$a)$

(i) Se cumple $F(x,y,z)=0$. Del teorema de la función Implicita se deduce :

$$\frac{\partial z}{\partial x}=-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial z}};\qquad \frac{\partial y}{\partial z}=-\frac{\frac{\partial f}{\partial z}}{\frac{\partial f}{\partial y}};\qquad \frac{\partial x}{\partial y}=-\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$$

$\Rightarrow$

$$\frac{\partial z}{\partial x}\frac{\partial y}{\partial z}\frac{\partial x}{\partial y}=-1$$

$ii)$ $\quad f(x+z(x,y),y)=0 \quad$ derivando c/r a x :

$$\frac{\partial f}{\partial u}(u,y)\frac{\partial u}{\partial x}=0 \qquad \text{con } u(x,y)=x+z(x,y)$$

$$\frac{\partial f}{\partial u}(u,y)(1+\frac{\partial z}{\partial x}(x,y))=0$$

$$\frac{\partial z}{\partial x}(x,y)=-1$$

por otra parte : $f(x+z(x,y),y)=0 \qquad$ derivando c/r a y :

$$\frac{\partial f}{\partial u}(u,y)\frac{\partial u}{\partial y}+\frac{\partial f}{\partial y}=0$$

$$\frac{\partial f}{\partial u}(u,y)\frac{\partial z}{\partial y}(x,y)+\frac{\partial f}{\partial y}=0$$

$$\frac{\partial z}{\partial y}(x,y)=-\frac{\frac{\partial f}{\partial y}(x,y)}{\frac{\partial f}{\partial u}(u,y)}=-\frac{\frac{\partial f}{\partial y}(x,y)}{\frac{\partial f}{\partial z}(u,y)}$$

Las derivadas de segundo orden quedan:

$$\frac{\partial^2 z}{\partial x^2}(x,y)=0;\frac{\partial^2 z}{\partial y\partial x}(x,y)=0;$$

$$\frac{\partial^2 z}{\partial y^2}(x,y)=-\frac{\frac{\partial^2 f}{\partial y^2}(x,y)\frac{\partial f}{\partial z}(u,y)-\frac{\partial f}{\partial y}(x,y)\frac{\partial^2 f}{\partial y\partial z}(u,y)}{\left(\frac{\partial f}{\partial z}(u,y)\right)^2}$$

$$\frac{\partial^2 z}{\partial x\partial y}(x,y)=-\frac{\frac{\partial^2 f}{\partial x\partial y}(x,y)\frac{\partial f}{\partial z}(u,y)-\frac{\partial f}{\partial y}(x,y)\frac{\partial^2 f}{\partial x\partial z}(u,y)}{\left(\frac{\partial f}{\partial z}(u,y)\right)^2}$$

$P3$

$b)\, i)\; f(x,y) = x^2 + y^2 + x + y \quad s.a. \qquad Q: x^2 + y^2 \le 1$

$Int\, Q$:

$$\frac{\partial f}{\partial x} = 2x+1 \qquad ; \qquad \frac{\partial f}{\partial y} = 2y+1$$

ptos estacionarios cumplen :

$$2x+1=0 \Rightarrow x_e = -\frac{1}{2}$$

$$2y+1=0 \Rightarrow y_e = -\frac{1}{2}$$

$$H_f = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} > 0 \Rightarrow x_e, y_e \in Int\, Q; \text{mínimos locales.}$$

$Fr\, Q$: método de los multiplicadores de Lagrange

$$L(x,y,t) = x^2 + y^2 + x + y + t(x^2 + y^2 - 1)$$

$$\frac{\partial L}{\partial x} = 2x+1+2xt = 2x(1+t)+1 = 0 \qquad (1)$$

$$\frac{\partial L}{\partial y} = 2y+1+2yt = 2y(1+t)+1 = 0 \qquad (2)$$

$$\frac{\partial L}{\partial t} = x^2 + y^2 - 1 = 0 \qquad (3)$$

$x, y \ne 0, t \ne -1$ (por (1) y (2))

$$\Rightarrow \frac{1}{2y} = \frac{1}{2x} \Rightarrow y = x \quad (4)$$

(4) en (3) entrega como ptos críticos:

$$(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}).$$

evaluando en la función objetivo se obtiene:

$$f(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 1+\sqrt{2}; f(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = 1; f(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 1; f(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}) = 1-\sqrt{2}$$

$f(-\frac{1}{2}, -\frac{1}{2}) = -\frac{1}{2}$. Se obtiene el máximo y el mínimo

$d)$ Planos paralelos tienen vectores normales l.d:

$z(x,y)=x+y$ Tiene por normal $n_1=(1,1,-1)$

Dllo de Taylor en torno a (x_0,y_0):

$$z=z_0+\langle \nabla z(x_0,y_0),(x-x_0,y-y_0)\rangle$$

$$z=\exp(x_0+y_0)+sen(x_0-y_0)+(\exp(x_0+y_0)+\cos(x_0-y_0))(x-x_0)$$
$$+(\exp(x_0+y_0)-\cos(x_0-y_0))(y-y_0)$$

desarrollando, el plano tangente tiene la forma:

$$cte=(\exp(x_0+y_0)+\cos(x_0-y_0))x+(\exp(x_0+y_0)-\cos(x_0-y_0))y-z$$

Que tiene por vector normal:

$$n_2=((\exp(x_0+y_0)+\cos(x_0-y_0)),(\exp(x_0+y_0)-\cos(x_0-y_0)),-1)$$

haciendo $n_1=k*n_2$ $\quad (l.d.)$

Se tiene:

$$x_0=-y_0; \qquad x_0=\frac{\pi}{4}+k\frac{\pi}{2}$$