**Box 7-3.** *(continued)*

$$\begin{bmatrix} A_x' \\ A_y' \\ A_z' \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Applying the usual rules for matrix multiplication gives the equations

$$\begin{aligned} A_x' &= R_{11}A_x + R_{12}A_y + R_{13}A_z \\ A_y' &= R_{21}A_x + R_{22}A_y + R_{23}A_z \\ A_z' &= R_{31}A_x + R_{32}A_y + R_{33}A_z \end{aligned}$$

To define the elements of the rotation matrix **R** we need to know the Cartesian coordinates of the Euler pole **E** = $(E_x, E_y, E_z)$ and the angle of rotation $\Omega$. The elements of the matrix are then given by

(first row of matrix)

$$\begin{aligned} R_{11} &= E_x E_x (1-\cos\Omega)+\cos\Omega \\ R_{12} &= E_x E_y (1-\cos\Omega)-E_z \sin\Omega \\ R_{13} &= E_x E_z (1-\cos\Omega)+E_y \sin\Omega \end{aligned}$$

(second row of matrix)

$$\begin{aligned} R_{21} &= E_y E_x (1-\cos\Omega)+E_z \sin\Omega \\ R_{22} &= E_y E_y (1-\cos\Omega)+\cos\Omega \\ R_{23} &= E_y E_z (1-\cos\Omega)-E_x \sin\Omega \end{aligned}$$

(third row of matrix)

$$\begin{aligned} R_{31} &= E_z E_x (1-\cos\Omega)-E_y \sin\Omega \\ R_{32} &= E_z E_y (1-\cos\Omega)+E_x \sin\Omega \\ R_{33} &= E_z E_z (1-\cos\Omega)+\cos\Omega \end{aligned}$$

If you decide to program this operation on your computer, the following numerical example may help in debugging your program. For the rotation

$$\mathrm{ROT}[\mathbf{E}, \Omega] = \mathrm{ROT}[(-37°, 312°), 65°]$$

the values of the elements of the matrix are

$$\mathbf{R} = \begin{bmatrix} 0.588 & 0.362 & -0.724 \\ -0.729 & 0.626 & -0.278 \\ 0.352 & 0.691 & 0.632 \end{bmatrix}$$

The point **A** = (20°, 130°) has the Cartesian components $(A_x, A_y, A_z)$ = (−0.604, 0.720, 0.342). Performing the matrix multiplication yields the Cartesian components $(A_x', A_y', A_z')$ = (−0.342, 0.796, 0.500), which when converted to spherical coordinates is (30.0°, 113.2°).

*(continued)*