Lectures on Monetary Policy, Inflation, and the Business Cycle

Chapter 3. *The Basic New Keynesian Model*

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August 2006

In the present chapter I describe the key elements of a baseline sticky price model. In doing so I depart from the assumptions of the classical monetary economy discussed in chapter 2 in two ways. First, I introduce imperfect competition in the goods market, by assuming that each firm produces a differentiated good, for which it sets the price (instead of taking the price as given). Second, I impose some constraints on the price adjustment mechanism, by assuming that only a fraction of firms can reset their prices in any given period. While the resulting inflation dynamics can also be derived under the assumption of quadratic costs of price adjustment, I have chosen instead to present a derivation based on the formalism introduced by Calvo (1983), characterized by staggered price setting with random price durations. The resulting framework constitutes what I will refer to as the basic New Keynesian (NK) model. As discussed in the introduction, that model has become in recent years the workhorse model for the analysis of monetary policy, fluctuations and welfare, and the core framework on which many extensions have been built.

The introduction of differentiated goods requires that the household problem be modified slightly relative to the one considered in the previous chapter. I discuss first that modification, before turning to the firms' optimal price setting problem and the implied inflation dynamics.

1 Households

Once again I assume a continuum of identical, infinitely-lived households. Each household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \ U(C_t, N_t)$$

where C_t is now a consumption index given by

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \ di \right)^{\frac{\epsilon}{\epsilon-1}}$$

with $C_t(i)$ representing the quantity of good i consumed by the household in period t, for $i \in [0,1]$. The period budget constraint now takes the form

$$\int_0^1 P_t(i) \ C_t(i) \ di + Q_t \ B_t \leq B_{t-1} + W_t \ N_t + J_t$$

for $t = 0, 1, 2...$, where $P_t(i)$ is the price of good i, and where the remaining variables are defined as in the previous chapter: N_t denotes hours of work, W_t is the nominal wage, B_t represents purchases of one-period bonds (at a price Q_t), and J_t is a lump-sum component of income (which may include, among other items, dividends from ownership of firms). In addition to the sequence of period budget constraint I assume a solvency condition $\lim_{T \longrightarrow \infty} E_t\{Q_{t,t+T} \ B_{t+T}\} \geq 0$.

In addition to the consumption/savings and labor supply decision analyzed in the previous chapter, the household now must decide how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t be maximized for any given level of expenditures $\int_0^1 P_t(i) \ C_t(i) \ di$. As shown in the appendix, the solution to that problem yields the set of demand equations

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t \tag{1}$$

for all $i \in [0, 1]$, where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon} \ di\right]^{\frac{1}{1-\epsilon}}$ is an aggregate price index. Furthermore, and conditional on such optimal behavior, we have

$$\int_0^1 P_t(i) \ C_t(i) \ di = P_t \ C_t$$

i.e., we can write total consumption expenditures as the product of the price index times the quantity index. Plugging the previous expression in the budget constraint we obtain

$$P_t \ C_t + Q_t \ B_t \leq B_{t-1} + W_t \ N_t + J_t$$

which is formally identical to the constraint facing households in the single good economy analyzed in the previous chapter. Hence, the optimal consumption/savings and labor supply decisions are identical to the ones derived therein, and are thus given by the conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta \ E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \ \frac{P_t}{P_{t+1}} \right\}$$

Under the assumption of a period utility given by $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$, and as shown in the previous chapter, the resulting log-linearized optimality conditions take the form

$$w_t - p_t = \sigma \ c_t + \varphi \ n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - \rho)$$

where $i_t \equiv -\log Q_t$ is the short-term nominal rate and $\rho \equiv -\log \beta$ is the discount rate, and where lower case letter are used to denote the logs of the original variables. As before, the previous conditions are supplemented when necessary with an ad-hoc log-linear money demand equation of the form:

$$m_t - p_t = y_t - \eta \ i_t \tag{2}$$

2 Firms

I assume a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the production function

$$Y_t(i) = A_t \ N_t(i)^{1-\alpha} \tag{3}$$

where A_t represents the level of technology, assumed to be common to all firms and to evolve exogenously over time.

All firms face an identical isoelastic demand schedule, with price elasticity ϵ, given by (1), and to take the aggregate price level P_t and aggregate consumption index C_t as given.

Following the formalism proposed in Calvo (1983), each firm may reset its price only with probability $1-\theta$ in any given period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1-\theta$ of producers reset their prices, while a fraction θ keep their prices unchanged. As a result, the average duration of a price is given by $(1-\theta)^{-1}$. In that context, θ becomes a natural index of price stickiness.

2.1 Aggregate Price Dynamics

As shown in the appendix, the above environment implies aggregate price dynamics described by the equation

$$\Pi_t^{1-\epsilon} = \theta \ + (1-\theta) \ \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \tag{4}$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate.and P_t^* is the price set in period t by firms reoptimizing their price in that period. Notice that, as shown below, all firms will choose the same price since they face an identical problem. It follows from (4) that in a steady state with zero inflation ($\Pi = 1$) we must have $P_t^* = P_{t-1} = P_t$, for all t. Furthermore, a log-linear approximation to the aggregate price index around the zero inflation steady state yields

$$\pi_t = (1-\theta) \ (p_t^* - p_{t-1}) \tag{5}$$

The previous equation makes clear that, in the present setup, inflation results from the fact that firms reoptimizing in any given period choose a price that differs from the economy's average price in the previous period. Hence, and in order to understand the evolution of inflation over time, one needs to analyze the factors underlying firms' price setting decisions, a question to which I turn next.

2.2 Optimal Price Setting

A firm reoptimizing in period t will choose a price P_t^* that maximizes the current market value of the profits generated while that price remains effective. Formally, it solves the following problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \ E_t \left\{ Q_{t,t+k} \ \left(P_t^* \ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} \ C_{t+k} \tag{6}$$

for $k = 0, 1, 2, ...$where $Q_{t,t+k} \equiv \beta^k \ (C_{t+k}/C_t)^{-\sigma} \ (P_t/P_{t+k})$ is the stochastic discount factor for nominal payoffs, $\Psi_t(\cdot)$ is the cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t.

The first order condition associated with the problem above takes the form:

$$\sum_{k=0}^{\infty} \theta^k \ E_t \left\{ Q_{t,t+k} \ Y_{t+k|t} \ \left(P_t^* - \mathcal{M} \ \psi_{t+k|t} \right) \right\} = 0 \tag{7}$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ denotes the (nominal) marginal cost in period $t+k$ for a firm which last reset its price in period t, and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ is the optimal markup in the absence of constraints on the frequency of price adjustment. Henceforth, I refer to $\mathcal{M}$ as the desired or frictionless markup.

Notice that in the limiting case of no price rigidities ($\theta = 0$) the previous condition collapses to the familiar optimal price setting condition under flexible prices

$$P_t^* = \mathcal{M} \ \psi_{t|t}$$

Next I log-linearize the optimal price setting condition above around the zero inflation steady state. Before doing so, however, it is useful to rewrite it in terms of variables that have a well defined value in that steady state. In particular, dividing by P_{t-1} and letting $\Pi_{t,t+k} \equiv (P_{t+k}/P_t)$, we can write

$$\sum_{k=0}^{\infty} \theta^k \ E_t \left\{ Q_{t,t+k} Y_{t+k|t} \ \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} \ MC_{t+k|t} \ \Pi_{t-1,t+k} \right) \right\} = 0 \tag{8}$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ is the real marginal cost in period $t+k$ for a firm whose price was last set in period t.

As shown above, in a zero inflation steady state we must have $P_t^*/P_{t-1} = 1$ and $\Pi_{t-1,t+k} = 1$ Furthermore, constancy of the price level implies that $P_t^* = P_{t+k}$ along that steady state, from which it follows that $Y_{t+k|t} = Y$ and $MC_{t+k|t} = MC$, in addition to $Q_{t,t+k} = \beta^k$, must hold in that steady state. Accordingly, we must have $MC = 1/\mathcal{M}$. A first-order Taylor expansion of (8) around that steady state yields:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\{\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\} \tag{9}$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from steady state.

In order to gain some intuition about the factors determining firms' price setting decision it is useful to rewrite the (9) as follows:

$$p_t^* = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{mc_{t+k|t} + p_{t+k}\}$$

where $\mu \equiv \log \frac{\epsilon}{\epsilon-1}$. Hence, firms resetting their prices will choose a price that corresponds to their desired markup over a weighted average of their current and expected (nominal) marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon, θ^k.

3 Equilibrium

Market clearing in the goods market requires

$$Y_t(i) = C_t(i)$$

for all $i \in [0,1]$ and all t. Letting aggregate output be defined as $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} \, di\right)^{\frac{\epsilon}{\epsilon-1}}$ it follows that

$$Y_t = C_t$$

must hold for all t. One can combine the market clearing condition with the consumer's Euler equation to yield the equilibrium condition.

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - \rho\right) \tag{10}$$

Market clearing in the labor market in turn requires

$$N_t = \int_0^1 N_t(i) \; di$$

Using (3) we have

$$\begin{aligned} N_t &= \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} \; di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} \; di \end{aligned}$$

where the second equality follows from (1) and goods market clearing. Taking logs,

$$(1-\alpha)\ n_t = y_t - a_t + d_t$$

where $d_t \equiv (1-\alpha)\log\int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}}\ di$ is a measure of price (and, hence, output) dispersion across firms. In the technical appendix it is shown that, in a neighborhood of the zero inflation steady state, d_t is equal to zero up to a first order approximation. Hence one can write the following approximate relation between aggregate output, employment and technology:

$$y_t = a_t + (1-\alpha)\ n_t \tag{11}$$

Next I derive an expression for an individual firm's marginal cost in terms of the economy's average real marginal cost. The latter is defined by

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (w_t - p_t) - (y_t - n_t) - \log(1-\alpha) \\ &= (w_t - p_t) - \frac{1}{1-\alpha}\ (a_t - \alpha y_t) - \log(1-\alpha) \end{aligned}$$

for all t, where the second equality defines the economy's average marginal product of labor, mpn_t, in a way consistent with (11). Using the fact that

$$\begin{aligned} mc_{t+k|t} &= (w_{t+k} - p_{t+k}) - mpn_{t+k|t} \\ &= (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}\ (a_{t+k} - \alpha y_{t+k|t}) - \log(1-\alpha) \end{aligned}$$

we have

$$\begin{aligned} mc_{t+k|t} &= mc_{t+k} + \frac{\alpha}{1-\alpha}\ (y_{t+k|t} - y_{t+k}) \\ &= mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}\ (p_t^* - p_{t+k}) \end{aligned} \tag{12}$$

where the second equality follows from the demand shedule (1) combined with the market clearing condition $c_t = y_t$. Notice that under the assumption of constant returns to scale ($\alpha = 0$) we have $mc_{t+k|t} = mc_{t+k}$, i.e. marginal cost is independent of the level of production and, hence, common across firms.

Substituting (12) into (9) and rearranging terms we obtain

$$\begin{aligned} p_t^* - p_{t-1} &= (1-\beta\theta)\sum_{k=0}^{\infty} (\beta\theta)^k \ E_t\left\{\Theta \ \widehat{mc}_{t+k} + (p_{t+k} - p_{t-1})\right\} \\ &= (1-\beta\theta)\Theta \ \sum_{k=0}^{\infty}(\beta\theta)^k \ E_t\{\widehat{mc}_{t+k}\} + \sum_{k=0}^{\infty}(\beta\theta)^k \ E_t\{\pi_{t+k}\} \end{aligned}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon} \leq 1$. Notice that the above discounted sum can be rewritten more compactly as the difference equation

$$p_t^* - p_{t-1} = \beta\theta \ E_t\{p_{t+1}^* - p_t\} + (1-\beta\theta)\Theta \ \widehat{mc}_t + \pi_t \tag{13}$$

Finally, combining (5) and (13) yields the inflation equation:

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \lambda \ \widehat{mc}_t \tag{14}$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \ \Theta$$

is strictly decreasing in the index of price stickiness θ, and in the measure of decreasing returns α.

Solving (14) forward, we can express inflation as the discounted sum of current and expected future deviations of real marginal costs from steady state:

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k \ E_t\{\widehat{mc}_{t+k}\}$$

Equivalently, and defining the average markup in the economy as $\mu_t = -mc_t$, we see that inflation will be high when firms expect average markups to be below their steady state (i.e. desired) level, for in that case firms that have the opportunity to reset prices will choose a price above the economy's average price level, in order to realign their markup with the latter's desired level.

It is worth emphasizing here that the mechanism underlying fluctuations in the aggregate price level and inflation laid out above has little in common with the one at work in the classical monetary economy. Thus, in the present model, inflation results from the aggregate consequences of purposeful price-setting decisions by firms, which adjust their prices in light of current and anticipated cost conditions. By contrast, in the classical monetary economy analyzed in chapter 2 inflation is a consequence of the changes in the aggregate price level that, given the monetary policy rule in place, are required in order to support an equilibrium allocation that is independent of the evolution of nominal variables, with no account given of the mechanism (other than an invisible hand) that will bring about those price level changes.

Next I derive a relation between the economy's real marginal cost and a measure of aggregate economic activity. Notice that independently of the nature of price setting, average real marginal cost can be expressed as

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (\sigma\ y_t + \varphi\ n_t) - (y_t - n_t) - \log(1-\alpha) \\ &= \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\ y_t - \frac{1+\varphi}{1-\alpha}\ a_t - \log(1-\alpha) \end{aligned} \tag{15}$$

Furthermore, and as discussed above, under *flexible prices* the real marginal cost is given by the constant $mc = -\mu$. Defining the *natural* level of output, y_t^n, as the equilibrium level of output under flexible prices we have:

$$mc = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\ y_t^n - \frac{1+\varphi}{1-\alpha}\ a_t - \log(1-\alpha) \tag{16}$$

thus implying

$$y_t^n = -\psi_{y0}^n + \psi_{ya}^n\ a_t \tag{17}$$

where $\psi_{y0}^n \equiv \frac{(1-\alpha)\ (\mu - \log(1-\alpha))}{\sigma+\varphi+\alpha(1-\sigma)} > 0$ and $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$. Notice that when $\mu = 0$ (perfect competition) the natural level of output corresponds to the equilibrium level of output in the classical economy, as derived in chapter 2. The presence of market power by firms has the effect of lowering that output level uniformly over time, without affecting its sensitivity to changes in technology.

Subtracting (16) from (15) we obtain

$$\widehat{mc}_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\ (y_t - y_t^n) \tag{18}$$

i.e., the log deviation of real marginal cost from steady state is proportional to the log deviation of output from its flexible price counterpart. Following convention, I henceforth refer to the latter deviation as the *output gap,* and denote it by $\widetilde{y}_t \equiv y_t - y_t^n$.

By combining (18) with (14) one can obtain an equation relating inflation to its one period ahead forecast and the output gap:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t \tag{19}$$

where $\kappa \equiv \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. Equation (19) is often referred to as the *New Keynesian Phillips Curve* (henceforth, NKPC), and constitutes one of the key building blocks of the basic NK model.

A second key equation describing the equilibrium of the NK model can be obtained by rewriting (10) in terms of the output gap as follows

$$\widetilde{y}_t = -\frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{20}$$

where r_t^n is the *natural rate of interest*, given by

$$\begin{aligned} r_t^n &\equiv \rho + \sigma \; E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma \psi_{ya}^n \; E_t\{\Delta a_{t+1}\} \end{aligned} \tag{21}$$

Henceforth I will refer to (20) as the *dynamic IS equation* (or DIS, for short). Note that one can solve that equation forward to yield:

$$\widetilde{y}_t = -\frac{1}{\sigma} \sum_{k=0}^{\infty} (r_{t+k} - r_{t+k}^n) \tag{22}$$

where $r_t \equiv i_t - E_t\{\pi_{t+1}\}$ is the expected real return on a one period bond (or the real interest rate, for short). The previous expression emphasizes the fact that the output gap is proportional to the sum of current and anticipated deviations between the real interest rate and its natural counterpart.

Equations (19) and (20), together with an equilibrium process for the natural rate r_t^n (which in general will depend on all the real exogenous forces in the model), constitute the non-policy block of the basic NK model. That block has a recursive structure: the NKPC determines inflation given a path for the output gap, whereas the DIS determines the output gap given a path for the (exogenous) natural rate *and* the actual real rate. In order to close the model, we need to supplement the NKPC and the DIS with one or more equations determining how the nominal interest rate i_t evolves over time, i.e. with a description of how monetary policy is conducted. Thus, and in contrast with the classical model analyzed in chapter 2, when prices are sticky the equilibrium path of real variables cannot be determined independently of monetary policy..In other words: monetary policy is non-neutral.

In order to illustrate the workings of the basic NK model, next I consider two alternative specifications of monetary policy and analyze some of their equilibrium implications.

4 Equilibrium Dynamics under Alternative Monetary Policy Rules

4.1 Equilibrium under an Interest Rate Rule

I first analyze the equilibrium under a simple interest rate rule of the form:

$$i_t = \rho + \phi_\pi \ \pi_t + \phi_y \ \widetilde{y}_t + v_t \tag{23}$$

where v_t is an exogenous (possibly stochastic) component with zero mean. I assume ϕ_π and ϕ_y are non-negative coefficients, chosen by the monetary authority. Note that the choice of intercept ρ makes the rule consistent with a zero inflation steady state.

Combining (19), (20), and (23) we can represent the equilibrium conditions by means of the following system of difference equations.

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \ (\widehat{r}_t^n - v_t) \tag{24}$$

where $\widehat{r}_t^n \equiv r_t^n - \rho$, and

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \qquad ; \qquad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

with $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$.

Given that both the output gap and inflation are non-predetermined variables, the solution to (24) is locally unique if and only if $\mathbf{A}_T$ has both eigenvalues within the unit circle.[1] Under the assumption of non-negative coefficients (ϕ_π, ϕ_y) it can be shown that a necessary and sufficient condition for uniqueness is given by:[2]

$$\kappa \ (\phi_\pi - 1) + (1-\beta) \ \phi_y > 0 \tag{25}$$

[1] See, e.g., Blanchard and Kahn (1980)

[2] See Bullard and Mitra (2002) for a proof.

which I assume to hold, unless stated otherwise. The previous condition will be given an economic interpretation in chapter 4.

Next I examine the economy's equilibrium response to two exogenous shocks–monetary policy and technology–when the central bank follows interest rate rule (23).

4.1.1 The Effects of a Monetary Policy Shock

I assume that the exogenous component of the interest rate, v_t, follows an AR(1) process

$$v_t = \rho_v \; v_{t-1} + \varepsilon_t^v$$

where $\rho_v \in [0,1)$. Note that a positive (negative) realization of ε_t^v should be interpreted as a contractionary (expansionary) monetary policy shock, leading to a rise (decline) in the nominal interest rate, *given* inflation and the output gap.

Since the natural rate of interest is not affected by monetary shocks we set $\widehat{r}_t^n = 0$, for all t. We guess that the solution takes the form $\widetilde{y}_t = \psi_{yv} \; v_t$ and $\pi_t = \psi_{\pi v} \; v_t$, where ψ_{yv} and $\psi_{\pi v}$ are coefficients to be determined. Imposing the guessed solution on (20) and (19) and using the method of undetermined coefficients, we find:

$$\widetilde{y}_t = -(1 - \beta\rho_v)\Lambda_v \; v_t$$

and

$$\pi_t = -\kappa\Lambda_v \; v_t$$

where $\Lambda_v \equiv \frac{1}{(1-\beta\rho_v)[\sigma(1-\rho_v)+\phi_y]+\kappa(\phi_\pi-\rho_v)}$. It can be easily shown that as long as (25) is satisfied we have $\Lambda_v > 0$.

Hence, an exogenous increase in the interest rate leads to a persistent decline in the output gap and inflation. Since the natural level of output is unaffected by the monetary policy shock, the response of output matches that of the output gap.

One can use (20) to obtain an expression for the real interest rate

$$\widehat{r}_t = \sigma(1 - \rho_v)(1 - \beta\rho_v)\Lambda_v \; v_t$$

which is thus shown to increase unambiguosly in response to an exogenous increase in the nominal rate.

The response of the nominal interest rate, which combines both the direct effect of v_t and the variation induced by lower output gap and inflation, is given by:

$$\widehat{i}_t = \widehat{r}_t + E_t\{\pi_{t+1}\} = [\sigma(1-\rho_v)(1-\beta\rho_v) - \rho_v\kappa]\ \Lambda_v\ v_t$$

Note that if the persistence of the monetary policy shock, ρ_v, is sufficiently high, the nominal rate will decline in response to a rise in v_t . This is a result of the downward adjustment induced by the decline in inflation and the output gap more than offsetting the direct effect of a higher v_t. In that case, and despite the lower nominal rate, the policy shock still has a contractionary effect on output, since the latter is inversely related to the real rate, which goes up unambiguously.

Finally, one can use (2) to determine the change in the money supply required to bring about the desired change in the interest rate. In particular, the response of m_t on impact is given by:

$$\begin{aligned} \frac{dm_t}{d\varepsilon_t^v} &= \frac{dp_t}{d\varepsilon_t^v} + \frac{dy_t}{d\varepsilon_t^v} - \eta\frac{di_t}{d\varepsilon_t^v} \\ &= -\Lambda_v\ [(1-\beta\rho_v)(1+\eta\sigma(1-\rho_v)) + \kappa(1-\eta\rho_v)] \end{aligned}$$

Hence, we see that the sign of the change in the money supply that supports the exogenous policy intervention is, in principle, ambigous. If Even though the money supply needs to be tightened to raise the nominal rate *given output and prices*, the decline in the latter induced by the policy shocks combined with the possibility of an induced nominal rate decline make it impossible to rule out a countercyclical movement in money in response to an interest rate shick. Note however that $di_t/d\varepsilon_t^v > 0$ is a sufficient condition for a procyclical response of money, as well as for the presence of a liquidity effect (i.e. a negative short-run comovement of the nominal rate and the money supply in response to an exogenous monetary policy shock).

The previous analysis can be used to quantify the effects of a monetary policy shock, given numerical values for the model's parameters. Next I briefly present a baseline calibration of the model, which takes the relevant period to correspond to a quarter.

In the baseline calibration of the model's preference parameters it is assumed that $\beta = 0.99$, implying a steady state real return on financial assets of about four percent. I also assume and $\sigma = 1$ (log utility) and $\varphi = 1$ (unit Frisch elasticity of labor supply), values commonly adopted in the literature. We set the interest semi-elasticity of money demand, η, to equal 4.[3]

[3]That calibration is based on estimates of an OLS regression of (log) M2 inverse velocity

In addition we assume $\theta = 2/3$, which implies an average price duration of three quarters, a value consistent with the empirical evidence.[4] As to the interest rate rule coefficients we assume $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$, which are roughly consistent with observed variations in the Federal Funds rate over the Greenspan era.[5] Finally, we set $\rho_v = 0.5$, a value associated with a moderately persistent shock.

Figure 1 illustrates the dynamic effects of an expansionary monetary policy shock. The shock corresponds to a decrease of 25 basis points in ε_t^v , which–in the absence of a further change induced by the response of inflation or the output gap, would imply a decrease on impact of 100 basis points in the annualized nominal rate. The responses of inflation and the two interest rates shown in the figure are expressed in annual terms (i.e. they are obtained by multiplying by 4 the responses of π_t, i_t and r_t in the model).

In a way consistent with the analytical results above we see that the policy shock generates an increase in the real rate, and an decrease in inflation and output (whose response corresponds to that of the output gap, since the natural level of output is not affected by the monetary policy shock). Note than under the baseline calibration the nominal rate goes up, though by less than its exogenous component–as a result of the downward adjustment induced by the decline in inflation and the output gap. In order to bring about the observed interest rate response, the central bank must engineer a reduction in the money supply. The calibrated model thus displays a liquidity effect. Note also that the response of the real rate is larger than that of the nominal rate as a result of the increase in expected inflation.

Overall, the dynamic responses to a monetary policy shock shown in Figure 1 are similar, at least in a qualitative sense, to those estimated using structural VAR methods, as described in Chapter 1. Nevertheless, and as shown in Christiano et al. (2005), matching some of the quantitative fea-

on the three month Treasury Bill rate (quarterly rate, per unit), using quarterly data over the period 1960:1-1988:1. That period is characterized by a highly stable relationship between velocity and the nominal rate, consistent with the model.

[4] See, in particular, the estimates in Galí, Gertler and López-Salido (2001) and Sbordone (2002), based on aggregate data. Using the price of individual goods, Bils and Klenow (2004) uncover a mean duration slightly shorter (7 months).

[5] See, e.g., Taylor (1999). Note that empirical interest rate rules are generally estimated using inflation and interest rate data expressed in annual rates. Conversion to quarterly rates requires that the output gap coefficient be divided by 4. As discussed later, the output gap measure used in empirical interest rate rules does not necessarily match the concept of output gap in the model.

tures of the empirical impulse responses requires that the basic NK model is enriched in a variety of dimensions.

4.1.2 The Effects of a Technology Shock

In order to determine the economy 's response to a technology shock one must first specify a process for the technology parameter $\{a_t\}$, and derive the implied process for the natural rate. I assume the following AR(1) process

$$a_t = \rho_a \, a_{t-1} + \varepsilon_t^a \tag{26}$$

where $\rho_a \in [0,1)$.. Given (21), the implied natural rate, expressed in terms of deviations from steady state, is given by

Setting $v_t = 0$, for all t (i.e., no monetary shocks), and guessing that output gap and inflation are proportional to $\widehat{r}_t^n$, we can apply the method of undetermined coefficients in a way analogous to previous subsection (or just exploit the fact that $\widehat{r}_t^n$ enters the equilibrium conditions in a way symmetric to v_t, but with the opposite sign), to obtain

$$\begin{aligned} \widetilde{y}_t &= (1-\beta\rho_a)\Lambda_a \, \widehat{r}_t^n \\ &= -\sigma\psi_{ya}^n(1-\rho_a)(1-\beta\rho_a)\Lambda_a \, a_t \end{aligned}$$

and

$$\begin{aligned} \pi_t &= \kappa\Lambda_a \, \widehat{r}_t^n \\ &= -\sigma\psi_{ya}^n(1-\rho_a)\kappa\Lambda_a \, a_t \end{aligned}$$

where $\Lambda_a \equiv \frac{1}{(1-\beta\rho_a)[\sigma(1-\rho_a)+\phi_y]+\kappa(\phi_\pi-\rho_a)} > 0$

Hence, and as long as $\rho_a < 1$, a positive technology shock leads to a persistent decline in both inflation and the output gap. The implied equilibrium responses of output and employment are thus given by

$$\begin{aligned} y_t &= y_t^n + \widetilde{y}_t \\ &= \psi_{ya}^n \, (1-\sigma(1-\rho_a)(1-\beta\rho_a)\Lambda_a) \, a_t \end{aligned}$$

and

$$\begin{aligned} (1-\alpha)\, n_t &= y_t - a_t \\ &= [(\psi_{ya}^n - 1) - \sigma\psi_{ya}^n(1-\rho_a)(1-\beta\rho_a)\Lambda_a] \, a_t \end{aligned}$$

Hence, we see that the sign of the response of output and employment to a positive technology shock is in general ambiguous, depending on the configuration of paameter values, including the interest rate rule coefficients. In our baseline calibration we have $\sigma = 1$ which in turn implies $\psi_a = 1$. In that case, a technological improvement leads to a persistent employment decline. Such a response of employment is consistent with much of the recent empirical evidence on the effects of technology shocks.[6]

Figure 2 shows the responses of a number of variables to a favorable technology shock, as implied by our baseline calibration and under the assumption of $\rho_a = 0.9$. Notice that the improvement in technology is partly accommodated by the central bank, which lowers nominal and real rates, while increasing the quantity of money in circulation. That policy, however, is not sufficient to close a negative output gap, which is responsible for the decline in inflation. Under the baseline calibration output increases (though less than its natural counterpart), and employment declines, in a way consistent with the evidence mentioned above.

4.2 Equilibrium under an Exogenous Money Supply

Next I analyze the equilibrium dynamics of the basic NK model under an exogenous path for the growth rate of the money supply, Δm_t.

As a preliminary step, it is useful to rewrite the money market equilibrium condition in terms of the output gap, as follows:

$$\widetilde{y}_t - \eta\, \widehat{i}_t = l_t - y_t^n \tag{27}$$

where $l_t \equiv m_t - p_t$. Substituting the latter equation into (20) yields

$$(1+\sigma\eta)\ \widetilde{y}_t = \sigma\eta\ E_t\{\widetilde{y}_{t+1}\} + l_t + \eta\ E_t\{\pi_{t+1}\} + \eta\ \widehat{r}_t^n - y_t^n \tag{28}$$

Note also that real balances are related to inflation and money growth through the identity

$$l_{t-1} = l_t + \pi_t - \Delta m_t \tag{29}$$

Hence, the equilibrium dynamics for real balances, output gap and inflation are described by equations (28), and (29), together with the NKPC equation (19). They can be summarized compactly by the system

[6] See Galí and Rabanal (2005) for a survey of that empirical evidence.

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ l_{t-1} \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ l_t \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ y_t^n \\ \Delta m_t \end{bmatrix} \tag{30}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

The system above has one predetermined variable (l_{t-1}) and two nonpredetermined variables ($\widetilde{y}_t$ and π_t). Accordingly, a stationary solution will exist and be unique if and only if $\mathbf{A_M} \equiv \mathbf{A_{M,0}^{-1}}\mathbf{A_{M,1}}$ has two eigenvalues inside and one outside (or on) the unit circle. The latter condition can be shown to be always satisfied so, in contrast with the interest rate rule discussed above, the equilibrium is always determined under an exogenous path for the money supply.[7]

Next I examine the equilibrium responses of the economy to a monetary policy shock and a technology shock.

4.2.1 The Effects of a Monetary Policy Shock

In order to illustrate how the the economy responds to an exogenous shock to the money supply, I assume that Δm_t follows the AR(1) process

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m \tag{31}$$

where $\rho_m \in [0,1)$ and $\{\varepsilon_t^m\}$ is white noise.

The economy's response to a monetary policy shock can be obtained by determining the stationary solution to the dynamical system consisting of (30) and (31) and tracing the effects of a shock to ε_t^m (while setting $\widehat{r}_t^n = y_t^n = 0$, for all t).[8]

Figure 3 displays the dynamic responses of several variables of interest to an expansionary monetary policy shock, which takes the form of positive realization of ε_t^m of size 0.25. That impulse corresponds to a one percent

[7]Since $\mathbf{A_M}$ is upper triangular its eigenvalues are given by its diagonal elements which can be shown to be $\sigma\eta/(1+\sigma\eta)$, β, and -1. Hence existence and uniqueness of a stationary solution is guaranteed under any rule implying an exogenous path for the money supply.

[8]See e.g. Blanchard and Kahn (1980) a description of a solution method.

increase, on impact, in the annualized rate of money growth, as shown in the Figure. The sluggishness in the adjustment of prices implies that real balances rise in response to the increase in the money supply. As a result, clearing of the money market requires either a rise in output and/or a decline in the nominal rate. Under the calibration considered here, output increases by about a third of a percentage point on impact, after which it slowly reverts back to its initial level. The nominal rate, however, shows a slight increase. Hence, and in contrast with the case of an interest rate rule considered above, a liquidity effect does not emerge here. Note however that the rise in the nominal rate does not prevent the real rate from declining persistently (due to higher expected inflation), leading in turn to an expansion in aggregate demand and output (as implied by (22)) and, as a result, a persistent rise in inflation (which follows from (19)).

It is worth noting here that the absence of a liquidity effect is not a necessary feature of the exogenous money supply regime considered here, but instead a property of the calibration used. To see this note that one can combine equations (2) and (20), to obtain the difference equation

$$i_t = \frac{\eta}{1+\eta}\ E_t\{i_{t+1}\} + \frac{\rho_m}{1+\eta}\ \Delta m_t + \frac{\sigma-1}{1+\eta}\ E_t\{\Delta y_{t+1}\}$$

whose forward solution yields:

$$i_t = \frac{\rho_m}{1+\eta(1-\rho_m)}\ \Delta m_t + \frac{\sigma-1}{1+\eta}\sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k\ E_t\{\Delta y_{t+1+k}\}$$

Note that when $\sigma = 0$, as in the baseline calibration underlying Figure 3, the nominal rate always comoves positively with money growth. Nevertheless, and given that quite generally the summation term will be negative (since for most calibrations output tends to adjust monotonically to its original level after the initial increase), a liquidity effect will emerge for values of σ sufficiently above one combined with sufficiently low (absolute) values of ρ_m.[9]

4.2.2 The Effects of a Technology Shock

Finally, I turn to the analysis of the effects of a technology shock under a monetary policy regime characterized by exogenous money supply. Once

[9] See Galí (2001) for a detailed analysis.

again, I assume the technology parameter a_t follows the stationary process given by (26). That assumption combined with (17) and (21) is used to determine the implied path of $\widehat{r}_t^n$ and y_t^n as a function of a_t, as needed to solve (30). In a way consistent with the assumption of exogenous money, I set $\Delta m_t = 0$ for all t for the purposs of the present exercise.

Figure 4 displays the dynamic responses to a one percent increase in the technology. A comparison with the responses shown in Figure 2 (corresponding to the analogous exercise under an interest rate rule) reveals many similarities: in both cases the output gap (and, hence, inflation) display a negative response to the technology improvement, as a result of output failing to increase as much as its natural level. Note, howewever, that in the case of exogenous money the gap between output and its natural level is much larger, which explains also the larger decline in employment. This is due to the upward response of the nominal and real rates implied by the unchanged money supply, which contrasts with their decline (in response to the negative response of inflation and the output gap) under the interest rate rule. Since the natural real rate also declines in response to the positive technology shock (in order to support the transitory increase in output and consumption), the response of interest rates generated under the exogenous money regime becomes overly contractionary, as illustrated in Figure 4.

5 Notes on the Literature

Precedents: Mankiw-Romer

An early version and analysis of the baseline new Keynesian model can be found in Yun (1996), which used a discrete-time version of the staggered price-setting model originally developed in Calvo (1983). King and Wolman (1996) provides a detailed analysis of the steady state properties of that model. King and Watson (1996).compare its predictions regarding the cyclical properties.of money, interest rates, and prices with those of flexible price models. Woodford (1996) incorporates a fiscal sector in the model and analyzes its properties under a non-Ricardian fiscal policy regime.

An inflation equation identical to the new Keynesian Phillips curve can be derived under the assumption of quadratic costs of price adjustment, as shown in Rotemberg (1982). Hairault and Portier (1993) developed and analyzed an early version of a monetary model with quadratic costs of price adjustment and compared its second moment predictions with those of the

French and U.S. economies.

Two main alternatives to the Calvo random price duration model can be found in the literature. The first one is given by staggered price setting models with deterministic durations, originally proposed by Taylor (1980) in the context of a non microfounded model. A microfounded version of the Taylor model can be found in Chari, Kehoe and McGrattan (2000) who analyzed the output effects of exogenous monetary policy shocks. A second alternative price-setting structure is given by state dependent models, in which which the timing of price adjustments is influenced by the state of the economy. A quantitative analysis of a state dependent pricing model can be found in Dotsey, King and Wolman (1999) and, more recently, in Golosov and Lucas (2003) and Gertler and Leahy (2006).

The empirical performance of new Keynesian Phillips curve has been the object of numerous criticisms. An early critical assessment can be found in Fuhrer and Moore (1986). Mankiw and Reis (2002) give a quantitative review of the perceived shortcomings of the NKPC and propose an alternative price setting structure, based on the assumption of sticky information. Galí and Gertler (1999), Sbordone (2002) and Galí, Gertler and López-Salido (2002) provide favorable evidence of the empirical fit the equation relating inflation to marginal costs, and discuss the difficulties in estimating or testing the NKPC given the unobservability of the output gap.

Rotemberg and Woodford (1999) and Christiano, Eichenbaum and Evans (2005) provide empirical evidence on the effects monetary policy shocks, and discuss a number of modifications of the baseline NK model aimed at improving the model's ability to match the estimated impulse responses.

Evidence on the effects of technology shocks and its implications for the relevance of alternative models can be found in Galí (1999) and Basu, Fernald and Kimball (2004). Recent evidence as well as alternative interpretations are surveyed in Galí and Rabanal (2005).

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Technical Appendix

Optimal Allocation of Consumption Expenditures

The problem of maximization of C_t for any *given* expenditure level $\int_0^1 P_t(i)\, C_t(i)\, di$ $\equiv Z_t$ can be formalized by means of the Lagrangean

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\ di\right]^{\frac{\epsilon}{\epsilon-1}} - \lambda\ \left(\int_0^1 P_t(i)\ C_t(i)\ di - Z_t\right)$$

The associated first order conditions are:

$$C_t(i)^{-\frac{1}{\epsilon}}\ C_t{}^{\frac{1}{\epsilon}} = \lambda\ P_t(i)$$

for all $i \in [0,1]$. Thus, for any two goods (i,j) we have:

$$C_t(i) = C_t(j)\ \left(\frac{P_t(i)}{P_t(j)}\right)^{-\epsilon}$$

which can be substituted into the expression for consumption expenditures to yield

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\ \frac{Z_t}{P_t}$$

for all $i \in [0,1]$. The latter condition can then be substituted into the definition of C_t to obtain

$$\int_0^1 P_t(i)\ C_t(i)\ di = P_t\ C_t$$

Combining the two previous equations we obtain the demand schedule:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\ C_t$$

Aggregate Price Level Dynamics

Let $S(t) \subset [0,1]$ represent the set of firms not re-optimizing their posted price in period t. Using the definition of the aggregate price level and the fact that all firms resetting prices will choose an identical price P_t^* we have

$$
\begin{aligned}
P_t &= \left[\int_{S(t)} P_{t-1}(i)^{1-\epsilon}\ di + (1-\theta)\ (P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \\
&= \left[\theta\ (P_{t-1})^{1-\epsilon} + (1-\theta)\ (P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}
\end{aligned}
$$

where the second equality follows from the fact that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t-1$, though with total mass reduced to θ.

Equivalently, dividing both sides by P_{t-1} :

$$
\Pi_t^{1-\epsilon} = \theta + (1-\theta)\ \left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} \tag{32}
$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. Notice that in a steady state with zero inflation $P_t^* = P_{t-1} = P_t$, for all t.

Log-linearization of (32) around that steady state implies:

$$
\pi_t = (1-\theta)\ (p_t^* - p_{t-1}) \tag{33}
$$

Price Dispersion

From the definition of the price index:

$$
\begin{aligned}
1 &= \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\varepsilon}\ di \\
&= \int_0^1 \exp\{(1-\epsilon)(p_t(i)-p_t)\}\ di \\
&\simeq 1 + (1-\epsilon)\int_0^1 (p_t(i)-p_t)\ di + \frac{(1-\epsilon)^2}{2}\int_0^1 (p_t(i)-p_t)^2\ di
\end{aligned}
$$

thus implying the second order approximation

$$
p_t \simeq E_i\{p_t(i)\} + \frac{(1-\epsilon)}{2}\int_0^1 (p_t(i)-p_t)^2\ di
$$

where $E_i\{p_t(i)\} \equiv \int_0^1 p_t(i)\ di$ is the cross-sectional mean of (log) prices.

In addition,

$$\begin{aligned}
\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di &= \int_0^1 \exp\left\{-\frac{\epsilon}{1-\alpha}\ (p_t(i)-p_t)\right\}\ di \\
&\simeq 1-\frac{\epsilon}{1-\alpha}\int_0^1 (p_t(i)-p_t)\ di + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2\ di \\
&\simeq 1+\frac{1}{2}\frac{\epsilon(1-\epsilon)}{1-\alpha}\int_0^1 (p_t(i)-p_t)^2\ di + \frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2 \int_0^1 (p_t(i)-p_t)^2\ di \\
&= 1+\frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\int_0^1 (p_t(i)-p_t)^2\ di \\
&\simeq 1+\frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\ var_i\{p_t(i)\} > 1
\end{aligned}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$, and where the last equality follows from the observation that, up to second order,

$$\begin{aligned}
\int_0^1 (p_t(i)-p_t)^2\ di &\simeq \int_0^1 (p_t(i)-E_i\{p_t(i)\})^2\ di \\
&\equiv var_i\{p_t(i)\}
\end{aligned}$$

Finally, using the definition of d_t we obtain

$$d_t \equiv (1-\alpha)\log\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di \simeq \frac{1}{2}\frac{\epsilon}{\Theta}\ var_i\{p_t(i)\}$$

Figure 1: Effects of a Monetary Policy Shock (Interest Rate Rule))

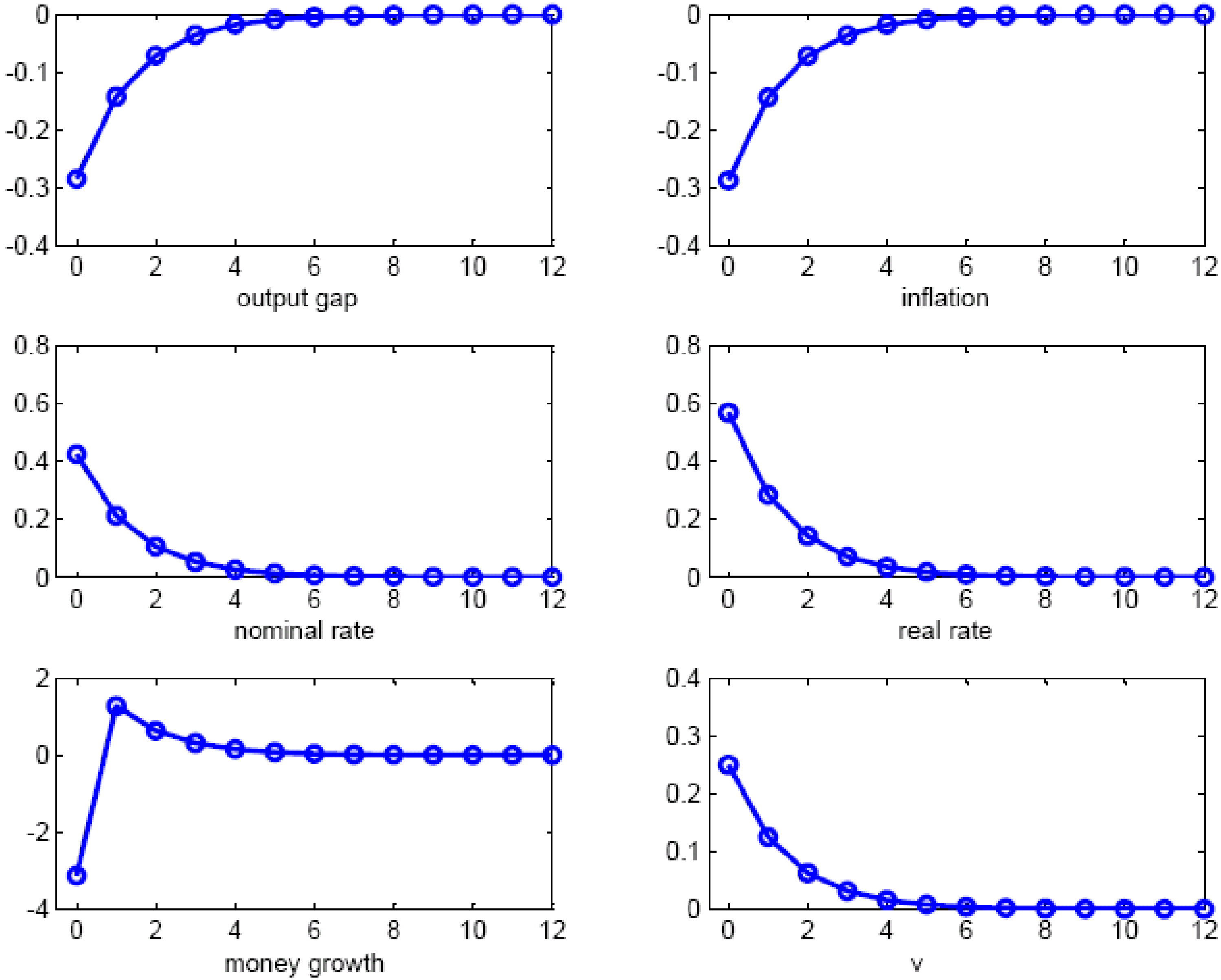

Figure 2: Effects of a Technology Shock (Interest Rate Rule)

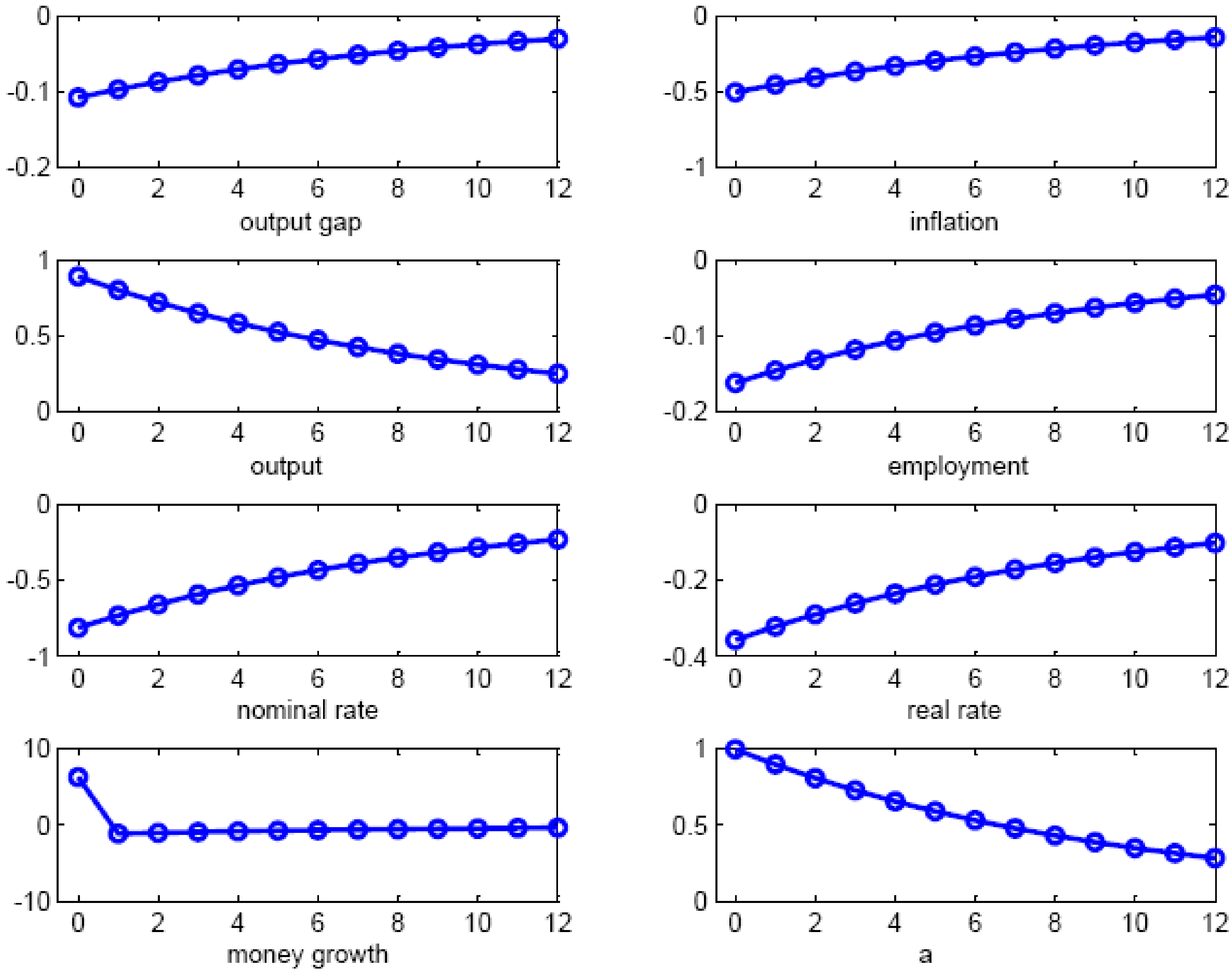

Figure 3: Effects of a Monetary Policy Shock (Money Growth Rule)

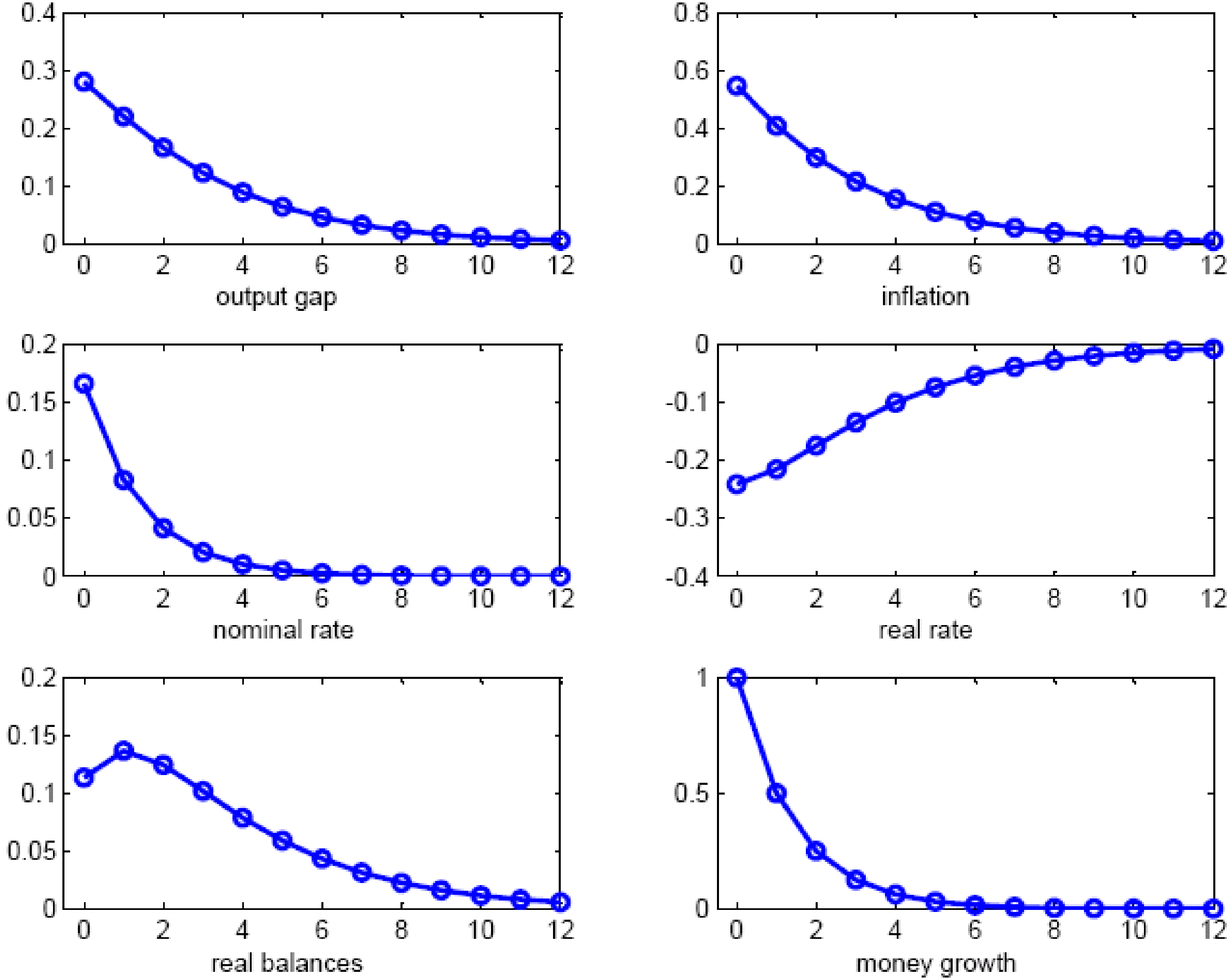

Figure 4: Effects of a Technology Shock (Money Growth Rule)