

Lectures on String/Brane Cosmology

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ABSTRACT: An overview is presented of some cosmological aspects of string theory. Recent developments are emphasised, especially the attempts to derive inflation or alternatives to inflation from the dynamics of branes in string theory. Time dependent backgrounds with potential cosmological implications, such as those provided by negative tension branes and S-branes and the rolling string tachyon are also discussed.[†]

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1. Introduction

This is an interesting time to think about cosmology from a string theory perspective. The two subjects: cosmology and strings, complement each other in several ways. Cosmology needs an underlying theory to approach its basic questions such as the initial singularity, if there was any, the origin of inflation or any alternative way to address the problems that inflation solves, such as the horizon and flatness problems and, more importantly, the origin of the density perturbations in the cosmic microwave background (CMB). The successes of inflation [1] in this regard makes us sometimes forget that it is only a scenario in search of an underlying theory. There are many variations of the inflationary scenario [2] but at the moment there is no

concrete derivation of inflation (or any of its alternatives) from a fundamental theory such as string theory.

On the other hand string theory, although lacking a full nonperturbative formulation, has been highly developed and understood in many respects and needs a way to be confronted with physics. One possibility is low-energy phenomenology, although it may be a long way before this can be tested, unless supersymmetry is discovered at low energies and its properties provide some hints to its high energy origin or we get lucky and the string scale is low enough to be probed in the future colliders such as LHC. Otherwise cosmology may turn out to be the main avenue to probe string theory. This has been reinforced in view of the recent observational discoveries that seem to indicate that the effective cosmological constant is not exactly zero and, furthermore, the accuracy of the CMB experiments [3, 4] that have provided a great deal of information on the scalar density perturbations of the cosmic microwave background. This has raised the status of cosmology to a science subject to precision experimental tests, with a very promising future due to the planned experiments for the not too far future, such as MAP and PLANCK.

Moreover, based mostly on string theory ideas, the brane world scenario has emerged in the past few years offering dramatic changes in our view of the universe [5]. The fact that we may be living on a hypersurface in higher dimensions does not only imply that the scale of string theory could be as small as 1TeV, but also provides completely new scenarios for the cosmological implications of string theory [6, 7]. Actually if the brane world scenario is realized, but with a string scale close to the Planck scale, the main place to look at its possible implications will be cosmology, rather than table top experiments or high energy accelerators.

Finally, the study of cosmological implications of string theory can shed some light into the better understanding of the theory itself. We already have the experience with the study of black hole backgrounds in string theory which has led to some of the main successes of the theory, namely the explicit calculation of the black hole entropy and the identification of the AdS/CFT correspondence [8], which not only provides a concrete realisation of the holographic principle [9] but has also led to important results in field and string theories. Cosmology is the other arena where nontrivial string backgrounds can be explored, some of the ideas developed from other studies can be put to test in cosmology and probably new insights may emerge. In particular the recent realisation that our universe could be in a stage with a nonzero vacuum energy gives rise to an important challenge for string theory, we need to be able to understand string theory in such a background [10]. Also previous ideas about quantum cosmology in general may find new realisations in the context of string theory. In summary, we may say that cosmology presents probably the most important challenges for string theory: the initial singularity, the cosmological constant, the definition of observables, the identification of initial conditions, realisation of de Sitter or quintessential backgrounds of the theory, etc.

It is then becoming of prime importance to learn the possible applications of string theory to cosmology. These lecture notes are an effort to put some of these ideas together for non-experts. Due to limitations of space, time and author's knowledge, the discussion is at a superficial level and incomplete. They were originally given to review the basic ideas on the subject, including brane cosmology in static and time dependent backgrounds to conclude with D-brane inflation and tachyon condensation, together with some details about the ekpyrotic scenario. However, right after the lectures were given, several interesting developments have occurred related with the subject of the lectures that have to be briefly included for completeness (rolling tachyon, S-branes, time dependent orbifolds). Fortunately there are several good reviews on the first part of the lectures that can be consulted for deeper insights [2, 11, 12, 13, 14, 15, 16, 17]. There are hundreds of articles on brane cosmology and I cannot make justice to everybody working in the field. I do apologise for omissions of important references. The presentation tries to include only the brane cosmology ideas formulated in the context of string theory or that have connections to it, therefore, many interesting developments in brane cosmology, which are not clearly related to string theory are omitted.

I first give an overview of the standard big-bang cosmology that introduces the physical parameters, notation and problems. Then I describe briefly the main ideas discussed in the past (before the year 2000) in string cosmology. These include the Brandenberger-Vafa scenario [18] where T-duality and winding modes could have an interesting implication for early universe cosmology, including the possible determination of the critical dimension of spacetime. Also the cosmology associated to the moduli fields, which could be candidates for inflaton fields [19], but also can cause a serious and generic cosmological problem once they get a mass, since they can either ruin nucleosynthesis by their decays, if they are unstable, or over close the universe, if they happen to be stable [20] (see also [21]). This has been called the cosmological moduli problem. Finally we mention the main ideas behind the Gasperini-Veneziano 'pre big-bang cosmology' that during the years has become the string cosmology scenario subject to more detailed study (see [17] for a very complete review on the subject with references to the earlier work).

In the third part of the lectures I concentrate on some recent developments. I will emphasise the role that string theory p -branes can play in cosmology. First we describe some of the interesting results coming out of a treatment of brane cosmology in the simple setting of 4D brane worlds moving in a 5D bulk [6, 7, 22, 23, 24, 25, 26, 27]. Two points are emphasised: the Einstein's equations in the 4D brane do not have the standard behaviour in the sense that the relation between the Hubble parameter and the energy density is different from the standard 4D cosmology. We also remark the interesting possibility for understanding cosmology in the brane world as just the motion of the brane in a static bulk. An observer on the brane feels his universe expanding while an observer in the bulk only sees the brane moving in

a static spacetime, this is usually known as mirage cosmology.

Then I discuss the possibility that the dynamics of D-branes may have direct impact in cosmology, in particular considering a pair of D-branes approaching each other and their subsequent collision could give rise to inflation [28]. For a D-brane/antibranes pair it is possible to compute the attractive potential from string theory and actually obtain inflation with the inflaton field being the separation of the branes [29, 30]. Furthermore, it is known from string theory that after the branes get to a critical distance, an open string mode becomes tachyonic thus providing an instability which is precisely what is needed to end inflation [29]. Obtaining then a realisation of the hybrid inflation scenario [31] with the two relevant fields having well defined stringy origin, *i.e.* the separation of the branes generates inflation and the open string tachyon finishes inflation and provides the mechanism to describe the process of the brane/antibranes collision and annihilation. Natural extensions of these ideas to include orientifold models, intersecting branes at angles and related constructions, [32, 33, 34, 35] will also be discussed, which illustrates that the realisation of hybrid inflation from the inter-brane separation as the inflaton field and the open string tachyon as the field responsible to end inflation and re-heat is very generic in D-brane models.

Tachyon condensation is one of the few physical process that has been studied in detail purely from string theory techniques [36, 37, 38, 39] and can have by itself important implications to cosmology, independent of its possible role in the brane inflation scenarios [40]. The rolling tachyon field has properties that have been uncovered just recently [41, 42], such as resulting in a pressure-less fluid at the end of its relaxation towards the minimum of the potential. Furthermore, its potential includes the D-branes as topological defects which also play an important role in cosmology (providing for instance dangerous objects such as monopoles and domain walls and less dangerous ones like cosmic strings). Finally it has partially motivated the introduction of a new type of branes known as space-like or S-branes [43] which can roughly be thought as kinks in time, rather than space, of the tachyon potential describing then the rolling of the tachyon. Just as for the case of D-branes, S-branes can also be obtained as solutions of supergravity equations, but these solutions being time dependent and therefore cosmological in nature [44, 45, 46, 47, 48, 49]. We illustrate in a simple example the interesting properties of these cosmological solutions. A general class of them have past and future cosmological regions, with a bounce, representing cosmologies with horizons and no spacelike singularities [50, 47, 51]. This can be interpreted as the spacetime due to the presence of negative tension branes with opposite charge [52, 53], similar to a pair of orientifold planes. These geometries are related to black holes and then the mass, charge, Hawking temperature and entropy can be computed in a similar way. The bouncing behaviour can be interpreted analogous to the Schwarzschild wormhole or Einstein-Rosen bridge, but this time connecting past and future cosmologies instead of the two static, asymp-

totically flat regions [53]. Stability of these solutions may be a potential problem for their full interpretation.

We also briefly discuss the probably more ambitious proposal of the ekpyrotic universe [54, 55, 56], in the sense that with the same idea of colliding branes, this time in the context of Horava-Witten compactifications rather than D-branes, it may not lead necessarily to inflation but could provide an alternative to it, approaching the same questions as inflation does, especially the inhomogeneities of the cosmic microwave background. This scenario has also lead to two interesting developments. First, resurrecting the idea of the cyclic universe [57, 56] and second, the realisation of cosmological string backgrounds by just orbifolding flat spacetime [58, 55, 59, 60]. This process guarantees an exact solution of string theory and has opened the possibility to approach issues concerning a big bang-like singularity performing explicit string calculations. Possible problems with this approach to time dependent backgrounds are briefly mentioned.

2. Cosmology Overview

2.1 Standard FRW Cosmology

The standard cosmological model has been extremely successful given its simplicity. The starting point is classic Einstein equations in the presence of matter. The requirements of homogeneity and isotropy of the 4D spacetime determines the metric up to an arbitrary function of time $a(t)$, known as the scale factor, which measures the time evolution of the Universe and a discrete parameter $k = -1, 0, 1$ which determines if the Universe is open, flat or closed, respectively. The Friedmann-Robertson-Walker (FRW) metric describing the evolution of the Universe can then be written as:

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] . \quad (2.1)$$

The scale factor $a(t)$ is given by solving Einstein's equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - 8\pi G T_{\mu\nu} . \quad (2.2)$$

In natural unites $\hbar = c = 1$, Newton's constant G can be written in terms of the Planck mass $8\pi G = 1/M_{Planck}^2$. The stress-energy tensor $T_{\mu\nu}$ is usually taken to correspond to a perfect fluid (latin indices are 3-dimensional):

$$T_{00} = \rho , \quad T_{ij} = p g_{ij} , \quad (2.3)$$

with the energy density ρ and the pressure p satisfying an equation of state of the form $p = w\rho$. Here w is a parameter which, for many interesting cases is just a

constant describing the kind of matter dominating in the stress-energy tensor. We present in the table the values of w for common cases corresponding to matter, radiation and vacuum domination.

Stress Energy	w	Energy Density	Scale Factor $a(t)$
Matter	$w = 0$	$\rho \sim a^{-3}$	$a(t) \sim t^{2/3}$
Radiation	$w = \frac{1}{3}$	$\rho \sim a^{-4}$	$a(t) \sim t^{1/2}$
Vacuum (Λ)	$w = -1$	$\rho \sim \frac{\Lambda}{8\pi G}$	$a(t) \sim \exp(\sqrt{\Lambda/3}t)$

Table 1: Behaviour of scale factor and energy density for matter, radiation and vacuum dominated universes. The solution for the scale factor is written for the case $k = 0$.

Einstein's equations for the ansatz (2.1) above reduce to the Friedmann's equations¹:

$$H^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} \quad (2.4)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p) , \quad (2.5)$$

with H the Hubble function $H \equiv \frac{\dot{a}}{a}$. These two equations imply the energy conservation equation $\nabla_\mu T^{\mu\nu} = 0$ or:

$$\dot{\rho} = -3H (\rho + p) . \quad (2.6)$$

Therefore, after using the equation of state $p = w\rho$ we are left with two equations which we can take as the first Friedmann equation and the energy conservation, for ρ and $a(t)$, which can be easily solved. Equation (2.6) gives immediately $\rho \sim a^{-3(1+w)}$ introducing this in Friedmann's equation gives the solution for $a(t)$. We show in the table the behaviour of ρ and $a(t)$ for typical equations of state. Notice that for the expressions for $a(t)$ we have neglected the curvature term k/a^2 from Friedmann's equation and therefore we write the solution for the flat universe case. The exact solutions for the other cases can also be found.

For the more general case we may say several things without solving the equations explicitly. First of all, under very general assumptions, based mainly on the positivity of the energy density ρ , it can be shown that the FRW ansatz necessarily implies an initial singularity, the big-bang, from which the universe starts expanding. For instance if $\rho + 3p > 0$, which is satisfied for many physical cases, the acceleration of the universe measured by \ddot{a} is negative, as seen from the second Friedmann's equation. For $k = -1, 0$ the first equation tells us that, for positive energy density,

¹The second equation is sometimes referred to as the Raychaudhuri equation.

the universe naturally expands forever whereas for $k = 1$ there will be a value of a for which the curvature term compensates the energy density term and $\dot{a} = 0$, after this time a decreases and the universe re-collapses. A word of caution is needed at this point, which is usually a source of confusion. Because of the previous argument, it was often claimed that, for instance, a closed universe ($k = 1$) will re-collapse. However we can see from the second Friedmann equation, which is independent of k , that if $\rho + 3p < 0$ the universe will always accelerate. This happens for instance for vacuum domination ($w = -1$) where for $k = 1$ the solution is $a(t) \sim \cosh(\sqrt{\Lambda/3}t)$, which is clearly accelerating.

We can illustrate the structure of the big-bang model in terms of a spacetime diagram, known as Penrose or conformal diagram, see for instance [61]. This diagram not only pictures the relevant parts of the spacetime, in this case the initial singularity, but it is such that by a conformal transformation, it represents the points at infinity in a compact region and furthermore, even though the spacetime is highly curved, especially close to the singularity, light rays follow lines at 45 degrees just as in standard Minkowski space. These diagrams are usually, but not always (depending on the symmetries of the metric) two-dimensional. In the FRW case it includes the $t - r$ plane, so each point in the diagram represents a 2-sphere for $k = 1$ or the 2D flat and hyperbolic spaces for $k = 0, -1$ respectively. The wiggled line of figure 1 is the spacelike surface at $t = 0$ representing the singularity. At this point the scale factor $a = 0$ and the radius of the sphere (for $k = 1$) is zero and $\rho \rightarrow \infty$. The Penrose diagrams extract in a simple way the causal structure of the spacetime. In this case we can see that extrapolating to the past from any point in the diagram necessarily hits the big-bang singularity.

A useful concept to introduce is the critical density

$$\rho_{critical} \equiv \frac{3H^2}{8\pi G} . \quad (2.7)$$

Which, for the present time $H = H_0 \sim 65\text{Km/s/Mpc}^{-1}$ gives $\rho_c \sim 1.7 \times 10^{-29} \text{g/cm}^3$ (1Mpc (mega-parsec) = 3×10^{22} meters).

This allows us to define a dimensionless parameter Ω which corresponds to the ratio of the energy density of a given system to the critical density: $\Omega_i \equiv \rho_i / \rho_{critical}$, with the index i labelling the different con-

tributions to the energy density, and the total ratio is $\Omega = \sum_i \Omega_i$. With these definitions we can write the first Friedmann equation as:

$$\Omega = 1 + \frac{k}{H^2 a^2} . \quad (2.8)$$

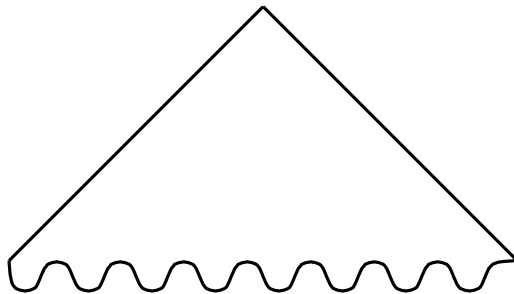


Figure 1: Penrose diagram for a FRW spacetime, the initial spacelike singularity is represented by the wiggled lines, and future infinity is mapped to finite values.

From this we can see the clear connection between the curvature of the spatial sections given by k and the departure from critical density given by Ω . A flat universe ($k = 0$) corresponds to a critical density ($\Omega = 1$) whereas open ($k = -1$) and closed ($k = 1$) universes correspond to $\Omega < 1$ and $\Omega > 1$ respectively.

With all this information in mind, we just assume that the early universe corresponds to an expanding gas of particles and, with the input of the standard model of particle physics, and some thermodynamics ² we can trace the evolution of the system. We present in table 1 some of the important points through the evolution and refer to the standard literature for details [2]. There are few things to keep in mind: the gas is considered to be in equilibrium. The main two reasons for a particle to leave equilibrium is that its mass threshold is reached by the effective temperature of the universe and so it is easier for this particle to annihilate with its antiparticle than being produced again, since, as the universe cools down, there is not enough energy to produce such a heavy object. Also, if the expansion rate of the relevant reactions Γ is smaller than the expansion rate of the universe, measured by H , some particles also get out of equilibrium. For instance, at temperatures above 1 MeV the reactions that keep neutrinos in equilibrium are faster than the expansion rate but at this temperature $H \geq \Gamma$ and they decouple from the plasma, leaving then an observable, in principle, trace of the very early universe. Unfortunately we are very far from being able to detect such radiation.

At the atomic physics scale, the universe is cold enough for atoms to be formed and the photons are out of equilibrium, giving rise to the famous cosmic microwave background. At approximately the same time also the universe changes from being radiation dominated ($w = 1/3$) to matter dominated ($w = 0$). After this, the formation of structures such as clusters and galaxies can start, probably due to the quantum fluctuations of the early universe, leading to our present time.

The standard cosmological model has strong experimental evidence which can be summarised as follows:

- The original observation of Hubble and Slipher at the beginning of the 20th century, that the galaxies are all separating from each other, at a rate that is roughly proportional to the separation, is clearly realised for H approximately constant at present, $H = H_0 > 0$. This has been overwhelmingly verified during the past few decades.
- The relative abundance of the elements with approximately 75% Hydrogen almost 24% Helium, and other light elements such as Deuterium D and helium-4 ${}^4\text{He}$, with small fractions of a percent, is a big success of nucleosynthesis, and at present is the farther away in the past that we have been able to compare theory and observation.

²As long as gravity is weak we can safely define the concept of thermal equilibrium and therefore a temperature.

Temperature	Time	Particle Physics	Cosmological Event
10^{19} GeV	10^{-43} s	String Theory?	Gravitons decouple ?
10^2 GeV - 10^{19} GeV	10^{-43} s - 10^{-12} s	Grand Unification? Desert? String Theory? Extra dimensions?	Topological defects? Baryogenesis? Inflation?
10^2 GeV	10^{-12} s	Electroweak Breaking	Baryogenesis?
0.3 GeV	10^{-5} s	QCD scale	Quark-Hadron transition
$10 - 0.1$ MeV	$10^{-2} - 10^2$ s	Nuclear Physics scale	Nucleosynthesis, Neutrinos decouple
10 eV	10^{11} s	Atomic Physics scale	Atoms formed, CMB Matter domination

Table 2: A brief history of the universe (or time in a nutshell). The temperature units can be translated to K by using $1 \text{ GeV} = 1.16 \times 10^{13} K$.

- The discovery of the cosmic microwave background, signalling the time of last photon scattering, by Penzias and Wilson in 1964 was perhaps the most spectacular test of the model. Starting in the 1990's with the discoveries of the COBE satellite and more recent balloon experiments such as BOOMERANG, Maxima and DASI, cosmology has been brought to the status of precision science. In particular, the confirmation of the black body nature of the CMB is known with excellent precision, but, more importantly, the fluctuations in the temperature $\frac{\delta T}{T}$ signalling density fluctuations $\frac{\delta \rho}{\rho}$ in the early universe provide a great piece of information about the possible microscopic origin of the large scale structure formation. The temperature fluctuations are analysed in terms of their spherical harmonics decomposition $\frac{\delta T}{T} = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$, with the power spectrum $C_l = \langle |a_{lm}|^2 \rangle$ showing a peak structure (see figure 2). The higher the multi-pole moment the smaller angular separation in the sky. The location and height of the peaks provides precise information about the fundamental parameters of FRW cosmology, such as Ω , Ω_{baryon} , the cosmological constant, etc. See for instance [62, 63]. In particular the first peak being at approximately $l = 200$ provides a very strong evidence in favour of a flat universe.

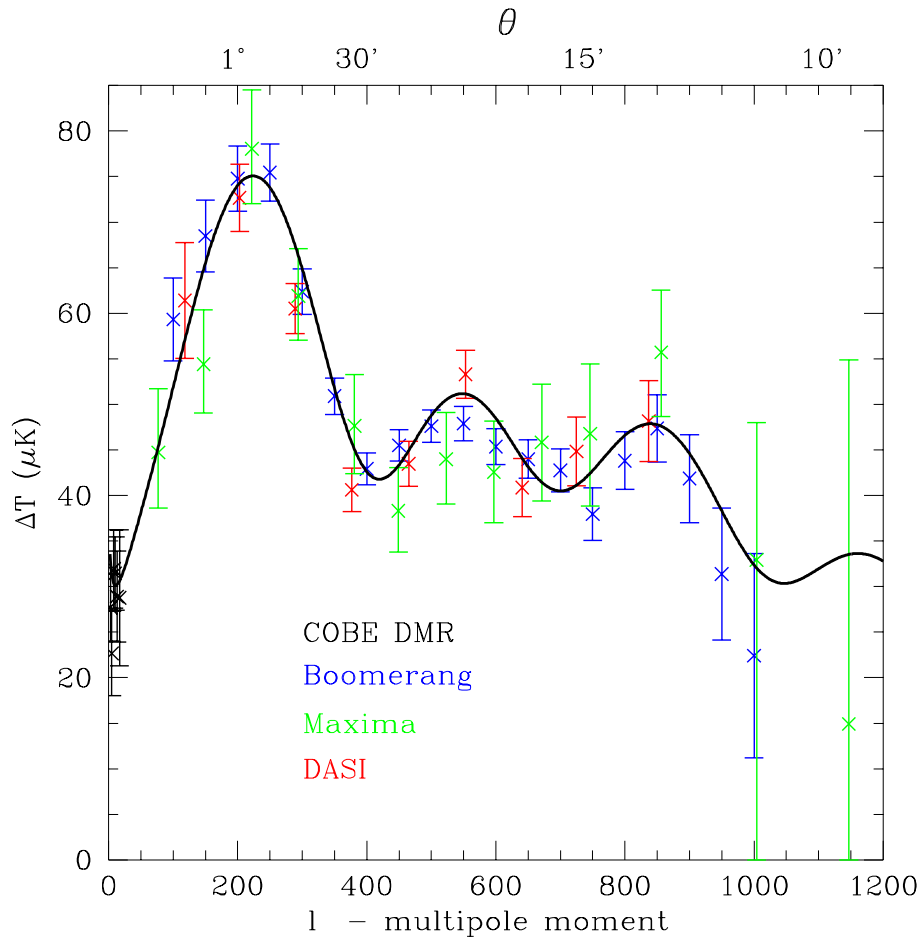


Figure 2: The cosmic microwave background spectrum of temperature fluctuations. The different location and height of the peaks as a function of multi pole moments put strong constraints on the parameters of the standard model of cosmology. The solid line represents the best fit of the combined observations. Courtesy of Rob Crittenden.

Further observations and theoretical developments point also to some problems of this model which we can summarise as follows

- The breakdown of classical relativity near the initial singularity is certainly the main conceptual problem in cosmology.
- The horizon problem. The isotropy of the universe reflected by the CMB is actually the source of a problem. Assuming the standard expansion of the universe we receive the same information from points in the space that do not appear to be in causal contact with each other. Therefore it is actually a puzzle why the radiation is so uniform.
- Origin of CMB anisotropies. The definitely observed anisotropies in the CMB are expected to be produced from physics of the early universe which is not explained in the standard model.
- Flatness. The universe is almost flat in the sense that $0.2 < \Omega < 2$. This evidence is being strengthened by the more recent results on the CMB, essentially the position and height of the first acoustic peak on the spectrum of the CMB precisely provides evidence for $\Omega \sim 1$ at present. See for instance [62, 63]. The flatness problems refers to the fact that for Ω to be so close to one at present it had to be essentially one in the early universe with a precision of many significative figures. There is no explanation for this.
- Baryogenesis. Combining the standard models of particle physics and cosmology we cannot explain why there seems to be an excess of matter over antimatter. The requirements for this to happen, *ie* process out of equilibrium, baryon number violation and CP violation need to be combined in a model beyond the standard models but at the moment has no explanation.
- Dark matter. The survey and study of the behaviour of matter, such as rotation curves for galaxies, at many different scales, has given evidence that there should be a new kind of matter not present in the standard model of particle physics. This should play an important role in the explanation for the large scale structure formation.
- Dark energy. Recent results from the study of high redshifted supernovae, combined with the CMB, has provided strong evidence for the fact that the universe is actually accelerating at present. This, as mentioned before, indicates that there should be a form of ‘dark energy’ which provides $\rho + 3p < 0$ and causes the universe to accelerate. An effective cosmological constant or a time varying scalar field are the main proposals for this dark energy. In any case this stresses the cosmological constant problem (why is the cosmological constant

almost zero?) and makes it more interesting to explain why it has the value it seems to have at present $\Lambda = 10^{-120} M_{Planck}^4 = (10^{-3} \text{eV})^4$. Present observations point towards $\Omega = \Omega_\Lambda + \Omega_B + \Omega_{DM} = 1$ with the contribution from dark energy $\Omega_\Lambda \sim 0.7$ whereas the dark matter and baryonic contributions together only make $\Omega_B + \Omega_{DM} \sim 0.3$. Another way to rephrase this challenge is by calling it the coincidence problem which essentially states: why each of these contributions happen to be of the same order by the time of galaxy formation. For a recent review see [64]

All of these problems are strong motivations to guide us into the possible ways to modify both the standard models of particle physics and cosmology. Furthermore, their extensions could not only offer solutions to these problems but generate new ones also. For instance grand unified models typically imply the existence of topological defects such as domain walls, cosmic strings and monopoles which could have an important impact in cosmology. In particular the existence of monopoles and domain walls would over-close the universe and therefore cause new problems, named the domain wall and monopole problems. Cosmic strings on the other hand were thought to be useful for galaxy formation, although by themselves would predict an spectrum of density perturbations which do not fit the CMB results.

2.2 Inflation

More than 20 years ago, the inflationary universe was proposed [1], offering a possible solution to the flatness, horizon and monopole problems. It was soon realised that, more importantly, it could also provide an explanation for the possible CMB anisotropies and therefore for structure formation.

The main idea behind inflation is that in the early universe there is a short time when the universe expanded very fast, usually an exponential expansion. If the inflationary period is long enough, it would flatten the universe quickly (solving the flatness problem), it would also explain why some regions could be in causal contact with each other, solving the horizon problem. Finally the fast expansion would dilute many objects, such as monopoles and other unwanted massive particles in such a way as to make them harmless for the over-closure of the universe.

The simplest realization of inflation is to introduce a scalar field ψ with a potential $V(\psi)$, the value of the potential provides an effective cosmological constant. If it is flat enough then we would be in a situation similar to the case $w = -1$ for which we already saw that the scale factor $a(t)$ increases exponentially.

The Friedmann's equation for this system becomes:

$$H^2 = \frac{8\pi G}{3} \left(V + \frac{\dot{\psi}^2}{2} \right) - \frac{k}{a^2}, \quad (2.9)$$

whereas the scalar field equation is

$$\ddot{\psi} - 3H\dot{\psi} = -V' . \quad (2.10)$$

Where $V(\psi)$ is the scalar field potential and $V' \equiv dV/d\psi$. Notice that the second term in this equation acts like a friction term for a harmonic oscillator (for a quadratic potential) with the friction determined by the Hubble parameter H .

The right hand side of equation (2.9) is the energy density due to the scalar field ψ . We can easily see that if the potential energy dominates over the kinetic energy and $V \sim \Lambda > 0$ we have the $w = -1$ case with exponential expansion $a \sim e^{Ht} \sim \exp(\sqrt{\Lambda/3})$ in units of the Planck mass, for $k = 0$, and similar expressions for other values of k . The important point is that the scale factor increases exponentially, therefore solving the horizon, flatness and monopole problems.

The conditions for inflation to be realised can be summarised in two useful equations, known as the slow roll conditions:

$$\epsilon \equiv \frac{M_{Planck}^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1 , \quad (2.11)$$

$$\eta \equiv M_{Planck}^2 \frac{V''}{V} \ll 1 . \quad (2.12)$$

The parameters ϵ and η have become the standard way to parametrise the physics of inflation. If the first condition is satisfied the potential is flat enough as to guarantee an exponential expansion. If the second condition is satisfied the friction term in equation (2.10) dominates and therefore implies the slow rolling of the field on the potential, guaranteeing the inflationary period lasts for some time.

If the potential were a constant we would be in a de Sitter universe expansion and the amount of inflation would be given by the size of H (since $a(t) \sim e^{Ht}$). More generally the number of e-foldings is given by:

$$N(t) \equiv \int_{t_{init}}^{t_{end}} H(t') dt' = \int_{\psi_{init}}^{\psi_{end}} \frac{H}{\dot{\psi}} d\psi = \frac{1}{M_{Planck}^2} \int_{\psi_{end}}^{\psi_{init}} \frac{V}{V'} d\psi . \quad (2.13)$$

A successful period of inflation required to solve the horizon problem needs at least $N \geq 60$. The recipe to a successful model of inflation is then to find a scalar field potential V satisfying the slow roll conditions in such a way that the number of e-foldings exceeds 60 (slightly smaller values are sometimes allowed depending on the scale that inflation occurs). It is of no surprise that many potentials have been proposed that achieve this. For a collection of models see for instance the book of Liddle and Lyth in [2]. Usually getting a potential flat enough requires certain amount of fine tuning unless there is a theoretical motivation for the potential. It is fair to say that at the moment there are no compelling candidates.

The scenario that has been used recently as a concrete paradigm is hybrid inflation, introduced by Linde in 1991 [31]. The idea here is to separate the inflaton with the ending of inflation. This means that there are at least two fields. The inflaton having a flat potential that satisfies the slow roll conditions and then a second field for which its mass depends on the inflaton field in such a way that before and during inflation the squared mass is positive but then after inflation the mass squared becomes negative and the field becomes tachyonic, signalling an instability in that direction. This means that the stationary point for this field is now a maximum instead of a minimum and the field wants to roll fast towards its true vacuum, ending inflation. Solving in this way the ‘graceful exit’ of inflation problem. This scenario permits that the inflaton field does not have to take values larger than the Planck scale as usually happens in single field potentials. Furthermore it is easier to realize in concrete examples either in supersymmetric theories and, as we will see, in string theory.

A typical potential for these fields takes the form:

$$V(X, Y) = a(Y^2 - 1)X^2 + bX^4 + c \quad (2.14)$$

with a, b, c suitable positive constants. We can easily see that for $Y^2 > 1$ the field X has a positive mass², at $Y^2 = 1$, X is massless and for $Y^2 < 1$ the field X is tachyonic, with a potential similar to the Higgs field. Therefore the potential in the Y direction can be very flat, see figure 3, and Y can be identified with the inflaton field ψ . The tachyon field X is responsible for finishing inflation since it provides the direction of maximum gradient after it becomes tachyonic. Notice that adding more fields usually does not help into improving the conditions for inflation, since the direction of maximum gradient is at the end the dominant. What helps in this case is that the field X changes from being massive to tachyonic and then allows Y to roll slowly and induce inflation.

Probably the most relevant property of inflation is that it can provide an explanation for the density perturbations of the CMB and therefore indirectly account for the large scale structure formation. Quantum fluctuations of the scalar field give rise to fluctuations in the energy density that at the end provide the fluctuations in

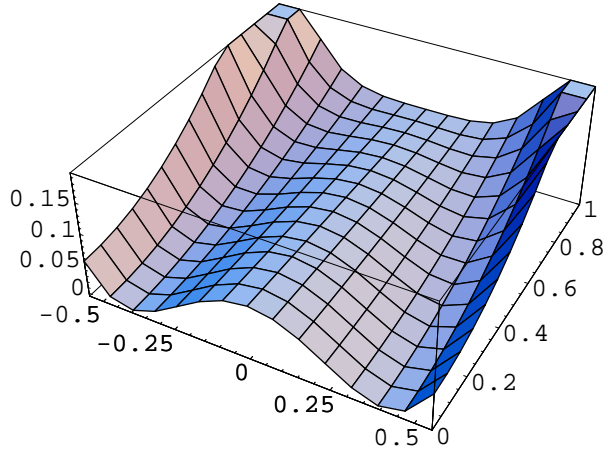


Figure 3: A typical potential for hybrid inflation, an inflaton field rolls slowly, for a critical value the other field becomes massless and then tachyonic constituting the direction of larger gradient and ending inflation.

the temperature observed at COBE [2, 12, 65]. Furthermore most of the models of inflation imply a scale invariant, Gaussian and adiabatic spectrum which is consistent with observations. This has made inflation becoming the standard cosmological paradigm to test present and future observations.

The typical situation is that any scale, including the perturbations, will increase substantially during inflation whereas the Hubble scale remains essentially constant. Therefore the scale will leave the horizon (determined essentially by H^{-1}) and the fluctuations get frozen. After inflation, the Hubble scale will increase faster and then the scales will re-enter the horizon. The amplitude of the density perturbation ($\delta\rho/\rho$) when it re-enters the horizon, as observed by Cosmic Microwave Background (CMB) experiments is given by:

$$\delta_H = \frac{2}{5} \mathcal{P}_{\mathcal{R}}^{1/2} = \frac{1}{5\pi\sqrt{3}} \frac{V^{3/2}}{M_p^3 V'} = 1.91 \times 10^{-5}, \quad (2.15)$$

where $\mathcal{P}_{\mathcal{R}}$ is the power spectrum computed in terms of the two-point correlators of the perturbations. Here the value of δ_H is implied by the COBE results [3].

In order to study the scale dependence of the spectrum, whatever its form is, one can define an effective *spectral index* $n(k)$ as $n(k) - 1 \equiv \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$. This is equivalent to the power-law behaviour that one assumes when defining the spectral index as $\mathcal{P}_{\mathcal{R}}(k) \propto k^{n-1}$ over an interval of k where $n(k)$ is constant. One can then work $n(k)$ and its derivative by using the slow roll conditions defined above [12, 2], and they are given by

$$n - 1 = \frac{\partial \ln \mathcal{P}_{\mathcal{R}}}{\partial \ln k} \simeq 2\eta - 6\epsilon, \quad \frac{dn}{d \ln k} \simeq 24\epsilon^2 - 16\epsilon\eta + 2\xi^2. \quad (2.16)$$

where $\xi^2 \equiv M_P^2 \frac{V' V'''}{V^2}$. Showing that for slow rolling ($\eta, \epsilon \ll 1$) the spectrum is almost scale invariant ($n \sim 1$).

The gravitational wave spectrum can be calculated in a similar way. The gravitational spectral index n_{grav} is given by

$$n_{grav} = \frac{d \ln \mathcal{P}_{grav}(k)}{d \ln k} = -2\epsilon. \quad (2.17)$$

Therefore we have a simple recipe to check if any potential can give rise to inflation: compute the parameters ϵ, η , check the slow roll conditions, if they are satisfied we can right away find the spectral indices and the COBE normalisation (2.15) puts a constraint on the parameters and scales of the potential.

Finally, quantum fluctuations can move the field up the potential, providing more inflation. This gives rise to eternal inflation since parts of the universe will keep expanding forever, each of them releasing the field to a lower value of the potential, which will lead to standard inflation, in a process that induces a self-reproducing universe (see for instance [66] and references therein).

3. Pre D-branes String Cosmology

Since the mid 1980's some effort has been dedicated to the cosmological implications of string theory. One possible approach was to look at cosmological solutions of the theory starting from a 10D effective action. A collection of many of these solutions can be seen in [67]. They correspond to solutions of Einstein's equations in the presence of dilaton and antisymmetric fields with a time dependent metric. Some solutions were found for which some dimensions expand and others contract. Furthermore, starting in 1991, Witten found an exact conformal field theory corresponding to a coset $SL(2, R)/U(1)$ that written in terms of the WZW action gave rise to the metric of a 2D black hole [67]. This opened the way towards looking for non trivial spacetimes as exact CFT's in 2D by investigating different cosets [68, 69, 70]. One interesting observation was made in [50] for which changing the sign of the Kac-Moody level provides a spacetime for which time and space were interchanged, and therefore a black hole geometry turned into a cosmological one. A similar structure has been found recently and we will mention it in section 4.6.

Besides looking for time-dependent solutions [13], there were several interesting issues discovered at that time. We can summarise the main results of those investigations as follows.

3.1 Brandenberger-Vafa Scenario

In 1987, T -duality was discovered in string theory (for a review see [71]). This refers to the now well known $R \rightarrow 1/R$ symmetry of the partition functions and mass spectrum of string theories compactified in a circle of radius R . In particular the bosonic and heterotic strings are known to be self-dual under this transformation. The mass formula takes the form

$$M^2 = \frac{n^2}{4R^2} + m^2 R^2 + N_L + N_R - 2. \quad (3.1)$$

In units of the inverse string tension $\alpha' = 1/2$. The integers n and m give the quantised momentum in the circle and the winding number of the string in the circle, $N_{L,R}$ are the left and right oscillator numbers. We can easily see that the spectrum is invariant under the simultaneous exchange $R \leftrightarrow 1/2R$ and winding and momenta $n \leftrightarrow m$. This symmetry has had many important implications in the development of string theory. Regarding cosmology, Brandenberger and Vafa soon realised that it could have interesting applications. First they emphasised that the concept of distance has different interpretation in the two dual regimes. There is a minimum distance in string theory.³ At large radius the position coordinate is

³This statement has been modified in the last few years due to the fact that for instance D0 branes can probe distances smaller than the string scale. This has no direct implications for the argument we are presenting here.

the conjugate variable to momentum $p = n/R$, as usual. But at distances smaller than the self-dual radius we have to use the dual coordinate which is the conjugate variable to winding $W = mR$. There is no sense to talk about distances smaller than the string scale, since they will be equivalent to large distances. Brandenberger and Vafa claimed that if the universe is thought to be a product of circles this may be a way to eliminate the initial singularity also ⁴.

The second interesting observation of BV was that this could provide a dynamical explanation of the reason why our universe looks four-dimensional. The argument goes like this: imagine that the universe starts with all spatial dimensions of the string size, then the existence of winding modes will prevent the corresponding dimension from expanding (imagine a rope wrapping a cylinder). However winding modes naturally annihilate with anti-winding modes. In the total ten dimensions a winding string will naturally miss to meet the anti-winding string just because their world-sheets can have many different trajectories in ten dimensions. However these world-sheets naturally overlap in a four-dimensional hypersurface of the total space in which winding and anti winding strings can annihilate and allow the expansion of the three spatial dimensions. This could then mean that the winding strings will prevent six of the dimensions from expanding and will leave three spatial dimensions to expand. This intuitive argument has been recently revived to include the presence of higher dimensional objects such as D-branes [18]. The claim is that even in the configuration of a gas of D-branes of different dimensionalities, the original BV argument still holds since the string winding modes are still the most relevant for the expansion argument [11].

Even though this is a very rough argument, that needs much refinement before it can be taken too seriously, it is essentially the only concrete proposal so far to approach the question of why we feel only three large dimensions. Only because of this reason it deserves further investigation. One criticism to this is the assumption that the dimensions are toroidal, something that generically is not considered very realistic for both our spatial dimensions and the extra ones. Regarding the assumptions in the extra dimensions, recently intersecting D6 brane models with realistic properties in toroidal compactifications were constructed. The chirality comes from the intersection even though the background space, being a torus, was not expected to give realistic models. Furthermore, there has been an attempt to extend the result of BV to more realistic compactifications such as orbifolds [72].

3.2 Moduli and Inflation

One of the few things that can be called a prediction of string theory is the existence of light scalar particles with gravitational strength interactions such as the complex dilaton field S and the moduli fields that describe the size and shape of the extra dimensions, which we can refer generically as T . In supersymmetric string models these fields are completely undetermined reflecting the vacuum degen-

⁴In their original article it was claimed that Einstein's equations were not symmetric under the T duality symmetry. However at that time it was not realized that the equations are rendered invariant by the appropriate transformation of the dilaton.

eracy problem. In the effective field theory this is realised by the fact that those fields have vanishing potential to all orders in perturbation theory due to the non-renormalisation theorems of supersymmetric field theories. This is usually taken to be an artifact of perturbation theory and the potentials are expected to be lifted hopefully breaking the continuous vacuum degeneracy and fixing the value of the moduli to the preferred phenomenologically (the dilaton leading to weak coupling and the radius larger than the string scale). However nonperturbative potentials are not well understood and the fixing of the moduli remains as one of the main open question in string models.

The fact that the potential is flat to all orders in perturbation theory may be an indication that after the breaking of supersymmetry the potential may be flat enough as to satisfy the slow roll conditions and generate inflation. Therefore string theory naturally provides good candidates for inflaton fields. However this has not been realised in practice.

A detailed study of the general properties expected for the dilaton potential was performed in [73] with very negative conclusions. The main problem is that nonperturbative superpotentials in string theory are expected to depend on e^{-aS} which give rise to runaway potentials, in specific scenarios such as the racetrack case on which the superpotential is a sum of those exponentials, similar to the one in figure 4, not only are too steep for inflation but also does not allow any other field to be the inflaton field since that will be the fastest rolling direction towards the minimum. Furthermore the nontrivial minimum creates a problem since if the field configuration starts to the left of the minimum it would naturally roll through the minimum towards the runaway vacuum which corresponds to zero string coupling. Instead of helping to solve cosmological problems, the dilaton potential actually creates a new one. A proper treatment of the situation in a cosmological background can cause enough friction as to stop the field in the nontrivial minimum as suggested in [74], see also [75].

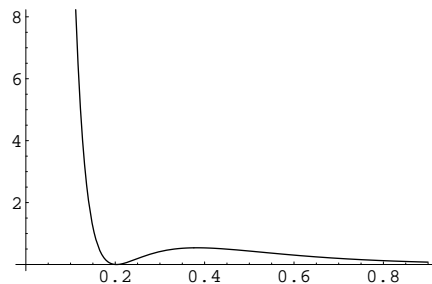


Figure 4: A typical potential for the dilaton field. It has a runaway minimum on which the theory is free and a nontrivial minimum at finite values. The potential is too steep to produce inflation and prevents any other field from becoming the inflaton. It is hard to stop the field from passing through the finite vacuum towards the one at infinity.

3.3 The Cosmological Moduli Problem

A more generic and probably more serious problem was pointed out in [20] (see also [21]). This is the so-called ‘cosmological moduli problem’. This refers in general to any scalar field that has gravitational strength interactions and acquires a nonzero mass after supersymmetry breaking. It was shown in general [20], that independent

of the mechanism for supersymmetry breaking, as long as it is mediated by gravity, the masses of the dilaton and moduli fields are expected to be of the same order as the gravitino mass. This causes a serious cosmological problem because if the scalar field happens to be stable, it will over-close the universe. Otherwise if the field decays, it will do it very late in the history of the universe due to the weakness of its couplings. This will then ruin the results of nucleosynthesis, like destroying ${}^4\text{He}$ and D nuclei and therefore changing their relative abundance.

Notice that we may say the same about the gravitino and the fermionic partners of the moduli, since they are also expected to get a mass of order $m \sim 1$ TeV. If they are stable, they could over-close the universe unless their mass is of order $m \sim 1$ keV. If unstable, their decay rate would then be $\Gamma \sim m^3/M_{Plank}^2$, imposing that they decay after nucleosynthesis, otherwise their decay products give unacceptable alteration of the primordial ${}^4\text{He}$ and D abundances, imposes a lower bound on their masses of $m > 10$ TeV, which is already a bit high. However for fermionic fields the problem can be easily cured by diluting the fields by inflation in a similar way that inflation solves the monopole problem. This can be done as long as the reheating temperature after inflation satisfies

$$T_{RH} \lesssim 10^8 \left(\frac{100\text{GeV}}{m_{3/2}} \right) \text{ GeV} . \quad (3.2)$$

The problem is more serious for scalar fields because even after inflation the scalar field ϕ will naturally be displaced from its equilibrium position by an amount $\delta\phi$ and oscillations around the minimum of its potential rather than thermal production are the main source for their energy. After inflation the field behaves like non-relativistic matter and therefore its energy density decreases with temperature $\rho \sim T^{-3}$ whereas radiation decreases faster $\rho_{rad} \sim T^{-4}$ so these fields dominate the energy density of the universe $\rho/\rho_{rad} \sim 1/T$ as the universe cools down. From their many couplings to the standard model fields it is expected that these moduli fields will decay, but again since their interactions are of gravitational strength this occurs very late, again their decay modes destroying the ${}^4\text{He}$ and D nuclei and the successful predictions of nucleosynthesis unless their mass satisfies $m > 10$ TeV. This again does not look very strong constraint, however now inflation does not help and furthermore, the decay of the scalar field leads to an entropy increase of the order

$$\Delta \sim \frac{\delta\phi^2}{mM_{Plank}} . \quad (3.3)$$

If Δ is very large this would erase any pre-existing baryon asymmetry, therefore this condition requires $\delta\phi \ll M_{Plank}$ which is not natural given that the moduli fields are expected to have thermal and/or quantum fluctuations that can be as large as the Planck scale.

Even though this problem has been discussed at length during the past 8 years, it is fair to say that there is not a completely satisfactory solution. Inflation at low

energies and low energy baryogenesis ameliorate the problem considerably [20, 19]. Probably the best proposal for the solution is thermal inflation [76]. A simple way out would be if the moduli fields are fixed at the string scale and then supersymmetry is broken at low energies, in that case these fields do not survive at low energies and the problem does not exist. Otherwise this is an important challenge for any realistic effort in string theory cosmology.

3.4 Pre Big-Bang Scenario

A natural next step from the BV proposal is to consider the possibility of T duality in backgrounds closer to the FRW type. For this let us first consider a low energy string effective action including the metric g_{MN} , the dilaton φ and the NS-NS anti-symmetric tensor of the bosonic and heterotic strings B_{MN} . In an arbitrary number of dimensions $D = d + 1$ the bosonic action takes the form:

$$S = \int d^D x \sqrt{-g} e^{-\varphi} \left(R + \partial_M \varphi \partial^M \varphi - \frac{1}{12} H_{MNP} H^{MNP} + \dots \right). \quad (3.4)$$

Where $H = dB$. For an ansatz of the type:

$$ds^2 = -dt^2 + \sum_i^d a_i^2(t) dx_i^2 \quad (3.5)$$

we can easily see that T duality is a symmetry of the equations of motion acting as:

$$a_i(t) \rightarrow \frac{1}{a_i(t)} \quad \varphi \rightarrow \varphi - 2 \sum_i \log a_i \quad (3.6)$$

Since $a_i(t)$ represent in this case the scale factors, like in FRW, this has been named scale factor duality. Thus we can see that expanding and contracting universes are related by this symmetry.

Furthermore, Veneziano and collaborators [77], realised that this symmetry can be combined with the standard symmetry for this kind of backgrounds corresponding to the exchange:

$$a(t) \leftrightarrow a(-t) \quad (3.7)$$

This simple observation opens up the possibility of considering a period before $t = 0$ for which the Hubble parameter increase instead of decrease. That is without duality the symmetry under $t \rightarrow -t$ would send $H(t) \rightarrow -H(-t)$ but combining this with duality provides four different sign combinations for $H(t)$. If the universe at late times is decelerating H would be a decreasing monotonic function of time for ‘positive’ t , then a combination of duality and the $t \rightarrow -t$ transformation can give rise to an $H(-t) = H(t)$ so that this function can be even, see figure 5. So we can see that there is a possible scenario in which the universe accelerates from negative times towards

the big bang and then decelerates after the big-bang. The acceleration would indicate a period of inflation before the big-bang without the need of an scalar potential.

A concrete solution for this system corresponds to the isotropic case $a_i = a_j \equiv a(t)$ for which:

$$a(t) = t^{1/\sqrt{d}} \quad t > 0 , \quad (3.8)$$

with a constant dilaton. For this $H(t) \sim 1/t$ decreases monotonically with time. By applying the transformation $t \rightarrow -t$ and duality we can generate the four different branches of solutions:

$$\begin{aligned} a(t) &= t^{\pm 1/\sqrt{d}} & t > 0 \\ &= (-t)^{\pm 1/\sqrt{d}} & t < 0 . \end{aligned} \quad (3.9)$$

With

$$\varphi_{\pm}(\pm t) = (\pm\sqrt{d} - 1) \log(\pm t) . \quad (3.10)$$

The two branches for which the universe expands $H > 0$ provide an interesting realization of the pre big-bang scenario, see figure 5. Notice that the solutions are such that have a singularity at $t = 0$ but also in this region the dilaton blows up implying strong coupling. It is expected that nonperturbative string effects would provide a smooth matching between these two branches. The weak coupling perturbative string vacuum appears as a natural initial condition in the pre big-bang era. Therefore the scenario consists of an empty cold universe in the infinite past that expands in an accelerated way towards a region of higher curvature until it approaches the region of strong coupling and large curvature which is assumed will match smoothly to the post big bang branch in which the universe continues expanding but decelerates.

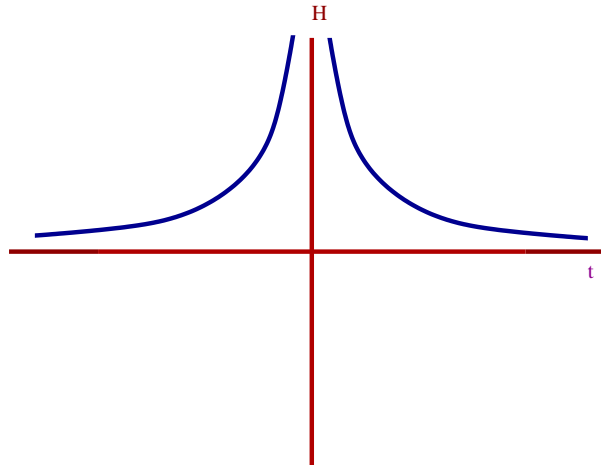


Figure 5: Possible realisation of the pre big bang scenario with the past and future regions expected to match at the singularity with strong coupling and large curvature

This cosmological string scenario is probably the one that has been subject to more detailed investigation during the past decade. It has several attractive features such as the possibility of having a period before the big-bang helping to provide the initial conditions and giving rise to an alternative to scalar field inflation, with the advantage of being motivated by string theory. Furthermore a study of the

density perturbations for this scenario has attracted alternatives to inflation. The spectrum of density perturbations has been estimated and claimed not to contradict the recent observations. Also it provides testable differences with respect to the tensor perturbations that could be put to test in the future.

The scenario has been also subject to criticism for several reasons. First, as the authors point out, the main problem to understand is the graceful exit question, that is how to pass smoothly from pre to post big-bang period. The argument is that close to the big-bang the perturbative treatment of string theory does not hold since the dilaton and the curvature increase, implying strong string coupling. Therefore there is no concrete way to address this issue in the framework that the theory is treated. Another important problem is the fact that the moduli are neglected from this analysis and there has to be a mechanism that stabilises the extra dimension. This is a standard problem in string theory so it is not particular of this scenario. Also the scale factor duality symmetry that motivated the scenario is not clearly realised in a more realistic setting with nontrivial matter content. The fact that the dilaton will eventually be fixed by nonperturbative effects may change the setting of the scenario.

Furthermore, issues of fine-tuning have been pointed out in the literature [78, 79] as well as not reproducing the CMB spectrum. For this, the density perturbations coming from the dilaton, are not scale invariant with a blue power spectrum with too small to account for the COBE data. Considering an axion field, dual to the NS-NS antisymmetric tensor of string theory, does not work in principle since even though the spectrum is scale invariant, the perturbations are isocurvature instead of adiabatic, implying a pattern of acoustic peaks different from what has been observed, especially at BOOMERANG and DASI. A possible way out has been proposed for which a nonperturbative potential for the axion is expected to be generated after the pre-big bang era. In this case, under the assumptions that the axion field is away from its minimum after the big-bang and it dominates the energy density before decaying, the perturbations change from isocurvature to adiabatic after the decay of the axion. This general mechanism has been named ‘curvaton’[80, 13, 17]. On the other hand tensor perturbations have a blue spectrum and could be eventually detected at antennas or interferometers. This could be a way to differentiate, observationally, this mechanism from standard scalar field inflation. Finally, electromagnetic perturbations are generically amplified in this scenario due to their coupling to the dilaton, something that may eventually be tested. This enhancement is interesting since it could be present in other string theory scenarios where the dilaton plays a cosmological role.

It is not clear if this scenario can be promoted to a fully realistic early cosmological framework. Nevertheless this is an interesting effort that deserves further investigation, it has kept the field of string cosmology active for several years, it has resurrected the old ideas of having cosmology before the big-bang [57] (remember

that even eternal inflation needed a beginning of time), in a string setting and has influenced in one way or another the recent developments in this area.

4. Brane Cosmology in String Theory

The discovery of D branes and the Horava-Witten scenario, have opened the way to the realisation of the brane-world scenario in string theory. The typical situation in type IIA, IIB and I strings is that there may be many D branes sustaining gauge and matter fields, at least one of them should include the Standard Model of particle physics where we would be living. All this is due to the open string sector of the theories for which the end points are constrained to move on the brane. The closed string sector including gravity and the dilaton probes all the extra dimensions. For spacetimes resulting from a product geometry, the effective Planck mass in 4D is given by

$$M_{Planck}^2 \sim M_s^8 R^6, \quad (4.1)$$

with R the size of the extra dimension and M_s the string scale. This is at the source of the claim that large extra dimensions allow the possibility of small M_s as long as the Planck mass is fixed to the experimentally known value.

In five dimensions, Randall and Sundrum generalised this adding a warp factor to the metric depending on the extra 5th dimension:

$$ds^2 = W(y) g_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (4.2)$$

where y is the fifth dimension and $W(y)$ is the warp factor. This metric shares the same symmetries as the direct product case $W = 1$ but allows now the possibility of $W(y)$ playing a role. In particular if there are branes at locations fixed in y , they will feel different scale in their metrics due to different values of $W(y)$, Randall and Sundrum found an exponential dependence in W that made the scales change very

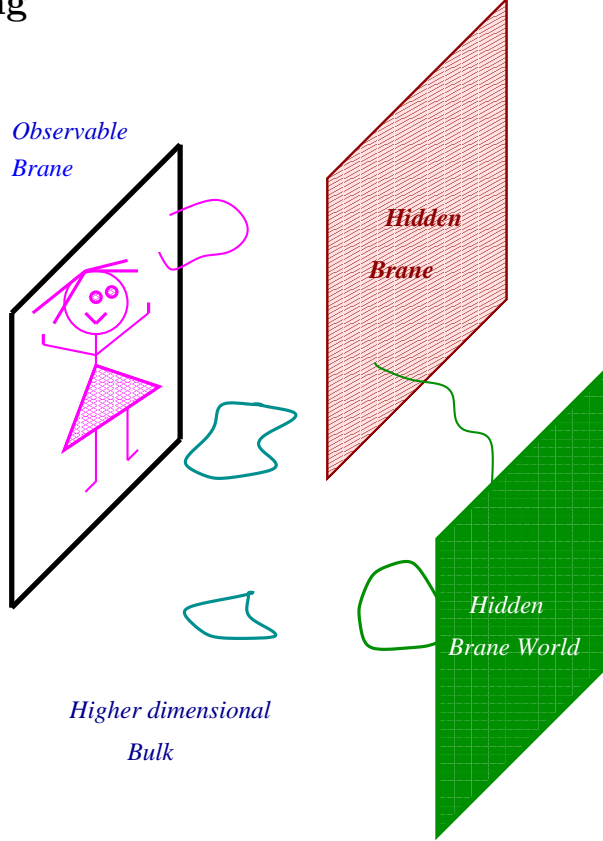


Figure 6: The Brane World. A picture of the brane-world scenario. Open strings (including matter and gauge fields) are attached to branes, whereas closed strings (including gravity and other fields, like the dilaton) can propagate in the bulk.

fast, allowing for the possibility of small fundamental scale even for not that large radii. Furthermore they even discovered that starting in 5D anti de Sitter space and fine tuning the cosmological constant it is possible to have infinitely large extra dimensions and having gravity localised in the brane. These discoveries triggered a large amount of interest, beyond the string community, on the physical implications of the extra dimensions and rapidly included cosmology.

In string theory it was found that there are explicit realisations of the brane world in at least three different ways.

- Having branes trapped in singularities supports chiral fermions and nonabelian gauge symmetries with $N = 1, 0$ supersymmetry and then allows for the possibility of having the standard model in one stack of branes at singularities [82]. Realistic models with $N = 1, 0$ supersymmetry have been found [83] with the small string scale $M_s \sim 10^{12}$ GeV or even $M_s \sim 1$ TeV.
- D-branes intersecting at nontrivial angles [84]. The intersection of the branes can have chiral fermions and therefore there can be found explicit realistic models in many possible bulk backgrounds, including torii. Again the standard model lives on D-branes and gravity in the bulk realising the brane world scenario. Nonsupersymmetric models require the string scale close to 1 TeV, whereas supersymmetric models may have a larger scale [85].
- Horava-Witten scenario in which 11D M-theory compactified in one interval [5]. The two 10D surfaces at the endpoints have E_8 gauge theories and when compactified to 4D give rise to a brane world also [86]. Compactifications of M-theory in terms of G_2 holonomy manifolds, although not constructed explicitly, are known that in order to have chiral fermions, the matter needs to be at singular points, therefore implying that if constructed they may have also a brane world structure [87].

Therefore the brane world appears naturally in string constructions and its cosmology can be studied bearing in mind each of the particular string realisations.

4.1 Brane Cosmology in 5D

Binetruy et al [6], made the first concrete description of a 5D cosmology with branes and found interesting results. We will briefly describe here some of the developments in this directions but only as an introduction to the real topic of these lectures which is brane cosmology in string theory. Therefore we will limit to mention the developments in that direction that will be used in the next sections.

Let us start with the simple action:

$$S = S_{Bulk} + S_{Brane} = \int d^5x \sqrt{-g_5} [R - \Lambda] - \int d^4x \sqrt{-g_4} (\mathcal{K} + \mathcal{L}_{matter}) \quad , \quad (4.3)$$

with \mathcal{K} the trace of the extrinsic curvature and the rest in a self explanatory notation. Looking for cosmological solutions from the bulk action we look at the most general solution which is homogeneous and isotropic in 4D. If we choose the most general metric depending on t and the 5th dimension y , we can take the brane to be at particular point in the space, say $y = 0$. We can work the bulk metric in a conformal frame (where the t, y part of the metric is conformally flat). In this case the metric can be written as [26]:

$$ds^2 = e^{2\nu(t', y)} B^{-2/3}(t', y) (-dt'^2 + dy^2) + B^{2/3} \left[\frac{d\chi^2}{1 - k\chi^2} + \chi^2 d\Omega^2 \right]. \quad (4.4)$$

Where the functions ν and B are completely arbitrary. Now we state Birkhoff's theorem which essentially says that the most general homogeneous and isotropic metric depends on only one variable. This can be seen as follows. Define the light cone coordinates:

$$u = \frac{t' - y}{2}, \quad v = \frac{t' + y}{2} \quad (4.5)$$

Einstein's equations reduce in these variables to

$$\begin{aligned} B_{uv} &= (2\Lambda B^{1/3} - 6kB^{-1/3}) e^{2\nu} \\ \nu_{uv} &= \left(\frac{\Lambda}{3} B^{-2/3} + kB^{-4/3} \right) e^{2\nu} \\ B_u [\log B_u]_u &= 2\nu_u B_u \\ B_v [\log B_v]_v &= 2\nu_v B_v \end{aligned} \quad (4.6)$$

This implies that $B = B[U(u) + V(v)]$ and $e^{2\nu} = B'U'V'$ where primes refer to derivatives with respect to the variable that the function depends on. Without loss of generality we can fix $V(v) = v$ and setting $r = B^{1/3}$, $t = 3(v - U)$ the first equation above can be integrated to give:

$$ds^2 = -h(r) dt^2 + \frac{dr^2}{h(r)} + r^2 d\Omega^2 \quad (4.7)$$

with $h(r) = k - \frac{\Lambda}{6}r^2 - \frac{\mu}{r^2}$. We can identify this metric clearly as Schwarzschild AdS! This shows that even though we started with a time dependent metric, the most general solution can be reduced to a static solution. As we said above this is just a statement of the general Birkhoff's theorem. So far we have not yet used the brane. However without performing any calculation we can see already the implications of this result for the brane cosmology. In our starting point the brane was assumed to be fixed at $y = 0$ and then the metric was a function of time and y . The time dependence providing for the cosmological nature of the metric. However we have found that the metric at the end is static. Where is the time dependence?. It so happens that with the change of variables we have made we can no longer claim that the brane

is at a fixed value of r . The location of the brane is determined by some function describing its trajectory in this static background $r = r(t)$. Therefore the brane is moving in time and it is this motion that will appear as a cosmological evolution for an observer on the brane. This is quite remarkable, since we can imagine that our present feeling that the universe expands, could only be an illusion and actually our universe would be moving with some non vanishing velocity in a higher dimensional, static bulk spacetime. This effect has been named mirage cosmology [22, 24, 26, 23].

Notice that we could have worked all the time with the original coordinates t, y and look for cosmological solutions. The end result is the same. In both procedures we need to take into account the presence of the brane by considering the Israel matching conditions, which can be derived in a straightforward way from standard techniques in general relativity, including the Gauss-Codazzi equations [90]. Or simply we can use standard Green function techniques to match the solutions in the brane, taken as a delta function source. It is usually simpler to assume a \mathbf{Z}_2 symmetry across the brane making the a cut and paste of the spacetime to make it symmetric around the brane. The Israel conditions then can be written as:

$$[\mathcal{K}_{ij}]_{-}^{+} = - \left(T_{ij} - \frac{1}{3} g_{ij} T_k^k \right) \quad (4.8)$$

Where \mathcal{K}_{ij} is the extrinsic curvature and T_{ij} the energy momentum tensor. The \pm refer to the two regions separated by the brane. Imposing these conditions we arrive at the Friedmann's equations for the 4D brane. They give the standard energy conservation

$$\dot{\rho} + 3(p + \rho) \frac{\dot{a}}{a} = 0 \quad (4.9)$$

with a_0 the scale factor on the brane. The first Friedman's equation takes an interesting form:

$$H^2 \sim \rho^2 + \dots \quad (4.10)$$

This instead of the standard behaviour $H^2 \sim \rho$ in FRW. It was then suggested that brane worlds would give rise to non-standard cosmologies in 4D. This has been addressed in several ways. The most accepted argument at the moment is that we have to have a mechanism to fix the value of the 5th dimension, before making the comparison. Once this is achieved, the standard behaviour is recovered. It is also claimed that in general ρ has to be substituted by the contribution to the energy density of the brane and then instead of just ρ^2 we get $(\rho + \Lambda_4)^2$ which when expanding the squares recovers the linear term in ρ which then, at late times, will be dominant over ρ^2 since ρ decreases in time.

Here we will follow Verlinde [27] who, in a very elegant way, recovered the Friedmann equation (see also [88]). The idea is to use the AdS/CFT correspondence where the brane takes the place of the boundary (therefore the CFT becomes interacting). The point is as follows. Start with the AdS_5 metric above and specify the location

of the brane in parametric form $r = r(\tau)$, $t = t(\tau)$. We can choose the τ parameter such that the following equation is satisfied:

$$\frac{1}{h(r)} \left(\frac{dr}{d\tau} \right)^2 - h(r) \left(\frac{dt}{d\tau} \right)^2 = -1 \quad (4.11)$$

this guarantees that the induced 4D metric in the brane takes the FRW form:

$$ds_4^2 = -d\tau^2 + a^2(\tau) d\Omega_3^2 \quad (4.12)$$

with the scale factor $a(\tau) = r(\tau)$. This is already an interesting piece of information that the scale factor of the brane is just the radial distance from the centre of the black hole (which is identified with the renormalisation group parameter in the field theory dual). Assuming that the matter Lagrangian in the brane is just a constant tension term with tension κ , the equation of motion for the brane action gives simply

$$\mathcal{K}_{ij} = \frac{\kappa}{3} g_{ij} \quad (4.13)$$

with g_{ij} the induced metric on the brane. This then gives us for the metric above: $\frac{dt}{d\tau} = \kappa r/h(r)$. Combining this with (4.11) and tuning the cosmological constant $\kappa^2 = \Lambda/6$, we get the Friedman equation:

$$H^2 = -\frac{1}{a^2} + \frac{\mu}{a^4} \quad (4.14)$$

which corresponds to the Friedmann equation for radiation ($\rho \sim 1/a^4$). In reference [27] this interpretation goes further by using the AdS/CFT correspondence identifying the radiation with the finite temperature CFT dual to the AdS solution. They also find general expressions for entropy and temperature. Finding in particular a general expression between the entropy and the energy density that generalises a result on 2D CFT by Cardy to the general dimensional case. This is known as the Cardy-Verlinde formula. This will not be used in what follows and we refer the reader to the literature for the details of this result.

Before finishing this section we may wonder if the general result used here about the Birkhoff's theorem holds in general in string theory. Unfortunately this is not the case once we introduce the dilaton field. It can be shown explicitly that Birkhoff's theorem does not hold in this case and therefore the cosmological evolution of a brane will have two sources, one the motion of the brane and two the time dependence of the bulk background [47, 91].

4.2 D-Brane Inflation

So far we have discussed string cosmology in a way that does not make contact with the scalar field inflation which has been the dominant topic from the cosmology point of view. The difficulty with this is that as we mentioned before, there is not very

much known about scalar potentials from string theory. In most cases under control the potentials are just zero and the lifting by nonperturbative effects usually leads to runaway potentials or in general potentials which are too steep to inflate. It is then an open question as how to derive inflating potentials from string theory.

The interactions between D-branes offer a new avenue to investigate these issues and has led to the first concrete examples of scalar field inflation from string theory, providing also a nice geometrical and stringy picture of the inflationary process, as well as the ending of inflation. Furthermore, these ideas lead to interesting new cosmological scenarios for which inflation is only a part.

In 1998, Dvali and Tye came up with a very interesting proposal to derive inflation from D-branes. They argued that two D-branes could generate inflation as follows. If both branes are BPS, meaning that they preserve part of the original supersymmetry of the system, and satisfy a Bogomolnyi-Prasad-Sommerfeld bound, the net force between them vanishes. The reason for this is that both have a positive tension and, therefore, are naturally attracted to each other by gravitational interactions. Also the exchange of the dilaton field naturally leads to an attractive interaction. However, both branes are also charged under antisymmetric Ramond-Ramond fields for which the interaction is repulsive, given that both branes have the same charge. Therefore the combined action of the three interactions cancels exactly if the branes are BPS.

This calculation can be done explicitly, the interaction amplitude corresponds to the exchange of closed strings between the two branes. The amplitude can be computed by calculating the one-loop open string amplitude corresponding to a cylinder [103]:

$$\mathcal{A} = 2 \int \frac{dt}{2t} \text{Tr} e^{-tH} = 2T_p \int \frac{dt}{2t} (8\pi^2 \alpha' t)^{-(p+1)/2} e^{-\frac{Y^2 t}{2\phi^2 \alpha'}} [Z_{NS} - Z_R] \equiv A_{NS} - A_R \quad (4.15)$$

With

$$\begin{aligned} Z_{NS} &= \frac{-16 \prod_n (1 + q^{2n})^8 + q^{-1} \prod_n (1 + q^{2n-1})^8}{\prod_n (1 - q^{2n})^8} \\ Z_R &= \frac{q^{-1} \prod_n (1 - q^{2n-1})^8}{\prod_n (1 - q^{2n})^8} \end{aligned} \quad (4.16)$$

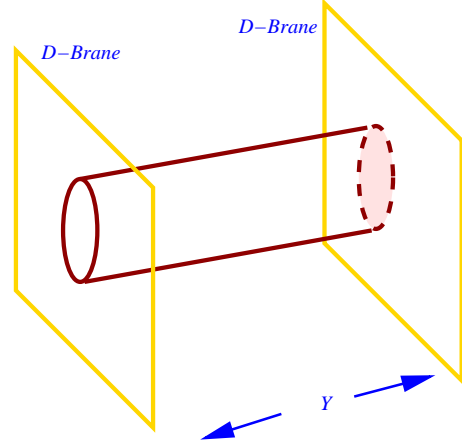


Figure 7: Cylinder interaction between two branes. It could be in two dual ways, as a tree-level exchange of closed strings, valid at large distances or as a one-loop exchange of open strings, dominant at small distances.

Here $q \equiv e^{-t/4\alpha'}$ and t is the proper time parameter for the cylinder. H is the Hamiltonian for each sector of the theory. It is easy to see that $Z_{NS} = Z_R$. Therefore the interaction potential just vanished. The cancellation between R-R and NS-NS sectors is a reflection of the BPS condition for the D-branes. (This is the reason that D-branes can generally be stacked together increasing the gauge symmetry.)

The proposal of Dvali and Tye was that, after supersymmetry gets broken, it is expected that the RR field and the dilaton may get a mass, whereas the graviton stays massless. Therefore the cancellation of the inter-brane force no longer holds and a nonzero potential develops which is generally attractive, given that gravity is the dominant force whereas the other interactions will have a Yukawa suppression due to the mass of the carrier modes. Therefore they propose that the effective potential would be a sum of two terms, the first one coming from the sum of the two tensions of the branes is like a cosmological constant term and the second is the uncanceled interaction potential. At distances large compared to the string length it takes the form:

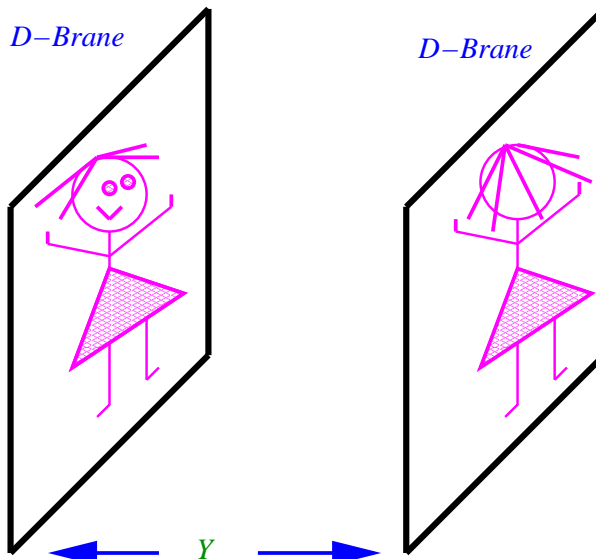


Figure 8: D brane- D brane interaction is zero as long as supersymmetry is unbroken.

$$V \approx 2T + \frac{a}{Y^{d-2}} \left(1 + \sum_{NS} e^{-m_{NS}Y} - 2 \sum_{RR} e^{-m_R Y} \right) \quad (4.17)$$

Where Y is the separation between the branes and a a dimension-full constant. In the limit of zero RR and NS-NS masses m_R, m_{NS} the interaction potential vanishes. But when they are massive we can see the potential takes a form that has the properties to lead to inflation since it is very flat, due to the exponential terms and has a positive value at infinity due to the tension term. The authors argued that this kind of potential satisfies the slow roll conditions and can give rise to inflation in a natural way. A minor problem of their scenario is that they assumed that the string scale was 1 TeV, therefore the density perturbations were of order $\delta_H \sim \frac{H}{\epsilon M_{Planck}}$ for an undetermined parameter ϵ which for $H \sim M_s^2/M_{Planck}$ and $M_s \sim 1$ TeV would imply an extremely small value of ϵ to get the COBE normalisation $\delta_H \sim 10^{-5}$. This problem can be easily solved by just assuming the string scale to be closer to the Planck scale or even an intermediate scale.

A more serious drawback of this scenario is the lack of computability. After all, the proposed potential is only motivated by physical intuition but does not correspond to an honest-to-God string calculation. This made it difficult to make explicit progress. Furthermore, the authors did not address the issue of what happens after the branes collide and how to finish inflation. Nevertheless, this scenario provided an interesting possibility of realising inflation from brane interactions, the shape of the potential looks naively correct and opened up the idea that the attraction and further collision of branes could have interesting cosmological implications.

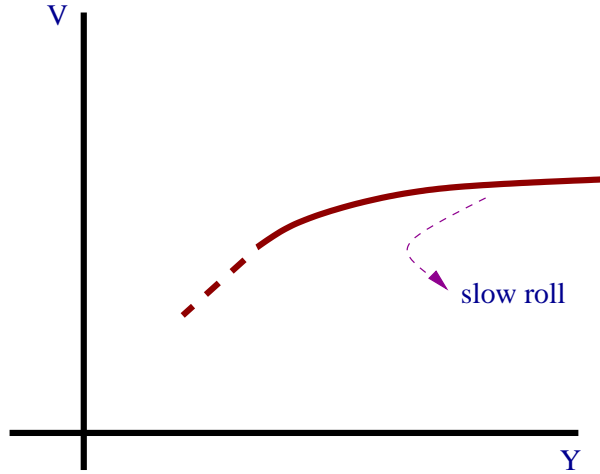


Figure 9: An expected shape of the inflation potential for D-branes, after supersymmetry breaking

4.3 D-Brane/Antibrane Inflation and Tachyon Condensation

Let us consider now a brane/antibrane pair, that means a pair of branes with opposite RR charge. We know that their interaction does not cancel since now, the cylinder diagram will give an amplitude for which the RR contribution changes sign and therefore we have:

$$\mathcal{A} = A_{NS} + A_R = 2A_{NS} \neq 0. \quad (4.18)$$

This gives rise naturally to an attractive force. Contrary to the case of the brane/brane potential, that required uncomputable nonperturbative corrections, this case is computable in an explicit way (from the cylinder diagram, taking the limit of $t \rightarrow \infty$ gives rise to the Newtonian potential at distances larger than the string scale) in perturbation theory.

Let us see in some detail how to compute the corresponding potential. We will start with one Dp -brane and an $\bar{D}p$ -brane in a large four-dimensional bulk with extra dimensions compactified in tori. The 4D effective action can be written as the sum of bulk and branes contributions:

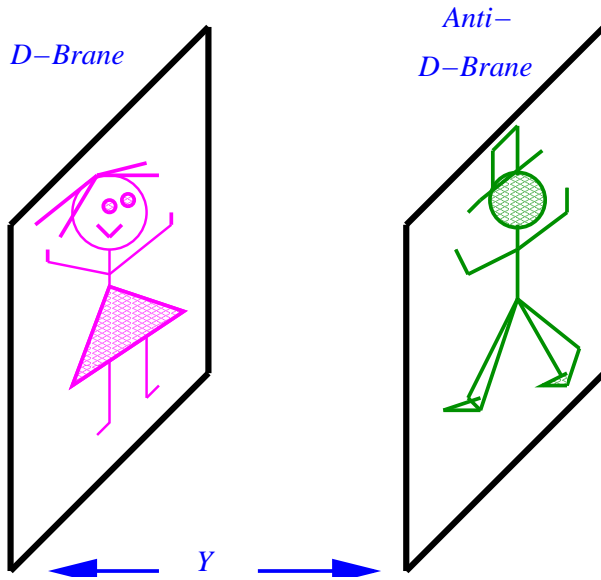


Figure 10: D branes and anti D branes attract each other.

$$S = S_{Bulk} + S_D + S_{\bar{D}}, \quad (4.19)$$

with the bulk action

$$S_{Bulk} = \frac{1}{2} \int d^4 x d^6 z \sqrt{-g} \{ M_s^8 e^{-2\varphi} R + \dots \}, \quad (4.20)$$

where we are denoting by x^μ the four spacetime coordinates and z^m the coordinates in the extra dimensions. The branes actions can be obtained expanding the Born-Infeld action as:

$$S_D = - \int d^4 x d^{p-3} z \sqrt{-\gamma} \{ T_p + \dots \}, \quad (4.21)$$

where $\gamma_{ab} = g_{\mu\nu} \partial_a x_i^\mu \partial_b x_i^\nu$ is the induced metric on the brane and $T_p = M_s^{p+1} e^{-\varphi}$ is the brane tension. We will assume the branes to be parallel and the separation is given by $Y^m \equiv (x_1 - x_2)^m$ where the sub-indices 1, 2 refer to the brane and antibrane respectively. Expanding in powers of $\partial_a Y^m$ we get:

$$S_D + S_{\bar{D}} = - \int d^4 x d^{p-3} Y \sqrt{-\gamma} T_p \left[2 + \frac{1}{4} g_{mn} \gamma^{ab} \partial_a Y^m \partial_b Y^n + \dots \right]. \quad (4.22)$$

The interaction part can be directly obtained by the calculation of the cylinder amplitude mentioned above. For large separations $M_s^{-1} \ll Y$ it simply takes the Newtonian form $V_{interaction} \sim Y^{d_\perp - 2}$ where $d_\perp \equiv 9 - p$ is the number of dimensions transverse to the branes. Combining this with the derivative dependent part of the branes action give us a potential of the form:

$$V(Y) = A - \frac{B}{Y^{d_\perp - 2}}, \quad (4.23)$$

where

$$A \equiv 2T_p V_{||} = \frac{2e^\varphi}{(M_s r_\perp)^{d_\perp}} M_s^2 M_{Planck}^2, \quad (4.24)$$

$$B \equiv \frac{\beta e^{2\varphi}}{M_s^8} T_p^2 V_{||} = \frac{\beta e^\varphi M_{Planck}^2}{M_s^{2(d_\perp - 2)} r_\perp^{d_\perp}}. \quad (4.25)$$

Here the symbols $||$ and \perp refer to parallel and perpendicular to the branes, then $V_{||}$ is the volume parallel to the brane and r_\perp is the radius of the space perpendicular to the brane, which is assumed constant. Also the constant β is given by $\beta = \pi^{-d_\perp/2} \Gamma\left(\frac{d_\perp - 2}{2}\right)$.

Notice also that equation (4.22) provides the normalisation of the kinetic energy and therefore we can work with the canonically normalised field $\Phi \equiv \sqrt{\frac{T_p V_{||}}{2}} Y$ when analysing the consequences of the potential.

Now that we have the full information for the scalar potential we can ask if this potential gives rise to inflation. The fact that in the limit $Y \rightarrow \infty$ it goes to a positive constant A is encouraging. To check the slow roll conditions we compute the constants ϵ and η and find that $\epsilon < \eta$ as usual, and:

$$\eta = -\beta(d_{\perp} - 1)(d_{\perp} - 2) \left(\frac{r_{\perp}}{Y} \right). \quad (4.26)$$

We can see that since β has a fixed value in string theory and the parameter η is proportional to the ratio $(d_{\perp} - 2) \left(\frac{r_{\perp}}{Y} \right) \gg 1$ (since the separation of the branes is assumed to be much smaller compared to the size of the extra dimension for the approximation of the potential to be valid). Therefore, we conclude that slow roll requires $\eta \ll 1$, implying that $Y \gg r_{\perp}$ which is inconsistent. This means that this potential does not give rise to inflation. The cases of D7 and D8 branes ($d_{\perp} = 1, 2$) have to be treated separately but the same conclusions hold.

We may wonder if this situation can be improved. We have realised that if the distance between the branes is $M_s^{-1} \ll Y \ll r_{\perp}$ then generically it does not give rise to inflation. Now we ask the following question. Relaxing the condition $Y \ll r_{\perp}$, is it possible to get inflation? For this we have to be able to compute the potential in a torus [29]. Let us consider the simplest case of a square torus (see figure 11). We assume the compactified transverse manifold to be a d_{\perp} -dimensional square torus with a uniform circumference r_{\perp} . When the brane-antibrane separation is comparable to r_{\perp} , we have to include contributions to the potential from p -brane images, *i.e.*, we have to study the potential in the covering space of the torus, which is a d_{\perp} -dimensional lattice, $(\mathbf{R}/\mathbf{Z})_{\perp}^d$.

The potential at the position of the antibrane is

$$V(\vec{r}) = A - \sum_i \frac{B}{|\vec{r} - \vec{r}_i|^{d_{\perp}-2}}, \quad (4.27)$$

where \vec{r} and \vec{r}_i are the vectors denoting the positions of the p -branes and antibranes in the d_{\perp} -dimensional coordinate space, and the summation is over all the lattice sites occupied by the brane images, labelled by i . We schematically show our set-up in Figs. 11, 12. It looks that the value of the potential diverges by simply summing over the infinite number of lattice sites, however this is just an artifact of working with the method of images and the value of the potential can be unambiguously

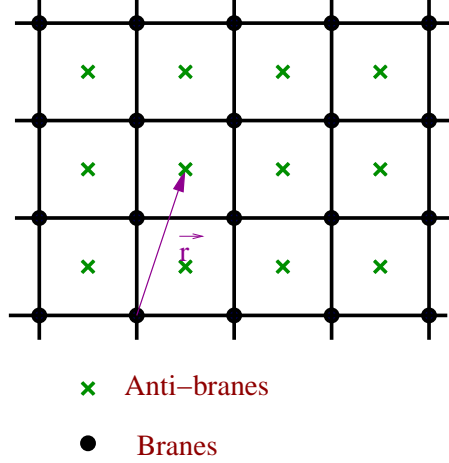


Figure 11: A lattice describing the equilibrium configuration of branes and antibranes.

computed taking into account the finite size of the torus (see appendix of [33] for a detailed calculation.).

Consider now the antibrane motion when the antibrane is near the centre of the hypercubic cell. In this case we may expand the potential in terms of power series of this small displacement \vec{z} from the centre. From a simple symmetry argument, one can easily see that all the odd powers in \vec{z} vanish. Furthermore, the quadratic terms in \vec{z} also vanish, so that the leading contribution to the potential is the quartic term in \vec{z} . We can therefore model the relative motion of the branes in a quartic potential

$$V(z) = A - \frac{1}{4} C z^4, \quad (4.28)$$

where A is defined as before and $C = \gamma M_s^{-8} e^{2\varphi} T_p^2 V_{\parallel} r_{\perp}^{-(2+d_{\perp})}$ with γ a constant of order $\mathcal{O}(1)$. It is straightforward to derive for this potential the slow roll parameter η and the density perturbation δ_H

$$\eta \approx -3\gamma \left(\frac{z}{r_{\perp}} \right)^2, \quad \delta_H \approx \frac{2}{5\pi} \sqrt{\frac{\gamma}{3}} \frac{N^{3/2}}{M_{Planck} r_{\perp}}, \quad (4.29)$$

where again we have used the standard slow-roll equations. We see now that slow-roll is guaranteed for sufficiently small z . Therefore we have succeeded in obtaining inflation from a completely computable string potential. This was the first example of such a string theory derived inflationary potential [29]. Furthermore, to obtain the minimum number of efoldings $N \geq 60$ and the COBE normalised value of the density fluctuations we can easily see that $\delta_H \approx 10^{-5}$ is obtained for a compactification radius $r_{\perp}^{-1} \approx 10^{12}$ GeV corresponding to an intermediate string scale $M_s \approx 10^{13}$ GeV. The spectral index $n \approx 1 - 3/N$ is in the favoured range. Therefore this is a string theory derived inflation that has all the properties of successful inflationary models, with the advantage of having a fundamental origin with a geometrical interpretation for the inflaton field as the distance, in the extra dimensions, of the colliding worlds, described by the brane and antibrane respectively.

Furthermore string theory also provides a way to end inflation [29]. This is probably the most interesting part of this scenario. We first recall that the string potential we have been discussing is valid for distances larger than the string scale. However the potential is attractive and at some point the branes get closer to each other and this approximation also will not be valid. We expect something different to happen at those separations and fortunately it happens to be understood. The point is that the amplitude (4.18) has a divergence appearing at a critical distance

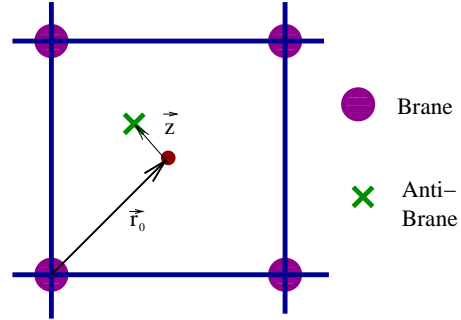


Figure 12: The location of the antibrane in a configuration that gives rise to inflation if the separation from the critical point in the middle is small enough.

$Y_c = \sqrt{2\alpha'}\pi$ [36]. What happens at this distance is that an open string mode that was massive at large separations becomes massless and, at separations smaller than this, it becomes tachyonic. The corresponding tachyon potential has been proposed to take an approximate form:

$$V(Y, T) = \frac{1}{4\alpha'} \left(\frac{Y^2}{2\pi^2\alpha'} - 1 \right) T^2 + CT^4 + \dots \quad (4.30)$$

With C a constant. Notice that this reproduces the change of the mass for the field T as a function of the separation Y . We can immediately see, that taking the effective potential as a function of both T and Y , gives us a potential precisely of the form proposed for hybrid inflation! Therefore string theory provides with a natural way to end inflation.

Moreover, the tachyon potential has been studied in some detail during the past few years and its general structure has been extracted. In particular, Sen conjectured that at the overlap point ($Y = 0$) the potential should be of the Mexican hat form with the height of the maximum equal to the sum of the brane tensions $2T_p$. The minimum would correspond to the closed string vacuum being supersymmetric where the potential vanishes. These conjectures have been verified with more than 90% accuracy using string field theory techniques [39, 37]. This allows us to estimate the reheating temperature after inflation which is essentially the energy difference between minimum and maximum, giving:

$$T_{RH} = \left(\frac{2e^\varphi}{(M_s r_\perp)^{d_\perp}} \right)^{1/4} \sqrt{M_s M_{Planck}}, \quad (4.31)$$

which for instance for $p = 5$ gives $T_{RH} \sim 10^{13}$ GeV.

Finally the tachyon potential also has topological defects which correspond to D $(p-2)$ branes (and antibranes) [37]. In fact all BPS D-branes are expected to appear as topological defects of a tachyon potential. This has implied an elegant classification of D branes from the mathematics of K -theory [101]. For the cosmological purposes that interests us here this can have very interesting implications in several ways. We have seen that inflation is possible but not generic. That means that only for distances between the branes close to the equilibrium position, the potential is flat enough as to give rise to inflation, otherwise the slow roll conditions are not satisfied and there is no inflation [29].

We can imagine a configuration of a gas of branes and antibranes. Most of the times they will interact and annihilate each other without giving rise to inflation but upon collision they will keep generating $p-2$ branes, these ones will generate $p-4$ branes and so on, implying a cascade of daughter branes out of the original ones. In this system at some point a pair of branes will be at a separation close to the equilibrium point and that will give rise to inflation, dominating the expansion of the universe and rendering the issue of initial conditions for inflation more natural.

Notice that as usual, once inflation is generated it dominates. Furthermore we may imagine the brane gas to originate from just one pair of a D9/ $\bar{\text{D}}9$ branes, acting as parent branes generating the cascade of daughter brane/antibrane systems. It remains to ask for the origin of the D9/ $\bar{\text{D}}9$ pair to start with, probably as some sort of quantum fluctuation, although this is not clear.

The cascade scenario also allows for some speculations about the dimensionality of space-time [29]. Starting in type IIB strings we know that branes of odd dimensionality (9, 7, 5, 3, 1) appear, therefore we can have D9 brane/antibrane annihilating immediately, also D7 branes annihilate their antibranes very easily because of their large dimensionality, as well as D5 branes. However D3 branes will have a harder time to meet their antibranes because of the difference in dimensionality. Remembering the argument of Brandenberger-Vafa for the dimensionality of spacetime argued that the world-sheets of two strings can meet in 4-dimensions but not in larger ones. This can be generalised to p branes in D dimensions for which the critical dimension is:

$$D_{\text{critical}} = 2p + 2. \quad (4.32)$$

Therefore we may say that D branes with $p = 3$ can meet in dimensions smaller or equal than 8 but miss each other in higher dimensions, whereas $p = 5$ branes meet in $D < 12$. This makes a rough argument why D3-brane worlds may survive annihilation in 10 dimensions and be preferred over higher dimensional ones. We may actually imagine a scenario where branes of all types are initially present and all dimensions are compact and small. The large dimension branes annihilate instantly, leading to a population of branes that include 3-branes and lower. The windings of these branes keep any dimensions from growing. Then the BV mechanism starts to act, making four dimensions large and six small, with no windings about the large spatial directions. After this we have the particular collision which causes inflation of the large dimensions. It would be very interesting to quantify this statement. For a further discussion and calculations on this regard see [92, 93].

We may still have to worry about D branes of dimensions smaller than 3. We know that domain walls ($p = 2$) and monopoles ($p = 0$) can be the source of serious cosmological problems, since they over-close the universe and therefore should not survive after inflation. Cosmic strings on the other hand are not ruled out (they have been ruled out as the main source of the density perturbations, but they could still exist and contribute at a minor scale [94]). Fortunately this scenario does not give rise

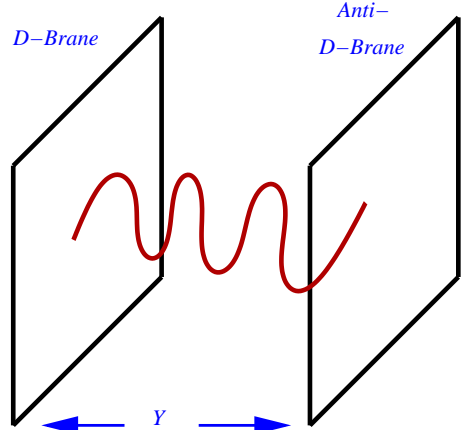


Figure 13: An open string state becomes tachyonic at a critical inter brane separation.

to monopoles nor domain walls. First of all, before inflation these defects may appear as some p branes wrapping different numbers of cycles in the extra dimensions, but they are diluted away by inflation. After inflation they may be produced from the standard Kibble mechanism. But they are not because of the following argument. Starting from type IIB strings we know that the dimensionality of the p branes has to be odd, which seems to eliminate those possibilities from the start. However we have to be more careful and a complete analysis requires thinking about the mechanism that produces topological defects in cosmology, namely the Kibble mechanism

In FRW the formation of topological defects appears since regions separated at distances larger than the particle horizon size H^{-1} are uncorrelated and therefore we expect one topological defect per Hubble radius. We know that in these models $H \approx M_s^2/M_{Planck}$, therefore $H^{-1} \gg r_\perp$ and there is correlation in the compact directions, implying no topological defects in those dimensions. We may then say that the Kibble mechanism is not at work in the extra dimensions and therefore that the topological defects that appear after inflation are the D $(p - 2k)$ branes wrapped around the same cycles as the original p branes that produced them (see figure 14). In this way, if inflation is generated by the collision of two p branes wrapped on an $n = p - 3$ cycle, the topological defects of dimension $p - 2k$ will also wrap the $p - 3$ cycle appearing as a $3 - 2k$ dimensional defect in the four large dimensions, excluding then monopoles and domain walls. Notice that this argument holds for p being even and odd.

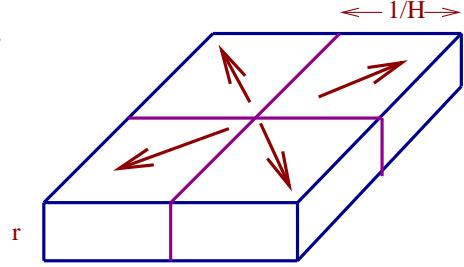


Figure 14: An illustration on how the Kibble mechanism is not in action in the extra dimensions since the inverse Hubble parameter is much bigger than the size of the extra dimension. Topological defects do appear in the noncompact dimensions.

The interesting point, as far as observations are concerned [93, 94], is that since cosmic strings will have a tension $\mu \sim M_s$, they may contribute to the CMB anisotropies. Current sensitivity rules out models for which $G\mu \geq 10^{-6}$. In this case $G\mu \sim 10^{-9}$ is still consistent and could be eventually tested in future experiments.

Finally, if inflation is caused by the collision of 3-branes we may ask where will our universe be. One possible answer to this is that the collision is between stacks of branes and anti-branes with different number of branes on each stack (remember that by being BPS we may have a stack of D-branes which do not interact with each other). Therefore we may have, say, 10 branes colliding with 4 antibranes leaving then 6 branes after the collision where the standard model can live.

This scenario is certainly very interesting. It provides the first example of a string-derived potential that gives rise to inflation and has also a stringy mechanism to end inflation by the appearance of the open string tachyon at a critical distance. Therefore hybrid inflation is realised in a stringy way. It then shares all the good

experimental success that inflation has at present, in terms of the spectrum of CMB fluctuations. It provides many other interesting features like the apparent critical dimensionality of 3-branes and the natural suppression of monopoles and domain walls after inflation. It was built however following several assumptions. First, it is assumed that an effective 4D FRW background is valid (implying that we have to be in a regime where the effective field theory description of string theory is valid). Second, the branes were assumed to be parallel and velocity effects were neglected⁵. The major assumption, however, is considering the moduli (r_\perp in this case) and dilaton to have been already fixed by some unknown stringy effect. This is a very strong assumption that prevents from claiming that this is a full derivation of inflation from string theory. A more dramatic way to see this problem is that since the configuration brane/antibrane breaks supersymmetry, there will naturally be NS tadpoles which generate a potential for the moduli. This potential is at the level of the disk (tree level) whereas the interaction described by the cylinder diagram was one-loop in open string terms. The assumption of having fixed the moduli refers to having found a mechanism that compensates the NS tadpole terms [96] and induces a minimum for the moduli potential. This is not impossible, but needs to be addressed and tree level terms are in principle dominant. Moreover the reheating mechanism is not completely understood, in particular, as we will see later, the tachyon's relaxation to its minimum is not standard in field theory. Furthermore, the scenario was presented without reference to realistic D-brane models. It is known that the way to get a chiral spectrum in D-branes corresponds to branes at singularities and intersecting branes. We will move now to these topics and mention how some of the problems mentioned above can be relaxed (although not completely solved).

4.4 Intersecting Branes, Orientifold Models and Inflation

The idea of the previous subsection can be extended to more general string constructions. Probably the simplest to consider is the intersection of branes at different angles. It is known that branes intersecting at nontrivial angles have chiral fermions in their spectrum, corresponding to open strings with endpoints on each of the branes. Again, as in the brane/antibrane case, we can compute the amplitude of the interaction between two branes at angles, which is given by:

$$\mathcal{A} = 2 \int \frac{dt}{2t} \text{Tr} e^{-tH} , \quad (4.33)$$

where H is the open string Hamiltonian. For two Dp -branes making n angles in ten dimensions this amplitude can be computed to give [104, 103, 34]:

⁵Velocity effects were considered by Tye and Shiu in [30]. Recent proposals for using the repulsive velocity effects in branes for cosmology are [95].

$$\mathcal{A} = V_p \int_0^\infty \frac{dt}{t} \exp^{-\frac{tY^2}{2\pi^2\alpha'}} (8\pi^2\alpha't)^{-\frac{p-3}{2}} \left(-iL\eta(it)^{-3} (8\pi\alpha't)^{-\frac{1}{2}} \right)^{4-n} (Z_{NS} - Z_R), \quad (4.34)$$

with $V_p L^{4-n}$ the volume of the common dimensions to both branes and

$$\begin{aligned} Z_{NS} &= (\Theta_3(0 | it))^{4-n} \prod_{j=1}^n \frac{\Theta_3(i\Delta\theta_j t | it)}{\Theta_1(i\Delta\theta_j t | it)} - (\Theta_4(0 | it))^{4-n} \prod_{j=1}^n \frac{\Theta_4(i\Delta\theta_j t | it)}{\Theta_1(i\Delta\theta_j t | it)}, \\ Z_R &= (\Theta_2(0 | it))^{4-n} \prod_{j=1}^n \frac{\Theta_2(i\Delta\theta_j t | it)}{\Theta_1(i\Delta\theta_j t | it)}, \end{aligned} \quad (4.35)$$

being the contributions coming from the NS and R sectors. Also in (4.35) Θ_i are the usual Jacobi functions and η is the Dedekind function and $Y^2 = \sum_k Y_k^2$ with Y_k the distance between the branes in the k th direction. This expression generalises the one we wrote before for parallel branes $\Delta\theta_i = 0$ and brane-antibranes for which one angle $\Delta\theta = \pi$.

In order to obtain the effective interaction potential at distances larger than the string scale $Y \gg l_s = M_s^{-1}$ we take the limit of $t \rightarrow 0$ and find that (for the compact extra dimensions being tori of radius r)⁶:

$$\begin{aligned} V_{int}(Y, \Delta\theta_j) &= -\frac{(2\pi r)^{p-5} F(\Delta\theta_j)}{2^{p-2} (2\pi^2\alpha')^{p-3}} \Gamma\left(\frac{7-p-n}{2}\right) Y^{(p+n-7)} \quad p+n \neq 7 \\ &= \frac{F(\Delta\theta_j)}{(4\pi^2\alpha')^{p-3}} \ln \frac{Y}{\Lambda_c} \quad p+n = 7. \end{aligned} \quad (4.36)$$

Where the function F contains the dependence on the relative angles between the branes, and is extracted from the small t limit of (4.35). The exact form of this function is given by

$$F(\Delta\theta_j) = \frac{(4-n) + \sum_{j=1}^n \cos 2\Delta\theta_j - 4 \prod_{j=1}^n \cos \Delta\theta_j}{2 \prod_{j=1}^n \sin \Delta\theta_j}. \quad (4.37)$$

The total potential will then be the sum of this interaction potential plus the part coming from the brane tensions, similar to the brane-antibrane case. If we consider the extra six dimensions as products of three two-tori (see figure 15), the i th ($i = 1, 2$) brane will wrap around each of the two cycles of the I th tori ($n_I^{(i)}, m_I^{(i)}$) times. The wrapping numbers $n_I^{(i)}$ and $m_I^{(i)}$ determine the angles between the branes. Also, the energy density of the two branes is given by

$$E_0 = E_1 + E_2 = T_p (A_1 + A_2) \quad (4.38)$$

⁶I am following closely the discussion of Gomez-Reino and Zavala in [34].

where A_i is the volume generated by the i th brane:

$$A_i = (2\pi r)^{p-3} \sqrt{\left((n_1^{(i)})^2 + (m_1^{(i)})^2\right) \left((n_2^{(i)})^2 + (m_2^{(i)})^2\right)} \quad (4.39)$$

Therefore the total potential is

$$V = E_0 + V_{int}. \quad (4.40)$$

This is a typical potential suitable for inflation with a constant piece plus an attractive interaction. We can see that, for small values of the angular parameter F , these potentials are inflationary with inflaton field Y . From here on the analysis is similar to the brane/antibrane system and we refer to the literature for the explicit numbers obtained by requiring the right number of e-foldings and the consistency with the COBE normalisation to fix the string scale ($M_s \approx 10^{13} - 10^{15}$ GeV). In reference [93], the intersecting models were compared with the brane/antibrane models regarding the amount of fine tuning. In both cases there is some fine tuning, in the brane/antibrane system inflation is obtained only in special configurations (like being close to the antipodal points of a circle) whereas in branes at angles the fine tuning requires a very small angular separation (of order 10^{-3} or so). Reference [93] argues for a less severe fine tuning in the case of intersecting branes.

The other difference is the way of ending inflation. In this system also there is a tachyon appearing at a critical separation realising again the hybrid inflation model. However the end result of tachyon condensation may differ. In some cases the two branes decay into a single brane and in other cases they recombine to produce a different configuration of intersecting branes but this time being a supersymmetric configuration. The decay product is always the configuration with the same charges but minimum energy. The second possibility is interesting because if the final configuration is still of intersecting branes, it has chiral fermions on the intersections and may allow a realistic model at the end of inflation. The reheating temperature can also be computed in terms of the difference in energy between the initial and

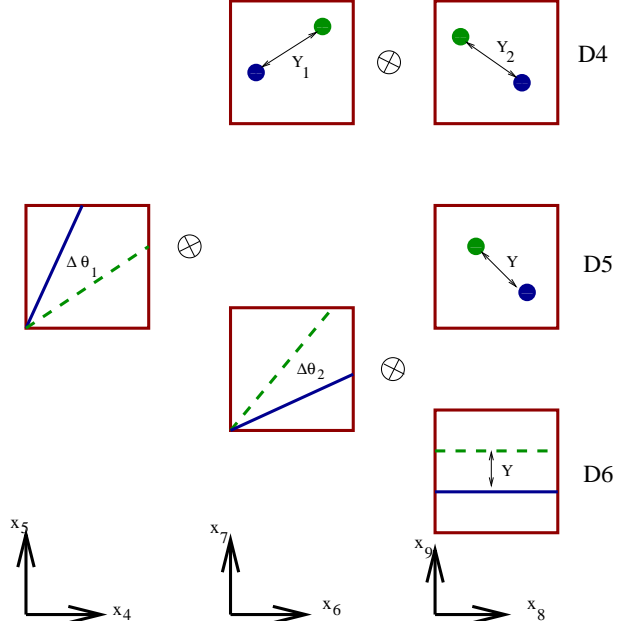


Figure 15: Possible configurations of intersecting D4, D5 and D6 branes when the extra dimensions are products of three two-tori.

the final configuration. Again topological defects will be produced in the process. After inflation no domain walls nor monopoles will survive but cosmic strings could, contributing to the CMB anisotropies at a rate that could be a few percent without contradicting experiment. This requires that $G\mu \lesssim 10^{-6}$ which for the ranges obtained for M_s (remember $\mu \sim M_s^2$) it is safe but relatively close to the limit as to be observable in the near future once Planck and MAP data are analysed. Notice that, since the string scale here tends to be a bit higher than in the brane-antibrane case, the brane at angles scenario is closer to be tested experimentally.

We must recall that the main problems of the brane/antibrane system are shared by the intersecting branes, namely the assumption of fixed dilaton and moduli by some unknown string effect as well as the details of reheating. For a recent discussion of reheating see [97].

Finally, a related, very interesting, scenario based on the attraction of D3 and D7 branes with magnetic fluxes, was proposed in [32]. Again, inflation was obtained with inflaton field being the separation between the branes. The tachyon condensation mechanism was further studied and found to lead to the D3/D7 supersymmetric bound state. This further illustrates the universality of the hybrid inflation realisation of D-brane models.

Let us finish this subsection by briefly considering the other extension of these models that addresses at least partially the issue of the moduli. The construction corresponds to the other way of constructing chiral models from D-branes, namely branes at singularities. In this scenario we typically have the extra dimensions being an orbifold or orientifold and have fractional branes and orientifold planes. Fractional branes correspond to branes which are attached to the fixed points of the orbifolds and cannot move from there (otherwise there is a problem with tadpole cancellations in the twisted sector and the model is inconsistent).

Once the branes are trapped at the singular fixed points of the orbifold it allows the existence of chiral fermions in its world volume and there exists realistic string models in which the standard model lives in one of these branes. An example of such a model is pictured in figure 18. It corresponds to the \mathbf{Z}_3 orbifold that has 27

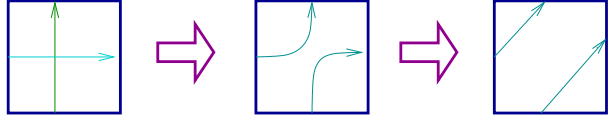


Figure 16: The decay of two intersecting branes to a single one wrapping around the torus.

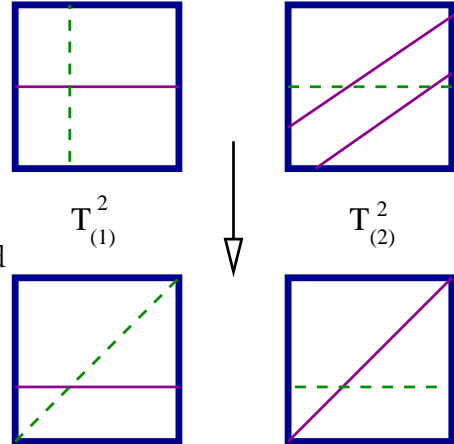


Figure 17: Recombination of two intersecting D branes to a supersymmetric state with also two intersecting D branes.

fixed points where we can put D3 branes to include the standard model. Consistency conditions coming from the cancellation of twisted R-R tadpoles require the appearance also of D7 branes. Untwisted tadpoles also imply that we have to have anti D7 branes as well as extra D3 branes in different fixed points. Therefore, for the six extra dimensions being a product of three two-tori we have three sets of parallel D7 branes and antibranes and several stacks of D3 branes located at several of the 27 fixed points, one of them includes the standard model. T -duality on all the dimensions map the D3 branes to D9 branes and D7 branes to D5 branes, a configuration that is usually simpler to deal with.

Having branes and antibranes trapped at fixed points allows for an extension of the brane/antibrane system discussed before in a way that the attraction between the brane and its antibrane corresponds to a potential for the size of the extra dimension (the distance Y is no longer a modulus because the branes cannot move from the fixed point). Therefore this allows to a natural reduction of the moduli, Y is just frozen $Y \sim r$, and the candidate for inflaton field is the modulus corresponding to the size of the extra dimensions [33], alleviating the issue of assuming it fixed.

Naively we can see that the interaction potential is proportional to $1/r^{d_\perp-2}$ so

$$V(r) = A - \frac{B}{r^{d_\perp-2}}, \quad (4.41)$$

we have to also know the kinetic term for r to work with the canonically normalised field. It is well known that the kinetic term takes the form $\partial r \partial r / r^2$ and so the canonically normalised field is $\Phi = \log r$, and the potential seems to be of the form $V = A - B e^{-a\Phi}$ which is very flat and would easily lead to inflation. However we have to be careful with this naive analysis because of two reasons. One is that this will provide the potential in a Brans-Dicke frame and not the Einstein frame because the Einstein Lagrangian will have a power of r multiplying the scalar curvature. Therefore to analyse inflation we have to go to the Einstein frame and then the potential above gets an overall factor of r^{-6} becoming a ‘repulsive’ rather than attractive potential in the sense that it has a runaway behaviour to the decompactification limit $r \rightarrow \infty$. Furthermore, in claiming that A and B were constants we were assuming that the ten-dimensional dilaton was fixed. However it is well known that in the effective 4D theory the dilaton combines with the radius r to make proper fields like the moduli S and T . More precisely, for a configuration of D9 and D5 branes in a compactification which is the product of three two-tori, the relevant fields are:

$$s \equiv \text{Re}S = e^{-\phi} M_s^6 r_1^2 r_2^2 r_3^2, \quad t_i \equiv \text{Re}T_i = e^{-\phi} M_s^2 r_i^2. \quad (4.42)$$

The potential in the Einstein frame then takes the form:

$$V = M_{Planck}^4 \left[\frac{A}{t_1 t_2 t_3} + \frac{B}{s t_2 t_3} \left(C - \frac{D}{a t_2 + b t_3} \right) + \dots \right], \quad (4.43)$$

where A, B, C, D, a, b are well defined constants. We can easily see that this is a runaway potential and does not give rise to inflation. The only way to get inflation is assuming that all of the moduli have been fixed by some higher energy effect except from one of them that we can focus. This actually happens in some models with fluxes in which the dilaton can be fixed but not the overall T field and it is then a plausible assumption. Once this is assumed there are several options at what the inflaton field which then has to be redefined in order to have canonical kinetic terms. The remnant potential is of the form:

$$V(X) = K_1 + K_2 \exp\left(\frac{-\sqrt{2}X}{M_{Planck}}\right) + \dots, \quad (4.44)$$

which depending of the relative signs of the constants $K_{1,2}$ it gives rise to inflation, without fine tuning. Again the numerical details can be found in the original literature.

A final interesting aspect of this scenario is the ending of inflation. Again when the size of the extra dimension is smaller than a critical value there appears a tachyon in the spectrum that can realize hybrid inflation. The end point after inflation is not the vacuum but actually a non BPS D-brane that is stable for smaller radii (and decays to the brane anti-brane at larger radii) [98]. The reheating temperature is then the difference between the tensions. It is worth remarking that this decay process is only known in detail for \mathbf{Z}_2 orbifolds for which the structure of non BPS branes and regions of stability has been understood [98]. For other orbifolds this has not been understood yet, in particular for the \mathbf{Z}_3 orbifold mentioned above. An extension

of this scenario for intersecting brane models has been considered in [35] with similar conclusions. Similar considerations were discussed previously by Tye *et al* in [6].

We may say that this scenario has several advantages. It naturally freezes the modulus related to the brane separation and makes the radius (or dilaton) the inflaton, reducing in a dynamical way the number of moduli. Inflation, due to the exponential dependence in the potential is obtained in a natural way without fine tuning. Still preserving the nice features of the tachyon field realising hybrid inflation. It has also some disadvantages, for instance it is clearly more complicated than the original scenario. The main weak point is the ad-hoc assumption of some moduli fixed by other string effects leaving the potential depending on one of them, the inflaton. We mentioned that there are models in the literature that can achieve this

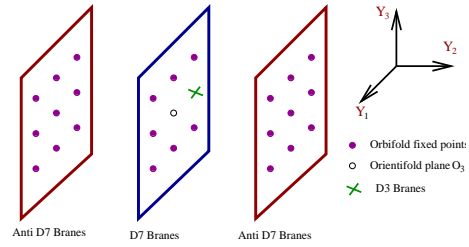


Figure 18: A configuration of a realistic model of branes and antibranes at singularities. The branes are trapped at fixed points. The three parallelograms represent D7 and anti D7 branes whereas D3 branes containing the Standard Model are trapped in one of the 27 fixed points.

partial fixing of the moduli, but at the moment there is no model in the literature that leaves unfixed the modulus that gives rise to inflation. Without this assumption the potential is runaway and surprisingly, although in the Brans-Dicke frame is attractive, in the Einstein frame is repulsive even though describes the brane/antibrane interaction.

4.5 The Rolling Tachyon

We have mentioned that the open string tachyon can play an important role in the D-brane inflation scenarios by providing the graceful exit of inflation and realising hybrid inflation. The effective action for the tachyon has been subject to intense study during the past several years. It has been probably the most important result that has emerged of string field theory and it provides, together with the potentials for D-brane interactions, one of the few concrete potentials derived from string theory. It is then worth investigating the possible implications of this potential in detail.

In the context of brane/antibrane or intersecting brane inflation, it is important to understand the reheating mechanism that the tachyon is responsible for. But more generally, it is interesting to isolate the tachyon by itself and ask what kind of cosmological implications it may have.

The results of different formalisms within string theory have provided an explicit expression for the tachyon effective Lagrangian which depending on the string theory it may take different forms. For the bosonic string, up to two derivatives:

$$\mathcal{L}_b = -\sqrt{-g}e^{-T}((1+k_b T)\partial_\mu T\partial^\mu T + (1+T)) . \quad (4.45)$$

With k_b a constant usually taken to be $k_b = 0$. For the supersymmetric NSR string, the boundary conformal field theory and other related formalisms have provided the expression:

$$\mathcal{L}_s = -\sqrt{-g}e^{-T^2}((1+k_s T^2)\partial_\mu T\partial^\mu T + 1) . \quad (4.46)$$

With k_s again usually taken to be $k_s = 0$ ⁷. And T stands here for the modulus of the tachyon field that is complex in this case.

Both Lagrangians provide interesting potentials for the tachyon field which run-away to $T \rightarrow \infty$. It can easily be seen, working with the canonically normalised field, that the tachyonic mass is of order $-M_s^2$ whereas in the minimum the second derivatives give us a mass² of order $1/k_b, 1/k_s$ respectively. That is, for those constants taken to zero the physical tachyon has an infinite mass. Otherwise the physical field would have a finite mass (a double well potential in the supersymmetric case).

More generally, the string calculations suggest that to all orders in derivative expansion these actions can take a Born-Infeld form.

$$\mathcal{L} = -V(T)\sqrt{1-g^{\mu\nu}\partial_\mu T\partial_\nu T} , \quad (4.47)$$

⁷It is usually argued that these constants can be set to zero by means of a field redefinition.

where $V(T)$ can take different forms depending on the type of string theory, namely bosonic or supersymmetric. It is this form of the Lagrangian that has been studied recently.

First, Sen studied the rolling of the tachyon to its asymptotic minimum $T \rightarrow \infty$ and concluded that even though the vacuum should correspond to the closed string vacuum and the unstable D-brane system (such as brane/antibrane pairs or non BPS D-branes) has decayed. The energy density is still localised. Furthermore he was able to prove that the resulting gas corresponded to a pressureless gas. This is easy to see from the effective action above for which the stress energy tensor give for a time dependent tachyon:

$$\rho = \frac{V(t)}{\sqrt{1 - \dot{T}^2}}, \quad p = -V(T)\sqrt{1 - \dot{T}^2}. \quad (4.48)$$

For constant energy density the pressure goes like $p = -V^2/\rho$ and at the minimum in $T \rightarrow \infty$ we know that $V \rightarrow 0$ and so $p \rightarrow 0$. The equation of state is $p = w\rho$ with $w = -(1 - \dot{T}^2)$ and therefore $-1 \leq w \leq 0$.

Having a time dependent tachyon field we should actually have considered a time dependent metric such as FRW. In [42] this was done obtaining the Friedmann's equations for this Lagrangian coupled to 4D gravity:

$$H^2 = \frac{8\pi G}{3} \frac{V(T)}{\sqrt{1 - \dot{T}^2}} - \frac{k}{a^2},$$

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3} \frac{V(T)}{\sqrt{1 - \dot{T}^2}} \left(1 - \frac{3}{2}\dot{T}^2\right). \quad (4.49)$$

Without the need to solve these equations it can be seen easily that the energy density decreases with time, while T increases, relaxing towards the asymptotic minimum of the potential. In the meantime the universe expands, accelerating first ($|\dot{T}| < 2/3$) and decelerating after ($|\dot{T}| > 2/3$). Depending on the value of the curvature $k = 0, 1, -1$, the scale factor $a(t)$ goes to a constant for $k = 0$, to a Milne universe $a(t) \rightarrow t$ for $k = -1$ and re-collapses for $k = 1$.

It is also natural to ask if this tachyonic potential can give rise to inflation by itself. In [40] it was proposed that the fact that the tachyon potential has topological defects in terms of lower dimensional D branes, they may be a source of topological inflation (inflation generated by the existence of a domain wall providing a cosmological constant due to its tension). More generally, in [29], it was looked if either of the above listed potentials would give rise to slow roll conditions with a negative result. The main reason is the absence of small parameters in the potential that can be tuned to give enough slow rolling. Similar conclusions were obtained by Kofmann and Linde in [42] when analysing the full action (including the higher derivative terms). In this case the density perturbations were also off scale. See

the last reference of [42] for a recent proposal to obtain tachyonic inflation adding a brane world.

Still, the more successful cosmological role for the tachyon, is providing the mechanism to end inflation in the hybrid inflation realisation in string theory. In this sense the decay of the tachyon has to provide the source for reheating. Two recent discussions in this direction have arrived at positive results, in the sense that this tachyonic reheating can work, although in a way different from standard reheating [97]. Also, there is much to be learned about the dynamics of the brane/antibrane annihilation process [100]. Certainly more work is needed in this direction before having a successful scenario.

4.6 S-Branes, dS/CFT and Negative Tension Branes

Recently a new class of objects were introduced in field and string theory named spacelike or S-branes. An S-brane is a topological defect for which all of its longitudinal dimensions are spacelike, and therefore it exists only for a moment of time. There are several reasons to introduce these objects. The simplest example in field theory corresponds to a (tachyon) potential of the form:

$$V(\phi) = (\phi^2 - a^2)^2, \quad (4.50)$$

with minima at $\phi_{\pm} = \pm a$. In 4D this has the standard domain wall topological defect extrapolating between the regions where the field is in the ϕ_+ and ϕ_- vacua. This domain wall would be a 2-brane. For a time dependent configuration in which we start at the maximum of the potential $\phi(x, t = 0) = 0$ but with nonzero velocity $\dot{\phi}(x, t = 0) = v$, we know that the field will roll towards ϕ_+ if v is positive, until it eventually arrives at the minimum. A time reversal situation would have the minimum starting in ϕ_- and going to $\phi = 0$ and therefore we can say that the field evolves from ϕ_- at $t = -\infty$ to ϕ_+ at $t = \infty$ looking as a kink in time filling all spatial dimensions, so this would correspond to an S2 brane (an S_p brane has $p + 1$ spatial dimensions to follow with the tradition of standard p branes notation). In practice this kind of process requires some fine tuned exchange of energy to the field to climb the barrier. The process of rolling tachyon is then a concrete realisation of S branes in string theory.

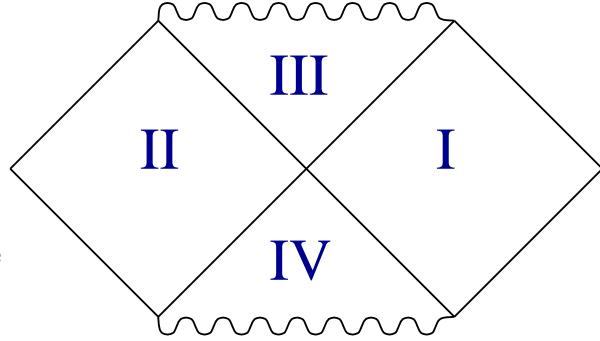


Figure 19: Penrose diagram for the Schwarzschild black hole. There are four regions separated by horizons. Regions I and II are static and asymptotically flat. Regions III and IV are time dependent with a spacelike singularity.

The main motivation for the introduction of the S-branes was the conjectured dS/CFT correspondence. The de Sitter (dS) space has become more interesting due to the indications that the universe seems to be approaching a de Sitter geometry in the future. The correspondence was proposed in [105] following some parallels with the well established AdS/CFT correspondence, given the close connection between dS and AdS spaces. (See however [106]). The point is that a boundary at infinity of, say, dS₄ corresponds to a Euclidean R_3 space for which the symmetry group of de Sitter space, $SO(4,1)$ acts as the conformal group of the Euclidean R_3 , suggesting that a conformal field theory on this boundary is dual to the full 4D gravity theory in de Sitter space. One of the interesting outcomes of this conjecture is that the renormalisation group parameter can be identified with time, in much the same way it was identified with the extra spatial coordinate in the AdS/CFT case. A simple way to see this possibility is by writing the dS₄ metric in FRW coordinates ($k=0$):

$$ds^2 = -dt^2 + e^{Ht} d\vec{x}^2, \quad (4.51)$$

with \vec{x} the spatial coordinates and H the Hubble parameter. The interesting observation is that this metric is invariant under $t \rightarrow t + \lambda$, $\vec{x} \rightarrow e^{-\lambda H} \vec{x}$ which generates time evolution in the 4D bulk and scale transformations in the Euclidean boundary. Late times (large values of λ) correspond to small distances (UV regime) whereas earlier times to IR regime. Generic expressions for the scale factor $a(t)$ will not have this symmetry but if we assume that $H(t)$ goes to a constant in the infinite past and infinite future we can see the time evolution between two fixed points under the renormalisation group, which could eventually be identified with early universe inflation and current acceleration. The monotonic evolution in time fits well with the expected c-theorem of field theories, shown to hold at least in 2D. The RG flow would correspond to the direction from future to past. This is a very tantalising proposal but unlike the AdS/CFT correspondence there is no much support yet for the dS/CFT one. S-branes are an attempt to bring this correspondence closer to the AdS/CFT one, with the S-branes playing the role of the D-branes in the boundary (the Euclidean R_3 in the example above).

Using the analogy with p branes, we expect that the S branes could also be found as explicit solutions of Einstein's equations (coupled to dilaton and antisymmetric tensor fields). In the same way that p brane solutions are black hole-like, we then expect that S brane solutions are time-dependent backgrounds of the theory, and therefore, they may have a cosmological interpretation. This is actually the case. Recently, solutions with these properties have been found, some of them were previously known. Rather than describing the general solutions of the Einstein-dilaton-antisymmetric tensor system, I will choose to describe one simple example and extract its physical properties. The reader is referred to the literature for the general cases [44, 46, 45, 47, 43, 48, 49].

The example we will concentrate on is the simple case of just 4D Einstein's equations in vacuum. Let us first recall the Schwarzschild black hole solution. We know the general static solution with spherical symmetry is just the Schwarzschild black hole solution, which in modern terminology is a black 0-brane. The solution is usually written as:

$$ds_I^2 = - \left[1 - \frac{2M}{r} \right] dt^2 + \left[1 - \frac{2M}{r} \right]^{-1} dr^2 + r^2 (\sin^2 \theta d\phi^2 + d\theta^2). \quad (4.52)$$

This metric is only valid in the region $r > 2M$ at $r = 2M$ there is a horizon which changes the relative signs of the metric and then for $r < 2M$ the role of t and r are exchanged and the metric becomes:

$$ds_{II}^2 = - \left[\frac{2M}{t} - 1 \right]^{-1} dt^2 + \left[\frac{2M}{t} - 1 \right] dr^2 + t^2 (\sin^2 \theta d\phi^2 + d\theta^2). \quad (4.53)$$

This is then a time dependent region that ends in the singularity $t = 0$ (usually called $r = 0$). Actually it is well known that there are two copies of each of these regions to have the complete causal structure of this spacetime. This is properly obtained by going to Kruskal coordinates. As in the FRW case we can write a Penrose diagram describing the structure of this spacetime.

In the Penrose diagram, Fig. 19 we illustrate the two copies of the asymptotically flat static regions separated by 45 degrees lines corresponding to the horizons. The spacelike singularity described by the wiggled line is in the time dependent regions. This ($p = 0$) brane can be used to look for a S0 brane. In this case we need the symmetries not to have the spherical symmetry $SO(3)$ of the black hole, but actually the hyperbolic symmetry $SO(2, 1)$ as suggested by the fact that S branes are kinks in time. We can then see that we need instead of a sphere ($k = 1$) a hyperbolic space ($k = -1$). This is very simple to obtain since both are related by an analytic continuation.

Therefore we can take the Schwarzschild solution and perform the following transformation $t \rightarrow ir, r \rightarrow it, \theta \rightarrow i\theta, \phi \rightarrow i\phi$ together with $M \rightarrow iP$. Both metrics above become:

$$d\hat{s}_I^2 = - \left[1 - \frac{2P}{t} \right]^{-1} dt^2 + \left[1 - \frac{2P}{t} \right] dr^2 + t^2 (\sinh^2 \theta d\phi^2 + d\theta^2), \quad (4.54)$$

whose surface of constant r and t is the hyperbolic plane \mathcal{H}_2 rather than the two-sphere, as expected. In addition to the symmetries of the hyperbolic space it has a spacelike Killing vector $\xi = \partial_r$ but is time dependent, again as expected for a S0 brane. The apparent singularity at $t = 2P$ is again a horizon. For $t < 2P$ the metric is:

$$d\hat{s}_{II}^2 = - \left[1 - \frac{2P}{r} \right] dt^2 + \left[1 - \frac{2P}{r} \right]^{-1} dr^2 + r^2 (\sinh^2 \theta d\phi^2 + d\theta^2), \quad (4.55)$$

which is now static with the timelike singularity at $r = 0$.

The corresponding Penrose diagram looks very interesting. It is just a 90 degrees rotation of the one for the Schwarzschild solution. Now the asymptotically flat regions (*I*, *III*) are time dependent. One can be thought to correspond to past cosmology and the other as future cosmology, looking as an appropriate metric for a pre big-bang scenario. Except that here there is no big-bang. Since extrapolation to the past for an observer in region *I* brings him to the horizon, which is identified with the position (in time) where the S0 brane is located [43]. The singularity is in the static region that connects the two time dependent regions. It is easy to see that the metric is asymptotically flat, in the time dependent regions, and the near horizon geometry is a 2D Milne universe ($ds^2 = -dt^2 + t^2 dr^2$) times the hyperbolic surface.

To see better the S brane interpretation we can go to a frame for which the metric in the cosmological regions takes the form:

$$ds^2 = C^2(\eta) [-d\eta^2 + d\Sigma_{k=-1}] + D^2(\eta) dr^2, \quad (4.56)$$

where $d\Sigma_{k=-1}$ is the metric for the hyperbolic space in 2 dimensions. The conformal time is defined by

$$C(\eta) = t(\eta) = P \cosh^2 \left[\frac{\eta}{2} \right], \quad (4.57)$$

and so η lies within the range $-\infty < \eta < \infty$. The scale factor for r becomes:

$$D(\eta) = \tanh \left[\frac{\eta}{2} \right]. \quad (4.58)$$

These expressions exhibit the bouncing structure of the 3 dimensional space and the timelike kink structure of the radial dimension. The position of the kink is precisely at the horizon, fitting very nicely the S brane interpretation. The bounce would seem to indicate that if we concentrate only on the time dependent part of the metric it looks like a bouncing cosmology with a contracting universe in the past, passing smoothly to an (exponentially) expanding universe in the future.

We know of course that the geometry includes also the singular static regions and therefore the transition from the past to the future cosmology has to pass through this region. Actually, remembering that the geometry is a rotated black hole we can borrow a nice interpretation from the Schwarzschild black hole, namely the wormhole or Einstein-Rosen bridge that connects the two asymptotically flat regions of the black hole. In our case, this is a timelike wormhole connecting the past and future cosmological regions. See the figures 22,23 that illustrate the wormhole as the bridge between the two cosmological regions and then producing the bouncing.

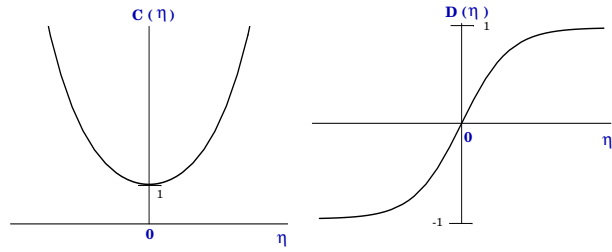


Figure 20: The bounce solution for the metric and the kink behaviour of the ‘extra dimension’ r . The kink corresponds to the location of the horizon that is identified with the S-brane.

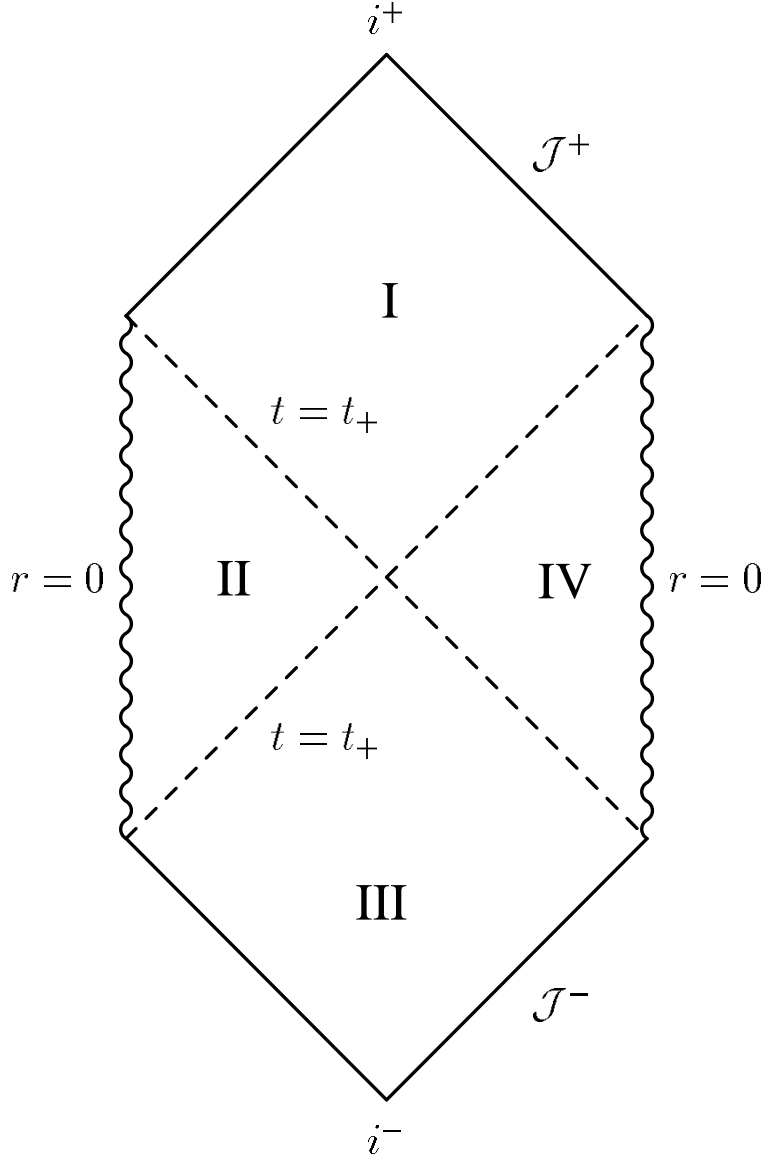


Figure 21: Penrose diagram for the $k = 0, -1$ brane solution. This diagram is very similar to the Schwarzschild black hole (rotated by $\pi/2$), but now region I (III) is not static, but time-dependent with a Cauchy horizon (at $t = t_+ \equiv P$) and region II (IV) is static.

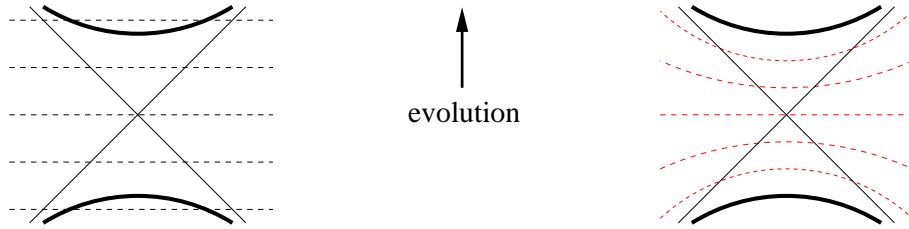


Figure 22: Different foliations for the timelike wormhole connecting past and future cosmologies.

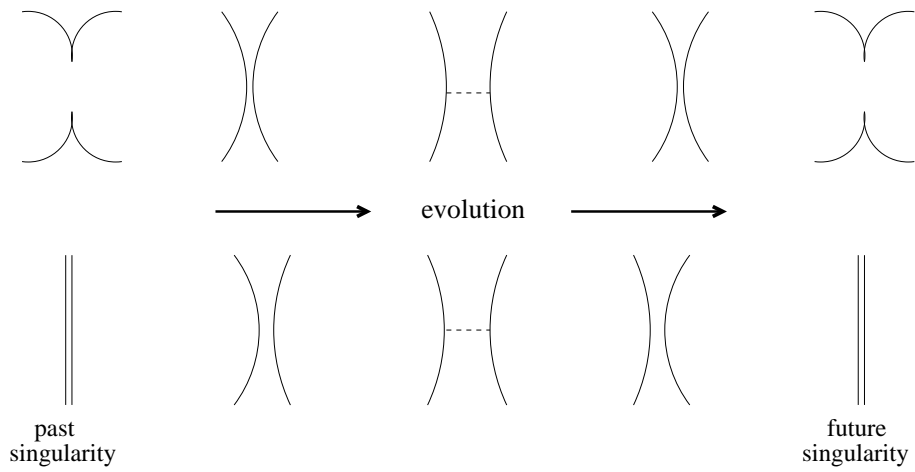


Figure 23: The time like wormhole.

As mentioned before this example is just a particular case of a general class of supergravity solutions representing S q -branes in d dimensions, found starting with the Lagrangian for the Einstein, dilaton, antisymmetric tensor $F_{q+2} = dA_{q+1}$

$$\mathcal{L} = \sqrt{-g} \left(R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2(q+2)!} F_{q+2}^2 \right) \quad (4.59)$$

There are solutions similar to the ones just discussed for hypersurfaces with $k = -1$ and also $k = 0$ with identical Penrose diagram.

In this case we can assign a charge with respect to the field F_{q+2} . However this charge, defined as an integral in the cosmological region will not be conserved in time (it will be ‘conserved’ in space, *i.e.* along a surface of constant t). A similar argument could be used to define a mass for this object. This is somehow unsatisfactory since there does not seem to be a really conserved quantity that identifies this geometry, contrary to the mass and charge of the black holes. Actually, this is only the case if we ignore the static regions. However, having the static region provides us

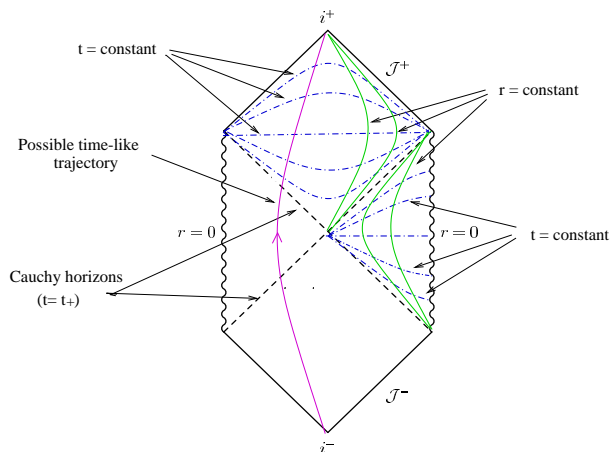


Figure 24: Penrose diagram illustrating the geodesics trajectories and the surfaces of constant time. The singularities are repulsive to timelike geodesics.

with a way to actually identify correctly this geometry. It turns out that the singularities are the physical objects (as it should be expected) to which mass (or tension) and charge can be assigned unambiguously. It is found that the two singularities correspond to negative mass objects with opposite charge [53, 52]. Furthermore, the similarity with black hole geometry indicates that there will be particle production and then we can also compute a generalised Hawking temperature, which could have interesting cosmological interpretation. Furthermore the entropy can be computed. It was found in [53] that the entropy density is proportional to the determinant of the metric in the hyperbolic or flat space, generalising then the famous $1/4$ area expression valid for $k = 1$, for which the entropy is finite. For $k = -1, 0$ the area of the horizon is infinite but the relation holds locally.

Finally in this kind of geometries we have to worry about stability. Even though the solution seems to be stable under small perturbations, the horizons may suffer some instability similar to the Reissner-Nordstrom black hole, in which the interior horizons are unstable. A naive calculation indicates that the past horizons seem to be unstable under some scalar field perturbations [53]. A more complete analysis is needed to confirm this is actually the case.

Other generalisations of Sp -branes exist. For $p \neq 0$, requiring the full $SO(n, 1) \times ISO(p + 1)$ symmetry, naturally generalises the D-brane solutions of $k = 1$ to $k = -1$. These solutions yield generically to different Penrose's diagrams than the one discussed here, with the location of the S brane corresponding to a singularity instead of a horizon, [48, 49]. However there are some general classes that have the same global structure as the S0-brane just described. The generalisations for which the symmetry is $SO(1, 1) \times O_k(n) \times ISO(p)$ where $O_k(n)$ refers to $SO(n - 1, 1)$ for $k = -1$ and $ISO(n)$ for $k = 0$, do have the Penrose diagram of Fig. 20 [53], for which the singularity is p dimensional and are the analogue for $k = 0, -1$ of the black p -brane solutions of Horowitz and Strominger ($k = 1$) for which the symmetry $O_k(n)$ is $SO(n)$.

4.7 Ekpyrotic/Cyclic Scenarios

So far we have mostly discussed cosmology associated with the physics of D-branes appearing in type IIA, IIB closed string theories and type I open strings. Let us now discuss brane cosmology in the other way of getting realistic string models, namely the Horava-Witten scenario. This scenario corresponds to M-theory compactified on an interval S^1/\mathbf{Z}_2 for which the two 10D end points have an E_8 gauge theory providing the strong coupling realisation of the heterotic string. Further compactification on a six-dimensional Calabi-Yau manifold then leave two 4D worlds at the ends of the interval in the 5D bulk. Again quasi realistic models can be obtained from this approach using mostly the topological properties of Calabi-Yau manifolds. It turns out that besides the end of the interval world (which we will refer to boundary branes) there are also in the compactifications 5 branes that are not restricted to live at the fixed points and can actually move through the bulk. These are called bulk branes. We have then configurations very similar to D-branes at orbifolds, although there are no D-branes in this construction.

A very interesting proposal was made in [54] regarding the collision of branes, this time not to obtain inflation but an alternative to inflation. The original idea was to assume that a bulk brane going from one boundary of the interval to the other end, would collide with the second boundary brane and produce the big-bang. The bulk brane would be almost BPS by which it was meant that it is essentially parallel to the boundary branes, moves slowly from one end to the other of the interval and small quantum fluctuations induce some ripples on this brane which when colliding with the visible brane would produce the density fluctuations measured in the CMB. There is no need of an inflation potential for this. A potential of the type $-e^{-\alpha Y}$ was proposed (although not derived) describing the attraction of the branes. The 5D metric is taken with a warp factor that implies that the motion is from smaller to larger curvature across the interval. Therefore the scale factor depends on the position of the brane in the interval.

Several criticisms have been made to this proposal. First, regarding the standard problems solved by inflation. The horizon and flatness problems require the branes to be very parallel before collision which may require fine tuning of initial conditions. Relics such as monopoles will not be present if the collision temperature is low enough, but this has to be quantified. There is no general natural dilution as in inflation, making the solutions of these problems more difficult in general. The issue of fine tuning the initial conditions has been very much debated [114]. However, the main difficulty of this scenario is the following [55]: it so happens that in a 4D description, $\dot{a} < 0$ before the collision and is expected that $\dot{a} > 0$ after the collision, which means passing from contraction to expansion, without crossing a singularity. This is a problem because it violates the null energy condition. Let us review this argument briefly. Starting with gravity coupled to a scalar field:

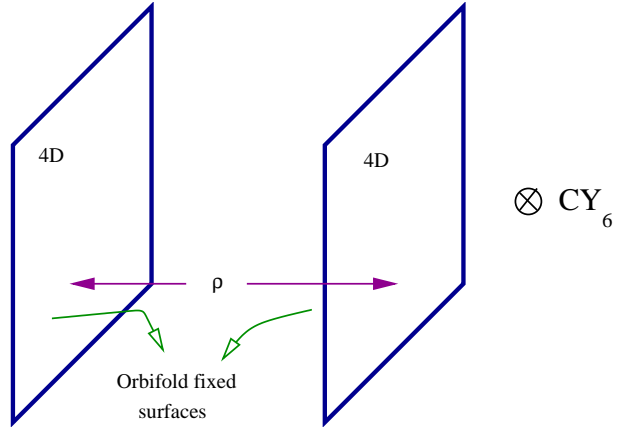


Figure 25: The Horava Witten scenario. Two surfaces each at the end of the interval provide chiral matter and possibly interesting cosmology.

$$\mathcal{L} = \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \quad (4.60)$$

we know that the energy density and pressure are given by:

$$\rho = \frac{1}{2} \dot{\phi}^2 + V \quad p = \frac{1}{2} \dot{\phi}^2 - V \quad (4.61)$$

Einstein's equations then give:

$$\dot{H} = -\frac{1}{4} (\rho + p) = -\frac{1}{4} \dot{\phi}^2 \leq 0. \quad (4.62)$$

Therefore H is monotonically decreasing and we cannot go from contraction ($H < 0$) to expansion ($H > 0$).

A way to avoid this problem is to simply get rid of the bulk brane and consider the collision between the two boundary branes. In this case, there is a singularity at the moment of collision, since the size of the fifth dimension reduces to zero, that could allow the transition from contraction to expansion. This is the second version of the ekpyrotic scenario. The singularity happens to be only in the extra dimension because the scale factors of the branes remain finite during the process. After the collision the two branes separate again and the scale factor increases. [55].

In a 4D effective action, this process can be understood in terms of the discussion we had in the pre big-bang section. Neglecting the scalar potential and identifying the separation between the branes with the string dilaton (as it happens in the Horava-Witten scenario) we can use equations (3.9)-(3.10) for the case $d = 4$. Out of the four possibilities provided by the choices of sign we can choose:

$$a(t) = |t|^{1/2} \quad \varphi = \varphi_0 \pm \sqrt{3} \log |t|. \quad (4.63)$$

The behaviour of the scale factor $a(t)$ is clearly from contraction at negative t to expansion at $t > 0$. But this still leaves the choice of sign for the dilaton open. Since the string coupling is proportional to $e^{-\phi}$, the $-$ sign choice (taken in the pre big-bang scenario) corresponds to strong string coupling whereas the $+$ choice (chosen in the ekpyrotic scenario) implies weak coupling at $t = 0$. Therefore they conjectured that the transition is smooth at the singular point and the branes start to separate again. We show in figure 27 a cartoon version of the process writing the sizes of the branes different just to remind that the warp factor changes with the separation of the boundary branes.

Finally this leads us to the third version of this scenario that corresponds to the cyclic universe [56]. In this case the two branes could keep separating and passing through each other an infinite number of times as long as the interacting potential has a very particular form. For instance for a potential like in figure 28. We may describe the history as follows. Let us start with the right hand side that would correspond to today. The potential is slightly positive and flat

reflecting the fact that the universe accelerates today (a mild inflation or quintessence). Since the slope of the potential is slightly negative the scalar field will start rolling towards smaller values: the branes approach each other. At some point the field will cross the $V = 0$ point and its energy density will be kinetic. The potential becomes negative very fast and at some point the energy density $\rho = V + \frac{1}{2}\dot{\varphi}^2$ touches zero, implying that the universe starts contracting. Since the kinetic energy is also large

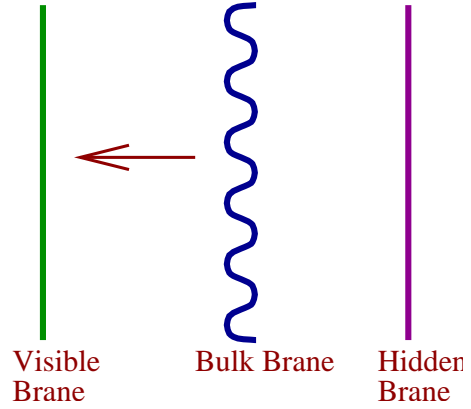


Figure 26: The first ekpyrotic scenario. The motion of the bulk brane with small ripples caused by quantum fluctuations, could be the origin of density perturbations after colliding with the other boundary brane.

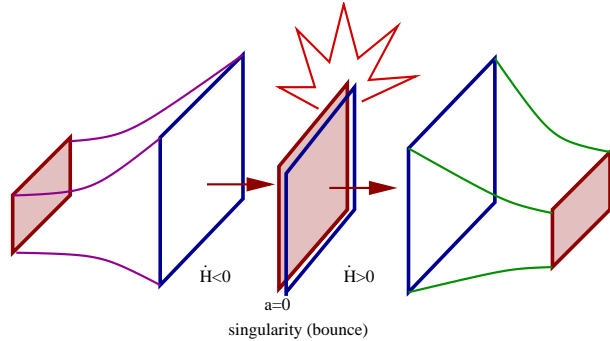


Figure 27: A cartoon of the collision between branes and the passing through of the branes with the different scale factors in each case.

the field easily passes through the minimum, towards the flat region at infinite φ (zero string coupling) where the branes collide and bounce back, with enough energy as to cross again the steep minimum and go to the right hand side of the potential, where it will get to the radiation dominated era, then repeat the whole cycle again.

These scenarios claim to approach the questions solved by inflation. For instance, the horizon problem does not exist if there is a bounce, since there will be clear causal contact between different points. In the cyclic version, the late period of mild inflation plays a similar role as the original inflationary scenario by dissolving some wanted objects, like magnetic monopoles, and emptying the universe for the next cycle, solving the flatness problem. Also the spec-

trum of perturbations has been claimed to be consistent with observations although there has been a debate on this issue, which I am not qualified to judge [115]. All parts seem to agree on the fact that the methods used so far are not conclusive one way or another. A full 5D treatment should be performed and then face the singularity at the collision.

There are very interesting aspects on these proposals, especially regarding the revival of the cyclic universe. Remember that this was proposed in the 1930's [57] but it was immediately realised that the entropy increases on each cycle meaning that the length of the cycles also increase and extrapolating back in time we hit again an initial singularity. Therefore making the model semi eternal. Similar to eternal inflation which also requires a beginning. The entropy problem is solved as follows. It is true that the total entropy increases with the cycles but the entropy of matter is always the same at the end of each cycle. This is due to the accelerated expansion at present which will dilute matter until bringing the universe essentially empty (one particle per Hubble radius), before restarting the cycle. Even though this idea has been found in the context of the ekpyrotic scenario, it is clearly independent of it and may have far reaching implications as well as different realisations.

Another interesting point of this scenario is that it connects the early universe and late universe in a coherent way. The current acceleration is used as a virtue to prepare the universe to the next cycle.

A weak point about these scenarios is the dynamics of the scalar field. Even though the scenarios are motivated in terms of string theory, the kind of potentials that work are relatively contrived and have not been derived from theory. This is definitely an urgent question to approach before these models can be considered genuine

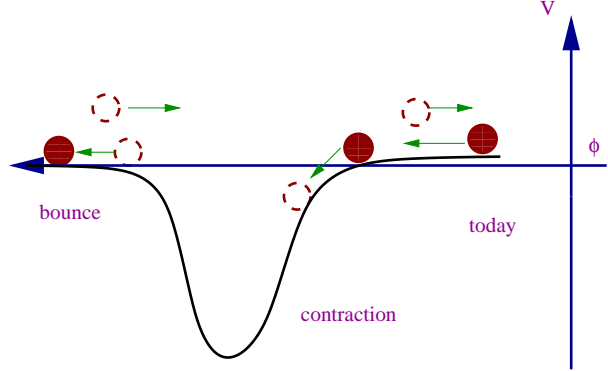


Figure 28: An illustration of the potential and trajectory of the field in the cyclic universe.

M-theory models. In this sense these scenarios are at present in the same stage as D-brane inflation was in 1998 where the scalar potential was only guessed, instead of explicitly calculated as in the brane/antibranes and intersecting brane models. Finding a potential with the proposed properties is certainly an interesting challenge.

The problems of the D-brane models also apply to this scenario. In particular the assumption of having fixed the moduli of the Calabi-Yau manifold is not justified. Although the main problem to deal with is the singularity giving rise to the bounce, which is a very strong assumption. Observationally, the important points to address refer to the spectrum of density perturbations since this is what could rule out the model. A criticism of this scenario and its comparison with inflation has been presented in [66].

4.8 Time Dependent Orbifolds

The major assumption of the ekpyrotic and cyclic universe scenario is the smooth passing through the singularity. This has motivated much recent effort in trying to describe field and string theory in such singular spaces. Given that the singularity can be associated to an orbifold singularity and the fact that orbifolds have proven to be backgrounds in which string theory is well behaved, despite the singularities, it is then natural to investigate the behaviour of string theory in orbifolds which are time-dependent. The simplest case illustrated in [58, 55] is the following. Take the spacetime to be the product of a 2D space and a $D - 2$ one, with the 2D metric

$$ds^2 = -dt^2 + t^2 dx^2. \quad (4.64)$$

With x a periodic coordinate $x \sim x + \alpha$. Before the identification this is just a realisation of flat 2D Minkowski space (similar to the polar coordinates representation of the flat metric in 2D) which corresponds to the Milne universe. We can define the Kruskal coordinates:

$$X_{\pm} = te^{\pm x} \quad (4.65)$$

for which the metric looks as $ds^2 = -dX_+ dX_-$. The identification $x \sim x + \alpha$ corresponds to multiplying X_{\pm} by a factor, which is a boost. This means we are orbifolding by a discrete element of the Poincare group in 2D. This defines an orbifold with $X_{\pm} = 0$ as a fixed point, which identifies the singularity. This space was studied in [58, 55]. This space has closed time like curves in the left and right sections of figure 29, and the singularity would be such that the space is not separable (non Hausdorff). We may still choose only the top and bottom ‘wedges’, see the figure, reflecting the singularity at the origin connecting past and future cosmologies. If the space defined by x is an interval (S_1/\mathbf{Z}_2 as in Horava-Witten) we can see the trajectories of the boundary branes (thick green and purple lines) which join and split again. It was conjectured in [55] that the branes could actually pass the singularity smoothly.

This simple space as well as other variations have been recently studied in the context of trying to formulate a consistent string theory in time dependent orbifold backgrounds and ask how sensible to the singularity string theory is. Several interesting results have been found, including the formulation of string amplitudes in this backgrounds and the construction of explicit time dependent backgrounds with at least one supersymmetry (something rare in time dependent backgrounds since the non-existence of timelike Killing vectors is usually an obstacle for supersymmetry).

Furthermore, combining boosts with shifts, in reference [51] cosmological backgrounds with precisely the same Penrose diagram as the one discussed above in the S-branes section, were discovered (see figure 20): with past and future cosmology separated

by the static region with the timelike singularity. Although this geometry has closed time-like curves in the static region which were absent in the S-brane solutions. The causal structure of the Penrose diagram motivated the interpretation of the singularities as orientifold planes, causing the universe to expand and a general discussion of the cosmological implications was done in [52, 51] (see also [53]). Several interesting cosmological issues were discussed including the avoidance of the horizon problem, given that it is a horizon rather than the big bang singularity, the starting point of the future cosmological region. It is interesting to see if these geometries have more explicit relation with the solutions of [47, 48, 49]. Also if the apparent instability of the horizon also happens in this case.

In the study of [59], even though some interactions seem to pass smoothly through the singularity, some divergences were also found ⁸. Moreover, a general argument was found in [60] in which the presence of a single particle in this kind of spaces would induce a black hole to develop immersing the whole space into a large

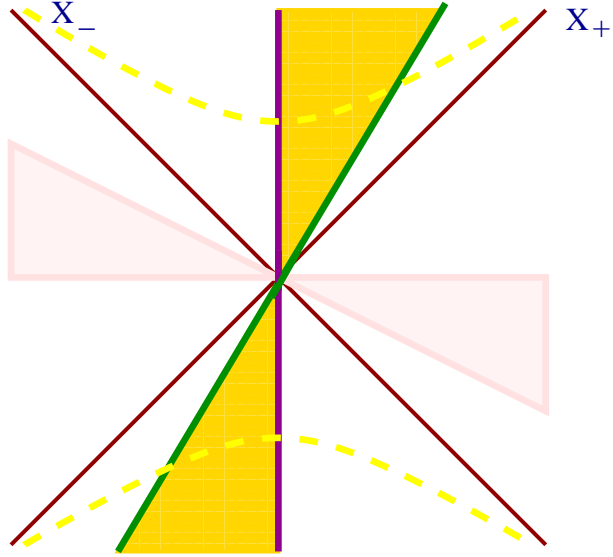


Figure 29: An orbifold of the 2D Milne universe. The horizontal wedges correspond to static regions with closed timelike curves. The vertical wedges correspond to past and future cosmological regions joined by the (non Hausdorff) singularity. If the radial direction is an interval, the heavy lines mark the trajectory of boundary branes, colliding and moving apart. The discontinuous lines are curves of constant time.

⁸See for instance the article of Liu, Moore and Seiberg in [59]. We need to keep in mind that this analysis was done in 10D string theory and not in 11D as in the Horava-Witten scenario of the previous subsection.

black hole and having to face the standard singularity problem. The argument goes as follows. Taking an orbifold like the one defined above, with

$$(X_+, X_-) \rightarrow (e^{n\alpha} X_+, e^{-n\alpha} X_-) \quad (4.66)$$

for any n and a constant α , and all transverse coordinates invariant. Two massless particles in the orbifold with impact parameter b smaller than the Schwarzschild radius would produce a black hole if:

$$G\sqrt{s} > b^{D-3}. \quad (4.67)$$

With G Newton's constant and s the squared center of mass energy of the particles in total D dimensions. In the original space they need to be very close to each other but in the orbifold we have to include all the image particles under the orbifold twist, since the twist is a Lorentz boost, each of the image particles has boosted energy (the momenta transform as the coordinates under the orbifold) whereas the impact parameter b does not change with n . Therefore for large enough n the condition above is satisfied and the particles produce a black hole. Actually the Schwarzschild radius becomes $R_s \sim Ge^{n\alpha}$ and for large enough n R_s can be as large as the whole space. Having the whole universe inside a black hole and then having a normal future big crunch singularity. Therefore it seems that time dependent orbifolds do not help into the problem of cosmological singularities and the regions of large curvature have to be dealt with in string theory. A possible exemption to this problem are the null branes introduced in [116].

5. Final Remarks

Probably the main ideas that were developed in our field in the 1980's are string theory and inflation. Twenty years after, both continue very lively but their connection, if any, is not yet understood. Many ideas have been discussed recently in string cosmology: from pre big-bang cosmology to mirage cosmology, D-brane inflation and rolling tachyon condensation, S-branes, dS/CFT correspondence, ekpyrotic/cyclic scenarios, time-dependent orbifolds, etc. The important questions related with initial conditions, the treatment of singularities, definition of observables and consistency of de Sitter space with string theory and the holographic principle, remain open and may remain open for some time [10, 118]. It is still useful to come up with a phenomenological attitude to these issues and hope to make little progress with time, before a possible breakthrough comes up. This at least can be a guide to what the relevant questions to be answered are. In some sense it is similar to the search for standard-like models from string theory, that has been very fruitful over the years, identifying new phenomenological avenues in string and field theories, as well as increasing our understanding of string theory itself. Eventually a common

property of several string cosmology scenarios could be identified that would be close to a prediction from the theory.

We have seen several interesting proposals for a consistent string cosmology. The S-brane ideas, dS/CFT correspondence and the rolling tachyon are certainly interesting avenues to explore, as well as higher dimensional cosmologies with a richer global structure than the standard big bang, as those described in section 4.6. But at the moment they are not developed enough as to have some phenomenological impact. On the other hand, there are three classes of scenarios that have been put forward that do have some phenomenological implications, namely: the pre big-bang scenario, the different versions of D-brane inflation and the ekpyrotic/cyclic scenarios. Both pre big-bang and ekpyrotic/cyclic scenarios are usually presented as alternatives to inflation and end up in disadvantage given that inflation is a scenario that has been evolving for more than 20 years and has many possible realisations, most of them not necessarily connected with a fundamental theory, such as strings. Whereas these alternative scenarios have some constraints from their original top-down formulation that make them less flexible. This is similar to field theory against string theory model building, which is more rigid. This is the reason why it has been very difficult to obtain fully phenomenologically realistic string models.

Probably a fairer comparison is between the different concrete string scenarios that have been proposed so far. In this sense, each scenario has its virtues and problems. From the top-down approach the D-brane inflation scenarios with subsequent rolling tachyon condensation have the advantage of being concrete and derivable from direct string perturbation theory calculations, as well as sharing the successes of standard inflationary models. It is still an interesting challenge to find string derived potentials with properties as those proposed in the pre big-bang and ekpyrotic scenarios. However, from the bottom-up view these two scenarios offer interesting alternatives to inflation that could be theoretically and experimentally tested in the not too far future (for string theory standards), in particular regarding the spectrum of density perturbations and gravitational waves. Both ekpyrotic and pre big-bang scenarios have the possibility of being tested, especially in their predictions for tensor perturbations, which would differ from inflationary models. For D-brane inflation models, the fact that they reproduce inflation is very good because they share the successful predictions of inflation. This also makes them difficult to be tested independently of inflation. Fortunately, the generic presence of cosmic strings after inflation may have important implications that could eventually put these proposals to test.

We do not have to forget that, as emphasised above, all these three classes of scenarios have assumptions. Their common weak point is regarding the problem of moduli fixing. Before addressing this issue (and others that vary from scenario to scenario) none of the proposals can be called a truly top-down approach towards cosmology. Probably the considerations of non-vanishing fluxes of antisymmetric tensor

fields [102] could help in finding models with, at least, the relevant moduli fixed, that could be used as starting point for a successful cosmology. The understanding of the singularity and possible bouncing in the pre big-bang and ekpyrotic/cyclic models is their fundamental challenge. It is then fair to say that even though there are now several interesting frameworks in the literature, it is still an open question to derive a realistic cosmological scenario from string theory, inflationary or non. In any case these concrete attempts, even if they prove not realistic, can serve as examples on what the typical problems to address are, in order to get a successful cosmology from string theory, and could help to eventually identify observable model-independent implications of string cosmology.

On this observational aspect, there has been recent interest on a model independent treatment of ‘trans-Planckian’ physics imprints in the CMB. The point is to realize that the inflation scale H (usually taken in the range $10^{13} - 10^{14}$ GeV) is smaller than the fundamental scale M (that could be the string scale M_s). Therefore the massive states, heavier than M (string modes) can be integrated out and an effective field theory description of the density perturbation can be done in an expansion in the small parameter $r = \left(\frac{H}{M}\right)^2$. If effects of order r are observable in the CMB they would then provide information about the fundamental theory. If the string scale is of the order of the Planck mass then $r \sim 10^{-11}$ which may be too small to be observed (still better than the present ratio of energies in collider experiments at 1 TeV which gives $\left(\frac{10^3 \text{ GeV}}{M_{\text{Planck}}}\right)^2 \sim 10^{-32}$). However if the string scale is smaller, like in some brane world models, their imprints may be observable. There may be special cases for which the leading term in the expansion is of the order $r^{1/2}$ that could have more detectable signals although the generic case of $\mathcal{O}(r)$ is based only on very general assumptions such as local effective field theory description. This is an important subject given the potential of testing string theoretical effects in a more or less model independent way.

The field of string/brane cosmology may no longer be in its infancy, it may be passing a turbulent adolescence time. Let us hope it will arrive at a successful and productive mature life.

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