25-9}{5-3}=8$	$\frac{13-8}{8-3}=1$	0	0		
5	25	$\frac{64-25}{8-5}=13$	$\frac{19-13}{11-5}=1$	0			
8	64	$\frac{121-64}{11-8}=19$	$\frac{22-19}{11-8}=1$				
11	121	$y'(11)=22$					
11	121						



$y(t)=0+0\times t+1t^2+0=t^2$

despejando $t\Longrightarrow t=\sqrt{y}$

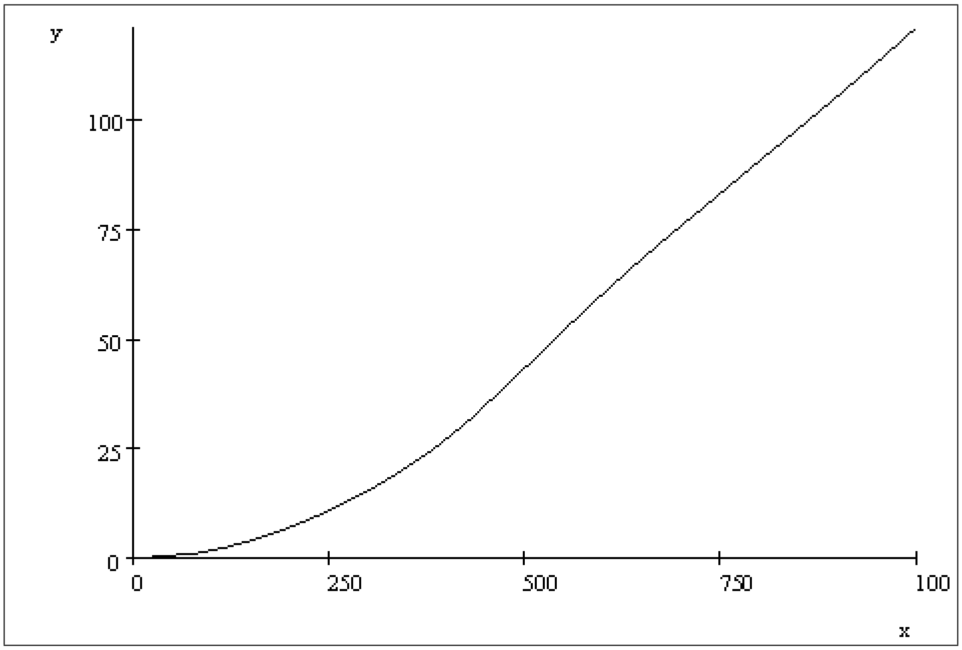
reemplazando en $S_x(t)$

$$S_x(t)=S_x(\sqrt{y})=\begin{cases}\frac{4309}{59}\sqrt{y}+\frac{116}{531}\sqrt{y}^3 & \text{if} \quad 0\le\sqrt{y}\wedge\sqrt{y}\le 3\\ \frac{2422}{59}\sqrt{y}+\frac{629}{59}\sqrt{y}^2-\frac{57}{59}\sqrt{y}^3+\frac{1887}{59} & \text{if} \quad 3\le\sqrt{y}\wedge\sqrt{y}\le 5\\ \frac{44\,116}{177}\sqrt{y}-\frac{5483}{177}\sqrt{y}^2+\frac{961}{531}\sqrt{y}^3-\frac{167\,267}{531} & \text{if} \quad 5\le\sqrt{y}\wedge\sqrt{y}\le 8\\ \frac{2695}{59}\sqrt{y}^2-364\sqrt{y}-\frac{245}{177}\sqrt{y}^3+\frac{233\,695}{177} & \text{if} \quad 8\le\sqrt{y}\wedge\sqrt{y}\le 11\end{cases}$$

transformando en una función que depende de y :

$$\Longrightarrow S_x(y)=\begin{cases}\frac{4309}{59}\sqrt{y}+\frac{116}{531}\sqrt{y}^3 & \text{if} \quad 0\le y\wedge y\le 9\\ \frac{2422}{59}\sqrt{y}+\frac{629}{59}y-\frac{57}{59}\sqrt{y}^3+\frac{1887}{59} & \text{if} \quad 9\le y\wedge y\le 25\\ \frac{44\,116}{177}\sqrt{y}-\frac{5483}{177}y+\frac{961}{531}\sqrt{y}^3-\frac{167\,267}{531} & \text{if} \quad 25\le y\wedge y\le 64\\ \frac{2695}{59}y-364\sqrt{y}-\frac{245}{177}\sqrt{y}^3+\frac{233\,695}{177} & \text{if} \quad 64\le y\wedge y\le 121\end{cases}$$

Graficando en un plano xy se obtiene el plano pedido.



5. Primero se hace una transformación para tener un sistema lineal. En este caso se usa el ln

   (a) $y = be^{ax} \Longrightarrow \ln(y) = \ln(b) + ax$

   Ahora buscamos el minimo del error cuadrático

   (b) $E = \int_\alpha^\beta (\ln(f(x)) - (\ln(b) + ax))^2 dx$

   (c) $\frac{\partial E}{\partial a} = \int_\alpha^\beta x\,(2\ln b + 2ax - 2\ln(f(x)))\,dx = 0 \Longrightarrow \int_\alpha^\beta (x\ln b + ax^2)dx = \int_\alpha^\beta x\ln(f(x))\,dx$

   (d) $\frac{\partial E}{\partial b} = \frac{1}{b}\int_\alpha^\beta (2\ln b + 2ax - 2\ln(f(x)))\,dx = 0 \Longrightarrow \int_\alpha^\beta (\ln b + ax)dx = \int_\alpha^\beta \ln(f(x))\,dx$

   El sistema de forma matricial queda de la forma

   (e) $\begin{bmatrix} \int_\alpha^\beta x\,dx & \int_\alpha^\beta x^2 dx \\ \int_\alpha^\beta dx & \int_\alpha^\beta x dx \end{bmatrix} \begin{bmatrix} \ln(b) \\ a \end{bmatrix} = \begin{bmatrix} \int_\alpha^\beta x\ln(f(x))\,dx \\ \int_\alpha^\beta \ln(f(x))\,dx \end{bmatrix}$

6. Primero, los ceros de $\tilde{T}_3$ se encuentran en $\bar{x}_k = \cos(\frac{2k-1}{2n}\pi), k = 1, 2, 3.$

   (a) $\bar{x}_1 = \cos(\frac{1}{6}\pi) = 0.866\,025\,403\,784\,438\,646\,76$

   (b) $\bar{x}_2 = \cos(\frac{3}{6}\pi) = 0.0$

   (c) $\bar{x}_3 = \cos(\frac{5}{6}\pi) = -0.866\,025\,403\,784\,438\,646\,76$

   Debemos usar una transformacion lineal para pasar de $[-1, 1]$ a $[1, 3]$.Esto es $\tilde{x}_k = 2 + \bar{x}_k$.

   (d) $\tilde{x}_1 = 2.\,866\,025\,403\,784\,438\,646\,8 \simeq 2.866$

   (e) $\tilde{x}_2 = 2$

   (f) $\tilde{x}_3 = 1.\,133\,974\,596\,215\,561\,353\,2 \simeq 1.134$

   Ahora debemos calcular los valores de $f(x)$ en $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$

   (g) $f(\tilde{x}_1) = f(2.866) = 3.\,017\,661\,066\,077\,538\,987\,9 \simeq 3.\,018$

   (h) $f(\tilde{x}_2) = f(2) = 1.\,386\,294\,361\,119\,890\,618\,8 \simeq 1.\,386$

   (i) $f(\tilde{x}_3) = f(1.134) = 0.142\,601\,866\,816\,505\,412\,01 \simeq 0.143$

   Las diferencias divididas son:

   (j)

x	$f(x)$	$f[x,x]$	$f[x,x,x]$
2.866	3.018	$\frac{1.386-3.018}{2-2.866} = 1.885$	$\frac{1.435-1.885}{1.134-2.866} = 0.26$
2	1.386	$\frac{0.143-1.386}{1.134-2} = 1.435$	
1.134	0.143		

   El polinomio interpolante de segunda grado es:

   (k) $\tilde{P}_3(x) = 3.\,018 + 1.\,885(x - 2.866) + 0.26(x - 2.866)(x - 2) = 0.619\,84x + 0.26x^2 - 0.894\,09$

   El error del polinomio está acotado de la forma

   (l) $\max_{x\in[1,3]} \left|\tilde{P}_3(x) - f(x)\right| \leq \frac{1}{2^3(3+1)!} \max_{x\in[1,3]} \left|f^{(4)}(x)\right| = \frac{1}{192} \max_{x\in[1,3]} \left|\frac{1}{2x^3}\right| = \frac{1}{192}\frac{1}{2} = \frac{1}{384} < 0.002\,604\,2$

7. Calculemos a_k y b_k

(a) $a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(kx)dx = \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos(kx)dx + \frac{1}{\pi}\int_{0}^{\pi} f(x)\cos(kx)dx = \frac{1}{\pi}\int_{0}^{\pi}\cos(kx)dx = \frac{1}{\pi k}\ sen(kx)|_0^{\pi} = 0$

(b) $b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sen(kx)dx = \frac{1}{\pi}\int_{0}^{\pi} sen(kx)dx = -\frac{1}{\pi k}\cos(kx)|_0^{\pi} = \frac{1}{\pi k}(1-(-1)^k)$

Luego, el polinomio general $S_n(x)$ es

(c) $S_n(x) = \frac{a_0}{2} + a_n\cos(nx) + \sum_{k=1}^{n-1} a_k\cos(kx) + b_k sen(kx) = \sum_{k=1}^{n-1}\frac{1}{\pi k}(1-(-1)^k)sen(kx)$

8. El polinomio es $S_n(x) = \frac{a_0}{2} + a_2\cos(2x) + a_1\cos(x) + b_1 sen(x)$. Calculemos los coeficientes

(a) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x(\pi - x)dx = \int_{-\pi}^{\pi} xdx - \frac{1}{\pi}\int_{-\pi}^{\pi} x^2dx = x^2\Big|_{-\pi}^{\pi} - \frac{1}{3\pi}\ x^3\Big|_{-\pi}^{\pi} = -\frac{2}{3}\pi^2$

(b) $a_1 = \frac{1}{\pi}\int_{-\pi}^{\pi} x(\pi - x)\cos(x)dx = \int_{-\pi}^{\pi} x\cos(x)dx - \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(x)dx = -\frac{1}{\pi}\left[x^2 sen(x)\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} 2x sen(x)dx\right] =$
$-\frac{2}{\pi}\int_{-\pi}^{\pi} x sen(x)dx = -\frac{2}{\pi}\left[-\ x\cos(x)|_{-\pi}^{\pi} + \int_{\pi}^{-\pi}\cos(x)dx\right] = \frac{2}{\pi}2\pi = 4$

(c) $a_2 = \frac{1}{\pi}\int_{-\pi}^{\pi} x(\pi - x)\cos(2x)dx = \frac{1}{\pi}\int_{-2\pi}^{2\pi}\frac{u}{2}(\pi - \frac{u}{2})\cos(u)\frac{du}{2} = \frac{1}{8\pi}\int_{-2\pi}^{2\pi} u(2\pi - u)\cos(u)du$
$= \frac{1}{4}\int_{-2\pi}^{2\pi} u\cos(u)du - \frac{1}{8\pi}\int_{-2\pi}^{2\pi} u^2\cos(u)du = -\frac{1}{8\pi}\int_{-2\pi}^{2\pi} u^2\cos(u)du = -\frac{1}{8\pi}\left[u^2 sen(u)\Big|_{-2\pi}^{2\pi} - \int_{-2\pi}^{2\pi} 2u sen(u)du\right]$
$= \frac{1}{4\pi}\int_{-2\pi}^{2\pi} u\sin(u)du = \frac{1}{4\pi}\left[-\ x\cos(x)|_{-2\pi}^{2\pi} + \int_{2\pi}^{-2\pi}\cos(x)dx\right] = -\frac{1}{4\pi}4\pi = -1$