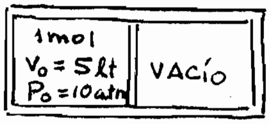
CONTROL N° 1 Otoño 17/08/2006

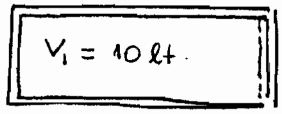
FI 22-A

1.



(a)

$n = 1$ mol



(b)

Exp. al vacío

(c)

compresión cuasi_ estática adiabática.

(a) Equilibrio inicial: $\Theta_0 = \dfrac{P_0 V_0}{R} = \dfrac{10\,\text{atm} \cdot 5\ell t}{0{,}082 \frac{\ell t\, at}{mol\, K}} \longrightarrow \boxed{\Theta_0 = 609{,}8\ K}$ (1)

(b) Expansión libre: $V_0 = 5\ell t \longrightarrow V_1 = 10\ell t$ Proceso no-cuasiest. adiab.

1ª Ley $\Delta U = Q - W$; Aquí: $\boxed{\begin{matrix} W = 0 \\ Q = 0 \end{matrix}} \Rightarrow \Delta U = U_1 - U_0 = 0 \rightarrow \boxed{U_1 = U_0}$ (2)

Como se trata de un gas ideal, donde $U(\Theta)$ solamente $\rightarrow \boxed{\Theta_1 = \Theta_0 = 609{,}8\ K.}$ (3)

En el estado de equilibrio final en (b): $P_1 V_1 = R\Theta_1$

$$\therefore\ P_1 = \frac{R\Theta_1}{V_1} = \frac{0{,}082 \cdot 609{,}8}{10} = 5\,\text{atm} \quad \therefore \quad \boxed{P_1 = 5\,\text{atm}} \ (4)$$

$H = U + PV = C_v\Theta + cte + R\Theta \Rightarrow \boxed{H = C_p\Theta + cte}$ $\therefore$ $\boxed{\Delta U = 0}$ y $\boxed{\Delta H = 0}$

c) Compresión adiabática cuasiestática : $\boxed{dQ = 0}$

De $P_2 V_2^\gamma = P_1 V_1^\gamma = cte.$, con $\gamma = \frac{5}{3}$ (gas monoatómico ideal)

$$P_2 \cdot 5^\gamma = P_1 \cdot 10^\gamma \longrightarrow P_2 = 5 \cdot \left(\frac{10}{5}\right)^\gamma = 5 \cdot 2^{5/3} \rightarrow \boxed{P_2 = 15{,}9\,at}$$

$$\therefore\ \Theta_2 = \frac{P_2 V_2}{R} = \frac{15{,}9 \cdot 5}{0{,}082} = 969{,}5\ K \rightarrow \Delta U \overset{C_v = \frac{3}{2}R}{=} 12{,}5\,(969{,}5 - 609{,}8)$$

$\therefore \boxed{\Delta U = 360\ J}$ $\Delta H = C_p\,\Delta\Theta = \frac{5}{2}R\,(969{,}5 - 609{,}8)$

De la 1ª Ley: $\Delta U = \cancelto{0}{Q} - W \longrightarrow \boxed{W = -360\ J}$

2. a) $dQ = C_v\, d\theta + P\, dv$ (1) 1 mol de gas ideal.

De $Pv = R\theta \rightarrow P dv + v dP = R\, d\theta$ (2)

Ahora, de $Pv^{\lambda} = cte \rightarrow P\lambda v^{\lambda-1} dv + v^{\lambda} dP$ $/ \div v^{\lambda-1}$

$\Rightarrow v\, dP = -\lambda P dv$ (3)

Reemplazando en (2): $P dv = R\, d\theta + \lambda P dv \Rightarrow P dv = \frac{R}{1-\lambda} d\theta$ (4)

Finalmente, de (1) y (4):

$$dQ = \left(C_v - \frac{R}{\lambda-1} \right) d\theta$$

$$\therefore \left.\frac{dQ}{d\theta}\right|_{proceso} = \boxed{C = C_v - \frac{R}{\lambda-1}}$$

b) De $\left.\begin{array}{l} Pv^{\lambda} = cte \\ \text{y} \quad Pv = R\theta \end{array}\right\}$ Eliminando P : $\boxed{\theta v^{\lambda-1} = cte}$

i) Si $\lambda > 1$; $\lambda - 1 = m > 0$ $\therefore$ $\theta = \frac{cte}{v^m}$ $\rightarrow$ gas se enfría en la expansión

ii) Si $\lambda < 1$; $\lambda - 1 = -m$ $\therefore$ $\theta = cte\, v^m \rightarrow$ gas se calienta.

c) $dQ = c\,\Delta\theta$

$$\therefore Q = \left(C_v - \frac{R}{\lambda-1} \right) \Delta\theta$$

Pero, $\Delta\theta = \theta_2 - \theta_0$

$$= 4^{\lambda-1}\theta_0 - \theta_0$$

$$= (4^{\lambda-1} - 1)\theta_0$$

$$\therefore \boxed{Q = \left(C_v - \frac{R}{\lambda-1} \right)(4^{\lambda-1} - 1)\theta_0}$$

3. $\left(\frac{\partial V}{\partial \Theta}\right)_P = \frac{nR}{P} + \frac{na}{\Theta^2}$ y $\left(\frac{\partial V}{\partial P}\right)_\Theta = -n\Theta\, f(P)$

a) De $V(\Theta, P) \longrightarrow dV = \left(\frac{\partial V}{\partial \Theta}\right)_P d\Theta + \left(\frac{\partial V}{\partial P}\right)_\Theta dP$

Por tratarse de la diferencial de una función de estado ella es necesariamente exacta. Por lo tanto, de la condición de exactitud:

$$\left[\frac{\partial}{\partial P}\left(\frac{\partial V}{\partial \Theta}\right)_P\right]_\Theta = \left[\frac{\partial}{\partial \Theta}\left(\frac{\partial V}{\partial P}\right)_\Theta\right]_P$$

o sea $$-\frac{\cancel{n}R}{P^2} = -\cancel{n}\, f(P) \Rightarrow \boxed{f(P) = \frac{R}{P^2}}$$

b) $\left(\frac{\partial V}{\partial \Theta}\right)_P = \frac{nR}{P} + \frac{na}{\Theta^2}$ (*) y $\left(\frac{\partial V}{\partial P}\right)_\Theta = -\frac{nR}{P^2}\Theta$ (**)

De (**): $dV = -nR\Theta \frac{dP}{P^2}$ $(\Theta = \text{cte})$

Integrando a $\Theta = $ cte: $\boxed{V = \frac{nR\Theta}{P} + \varphi(\Theta)}$ donde $\varphi(\Theta) = $ cte. de integración

Ahora, derivando esta última expresión parcialmente respecto de Θ:

$$\left(\frac{\partial V}{\partial \Theta}\right)_P = \frac{nR}{P} + \varphi'(\Theta) \quad (***)$$

Comparando (***) con (*): $\cancel{\frac{nR}{P}} + \frac{na}{\Theta^2} = \cancel{\frac{nR}{P}} + \varphi'(\Theta)$

$$\therefore \quad \varphi'(\Theta) = \frac{d\varphi(\Theta)}{d\Theta} = \frac{na}{\Theta^2}$$

o $$d\varphi(\Theta) = \frac{na}{\Theta^2}\, d\Theta \longrightarrow \boxed{\varphi(\Theta) = -\frac{na}{\Theta} + \text{cte}.}$$

Con esto se obtiene finalmente:

$$\boxed{V = \frac{nR\Theta}{P} - \frac{na}{\Theta} + \text{cte}.}$$