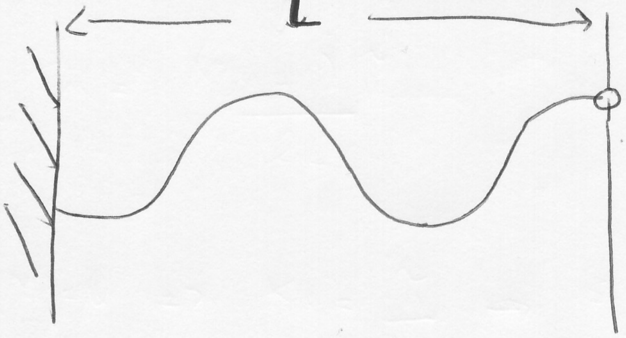


C.B:

$$y(x=0,t) = 0$$

$$\frac{\partial y}{\partial x}(x=L,t) = 0$$

C.I:

$$y(x,t=0) = \alpha x$$

$$\frac{\partial y}{\partial t}(x,t=0) = 0.$$

Solución:

Por separación de variables: $\dfrac{\partial^2 y}{\partial t^2} = \dfrac{\partial^2 y}{\partial x^2} \cdot c^2$

$$X(x)'' = -k^2 X(x)$$

$$\Rightarrow X(x) = A\cos(kx) + B\sin(kx)$$

y $$T'' = -\omega^2 T(t)$$

$$\Rightarrow T(t) = C\sin(\omega t) + D\cos(\omega t)$$

Con $c \cdot k = \omega$

CB $\dfrac{\partial y}{\partial x}(x=L,t) = \dfrac{\partial X}{\partial x}(x=L) \cdot T(t) = 0.$

$$\Rightarrow \frac{\partial X}{\partial x}(x=L) = 0 \Rightarrow B k\cos(kL) = 0$$

$$\Rightarrow \cos(kL) = 0 \Rightarrow kL = \frac{(2n+1)\pi}{2} \quad ; (n = 0, \ldots, \infty)$$

$$\Rightarrow \boxed{K_n = \frac{(2n+1)\pi}{2L}}$$

$$\Rightarrow y(x=0,t)=0 \quad \Rightarrow \quad X(x=0) = 0 = A\cdot$$

$$\Rightarrow \boxed{A=0}$$

$$\Rightarrow X_n(x) = B\cdot \sin(K_n x)$$

$$K_n \Rightarrow \omega_n$$

$$\Rightarrow y(x,t) = \sum_{n=0} \left\{ C_n \cdot \sin(\omega_n \cdot t) + D_n \cdot \cos(\omega_n \cdot t) \right\} \cdot \sin(K_n \cdot x)$$

C.I:

$$\frac{\partial y}{\partial t}(x,t=0) = 0 \quad \Rightarrow \boxed{C_n = 0}$$

$$\Rightarrow y(x,t) = \sum_{n=0} \left\{ D_n \cos(\omega_n \cdot t) \right\} \cdot \sin(K_n \cdot x)$$

$$\Rightarrow y(x,t=0) = \alpha x = \sum_{n=0} D_n \cdot \sin(K_n \cdot x)$$

$$\Rightarrow \alpha x = \sum_{n=0} D_n \cdot \sin\left(\frac{(2n+1)\cdot\pi}{2L}\cdot x\right) \qquad \cdot \Big/ \sin\left(\frac{(2n+1)\pi\cdot x}{2L}\right)\int_0^{2L}$$

$$\int_0^{2L} \alpha\cdot x \sin\left(\frac{(2n+1)\cdot x\cdot \pi}{2L}\right) dx = D_n \cdot L$$

$$I=\int_0^{2L} \underbrace{\alpha\cdot x}_{u}\cdot \underbrace{\sin\left(\frac{(2m+1)\pi x}{2L}\right)dx}_{dv} = -\alpha x\cdot\cos\left(\frac{(2m+1)\pi x}{2L}\right)\cdot\frac{2L}{(2m+1)\pi}\Bigg|_0^{2L}$$

$$+\frac{\alpha\, 2L}{(2m+1)\pi}\int_0^{2L}\cos\left(\frac{(2m+1)\pi x}{2L}\right)dx$$

$$= -\alpha\cdot 2L\cdot\frac{\cos((2m+1)\pi)\cdot 2L}{(2m+1)\cdot\pi} + \frac{2L\alpha}{(2m+1)\pi}\cdot\frac{2L}{(2m+1)\pi}\cdot\cancelto{0}{\sin(\ldots x)}\Bigg|_0^{2L}$$

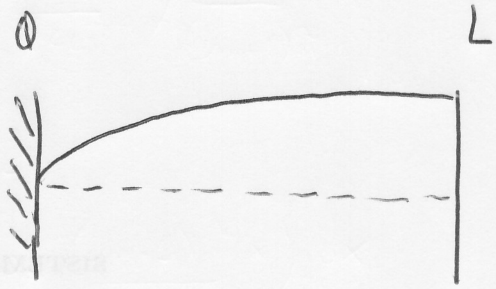
$$\Rightarrow \quad I = \frac{-\alpha\, 4L^2}{(2m+1)\pi}\cdot -1 = \frac{4L^2\alpha}{(2m+1)\pi} = D_m\cdot L$$

$$\Rightarrow \boxed{D_m=\frac{4L\alpha}{(2m+1)\pi}}$$

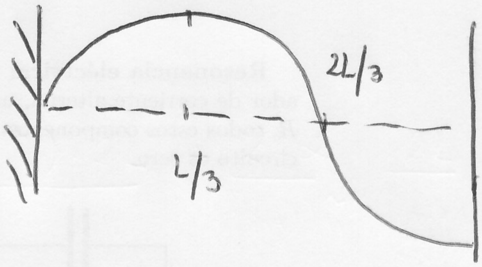
$$\Rightarrow \quad y(x,t)=\sum_{m=0}\frac{4L\alpha}{(2m+1)\pi}\cdot\cos\left(\frac{(2m+1)\,c\,\pi\, t}{2L}\right)\cdot\sin\left(\frac{(2m+1)\cdot\pi\cdot x}{2L}\right)$$

//

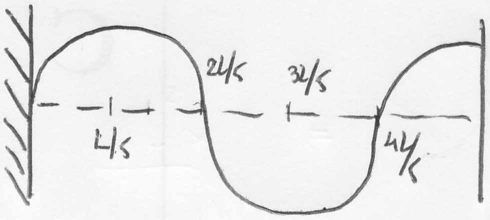
$$k_n = \frac{(2n+1)\pi}{2L}$$



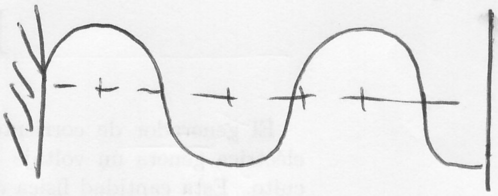
$$n=0 \Rightarrow k_0 = \frac{\pi}{2L} \rightarrow \sin\left(\frac{\pi x}{2L}\right) \rightarrow$$



$$n=1 \Rightarrow k_1 = \frac{3\pi}{2L} \rightarrow \sin\left(\frac{3\pi x}{2L}\right) \rightarrow$$



$$n=2 \Rightarrow k_2 = \frac{5\pi}{2L} \rightarrow \sin\left(\frac{5\pi x}{2L}\right) \rightarrow$$



$$n=3 \Rightarrow k_3 = \frac{7\pi}{2L} \rightarrow \sin\left(\frac{7\pi x}{2L}\right) \rightarrow$$