#1

$$Q' = \frac{1}{4} n \langle v \rangle \, dA \, dt$$

nº medio de choques con $d\Omega$ en un dt.

$dA \equiv \Omega$

$$\therefore \quad dN = -\frac{1}{4}\frac{N}{V}\Omega \, dt \qquad (1)$$

Pero, $\quad V = V_0 - A u_0 t \qquad (2)$

Luego,
$$\frac{dN}{N} = -\frac{\langle v \rangle \Omega}{4} \frac{dt}{V_0 - A u_0 t} \qquad (3)$$

Integrando:
$$\mathrm{Ln}\, N = -\frac{\langle v \rangle \Omega}{4} \int \frac{dt}{V_0 - A u_0 t} + C_1$$

$$= -\frac{\langle v \rangle \Omega}{4} \cdot \frac{1}{-A u_0} \int \frac{-A u_0 \, dt}{V_0 - A u_0 t} + C_1$$

$$\therefore \quad \mathrm{Ln}\, N = \frac{\langle v \rangle \Omega}{4 A u_0} \mathrm{Ln}\left(V_0 - A u_0 t\right) + C_1 \qquad (4)$$

Para $t = 0$, $\quad N(0) = N_0 \longrightarrow \mathrm{Ln}\, N_0 = \frac{\langle v \rangle \Omega}{4 A u_0} \mathrm{Ln}\, V_0 + C_1$

$$\therefore \quad C_1 = \mathrm{Ln}\, N_0 - \frac{\langle v \rangle \Omega}{4 A u_0} \mathrm{Ln}\, V_0 \qquad (5)$$

Luego, finalmente,
$$\boxed{\mathrm{Ln}\frac{N}{N_0} = \frac{\langle v \rangle \Omega}{4 A u_0} \mathrm{Ln}\frac{V_0 - A u_0 t}{V_0}} \qquad (6)$$

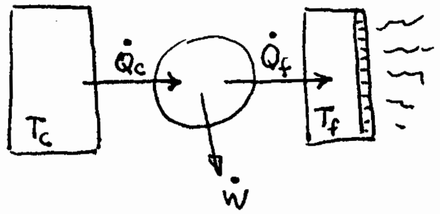
En el instante en que $V = \frac{V_0}{2}$, de (2) $\rightarrow A u_0 t^* = \frac{V_0}{2}$

$$\therefore \quad \mathrm{Ln}\frac{1}{2} = \frac{\langle v \rangle \Omega}{4 A u_0} \mathrm{Ln}\frac{1}{2} \rightarrow \frac{\langle v \rangle \Omega}{4 A u_0} = 1$$

$$\therefore \quad \boxed{\frac{\langle v \rangle}{u_0} = \frac{4A}{\Omega}}$$

2.

a)



Máquina reversible:

$$\eta = \frac{\dot{W}}{\dot{Q}_c} = 1 - \frac{T_f}{T_c} \longrightarrow \boxed{\dot{Q}_c = \frac{T_c}{T_c - T_f}\,\dot{W}} \quad (1)$$

Además:
$$\frac{\dot{Q}_c}{T_c} = \frac{\dot{Q}_f}{T_f} \longrightarrow \boxed{\dot{Q}_f = \frac{T_f}{T_c}\,\dot{Q}_c} \quad (2)$$

De (1) y (2):
$$\dot{Q}_f = \frac{T_f}{\cancel{T_c}} \cdot \frac{\cancel{T_c}}{T_c - T_f}\,\dot{W} \longrightarrow \boxed{\dot{Q}_f = \frac{T_f}{T_c - T_f}\,\dot{W}} \quad (3)$$

Esta es la potencia que debe ser disipada para mantener constante la temperatura de la fuente fría:

Luego,
$$\dot{Q}_f = \sigma_B\, \Omega\, T_f^4 \qquad (4)$$

Ahora, de (3) y (4):
$$\sigma_B\, \Omega\, T_f^4 = \frac{T_f}{T_c - T_f}\,\dot{W} \longrightarrow \boxed{\Omega = \frac{\dot{W}}{\sigma_B}\,\frac{1}{(T_c - T_f)\,T_f^3}} \quad (5)$$

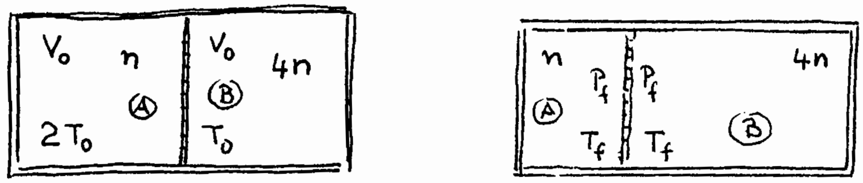
De (5)
$$\frac{d\Omega}{dT_f} = 0 \longrightarrow -\frac{\dot{W}}{\sigma_B}\,\frac{(T_c - T_f)\cdot 3T_f^2 - T_f^3}{[(T_c - T_f)\,T_f^3]^2} = 0$$

$$\therefore \boxed{T_f = \frac{3}{4}\,T_c} \quad (6)$$

$\therefore$ De (5) y (6):
$$\boxed{\Omega_{min} = \frac{9{,}5\,\dot{W}}{\sigma_B\, T_c^4}} \quad (7)$$

b) $\eta = 1 - \frac{T_f}{T_c} = 1 - \frac{3}{4} \rightarrow \boxed{\eta = 0{,}25}$

(3.)



Inicial Final

Inicialmente: $\begin{cases} P_0^A = \dfrac{2nRT_0}{V_0} & \text{gas (A)} \\ P_0^B = \dfrac{4nRT_0}{V_0} & \text{gas (B)} \end{cases}$ $\therefore \boxed{P_0^B = 2P_0^A}$

En el estado final hay equilibrio mecánico y equilibrio térmico (pared interior es diatérmana).

Esto significa que $\boxed{P_f^B = P_f^A = P_f}$ y que $\boxed{T_f^A = T_f^B = T_f}$

Cálculo de los volúmenes finales V_f^A y V_f^B.

$$V_f^A = \frac{nRT_f}{V_f} \quad y \quad V_f^B = \frac{4nRT_f}{V_f} \Rightarrow \frac{V_f^B}{V_f^A} = \frac{4}{1} \Rightarrow \frac{V_f^B}{V_f^A + V_f^B} = \frac{4}{4+1}$$

o sea, $\dfrac{V_f^B}{2V_0} = \dfrac{4}{5} \longrightarrow \boxed{V_f^B = \frac{8}{5}V_0}$ (1) y $\boxed{V_A = \frac{2}{5}V_0}$ (2)

Cálculo de T_f: Por ejemplo, si consideramos ambos gases en su conjunto, de la 1ª Ley: $\Delta U = \cancel{Q}^{=0} - \cancel{W}^{=0}$ pues no hay trabajo externo ~~W~~ entonces.

$$\therefore \Delta U = \Delta U_A + \Delta U_B = 0 \longrightarrow \cancel{n}\cancel{c_v}(T_f - 2T_0) + 4\cancel{n}\cancel{c_v}(T_f - T_0) = 0$$

$$\therefore \boxed{T_f = \frac{6}{5}T_0}$$

Variación de entropía del universo: $(\Delta S)_{univ} = (\Delta S)_A + (\Delta S)_B$

Para un gas ideal: $\Delta S = nc_v \ln\dfrac{T_f}{T_i} + nR\ln\dfrac{V_f}{V_i}$ y con $c_v = \frac{3}{2}R$ (gas id. monoat)

$$\therefore (\Delta S)_{univ} = \frac{3}{2}nR\ln\frac{6T_0}{5(2T_0)} + nR\ln\frac{2V_0}{5\cdot V_0} + 4n\frac{3}{2}R\ln\frac{6T_0}{5\cdot T_0} + 4nR\ln\frac{8V_0}{5\cdot V_0}$$

$$= nR\left[\frac{3}{2}\ln\frac{3}{5} + \ln\frac{2}{5} + 6\ln\frac{6}{5} + 4\ln\frac{8}{5}\right]$$

$$= 1{,}29\, nR$$