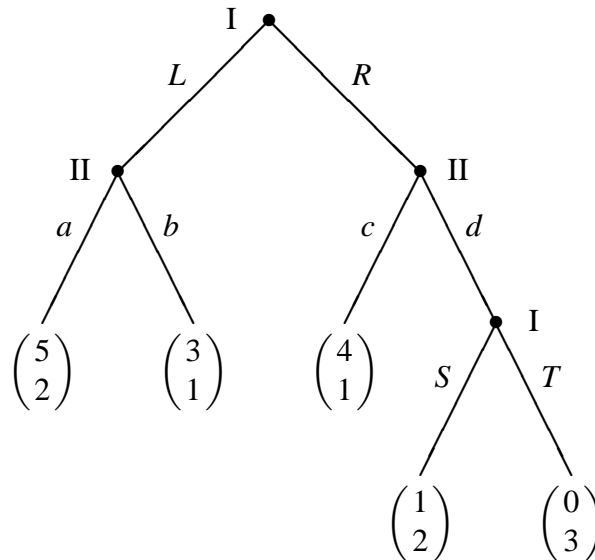

Exercises 4 Game Theory

MA300 / 301 / 402

due in week 5, Thursday 5pm

Exercise 4.1 – STRATEGIES AND REDUCED STRATEGIES IN GAME TREES

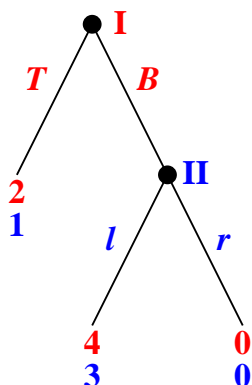
Consider the following game tree. At a leaf, the top payoff is for player I, the bottom payoff for player II.



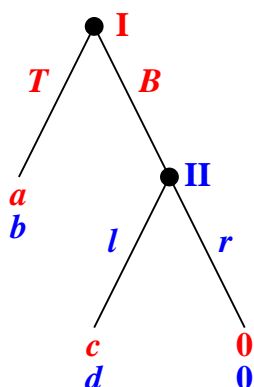
- (a) What is the number of strategies of player I and of player II? How many reduced strategies does each of the players have?
- (b) Give the reduced strategic form of the game.
- (c) What are the Nash equilibria of the game in reduced strategies? What are the subgame perfect equilibria of the game?
- (d) Identify every pair of reduced strategies where one strategy weakly or strictly dominates the other, and indicate if the dominance is weak or strict.

Exercise 4.2 – SUBGAME PERFECT EQUILIBRIA

- (a) For the following game tree, find all Nash equilibria (in pure strategies). Which of these are subgame perfect?



(b) In the following game tree, the payoffs a, b, c, d are positive real numbers.



For each of the following statements (i), (ii), (iii), decide if it is true or false, justifying your answer with an argument or counterexample; you may refer to any standard results. For any $a, b, c, d > 0$,

- (i) the game always has a subgame perfect Nash equilibrium (SPNE);
- (ii) the payoff to player II in any SPNE is always at least as high as her payoff in any Nash equilibrium;
- (iii) the payoff to player I in any SPNE is always at least as high as his payoff in any Nash equilibrium.

Exercise 4.3 – WEAK DOMINATION

		II		
		l	c	r
I	T	1 0	3 1	1 1
	M	1 1	3 0	0 1
	B	2 2	3 3	0 2

- (a) In the above two-player game, identify all pairs of strategies where one weakly dominates the other.
- (b) Assume you are allowed to remove a weakly dominated strategy of some player. Do so, and repeat this process (of iterated elimination of weakly dominated strategies) until you find a single strategy pair of the original game.
- (c) Find such an iterated elimination of weakly dominated strategies that results in a strategy pair other than the one found in (b), where *both* strategies, and the payoffs to the players, are different.
- (d) What are the Nash equilibria of the game?