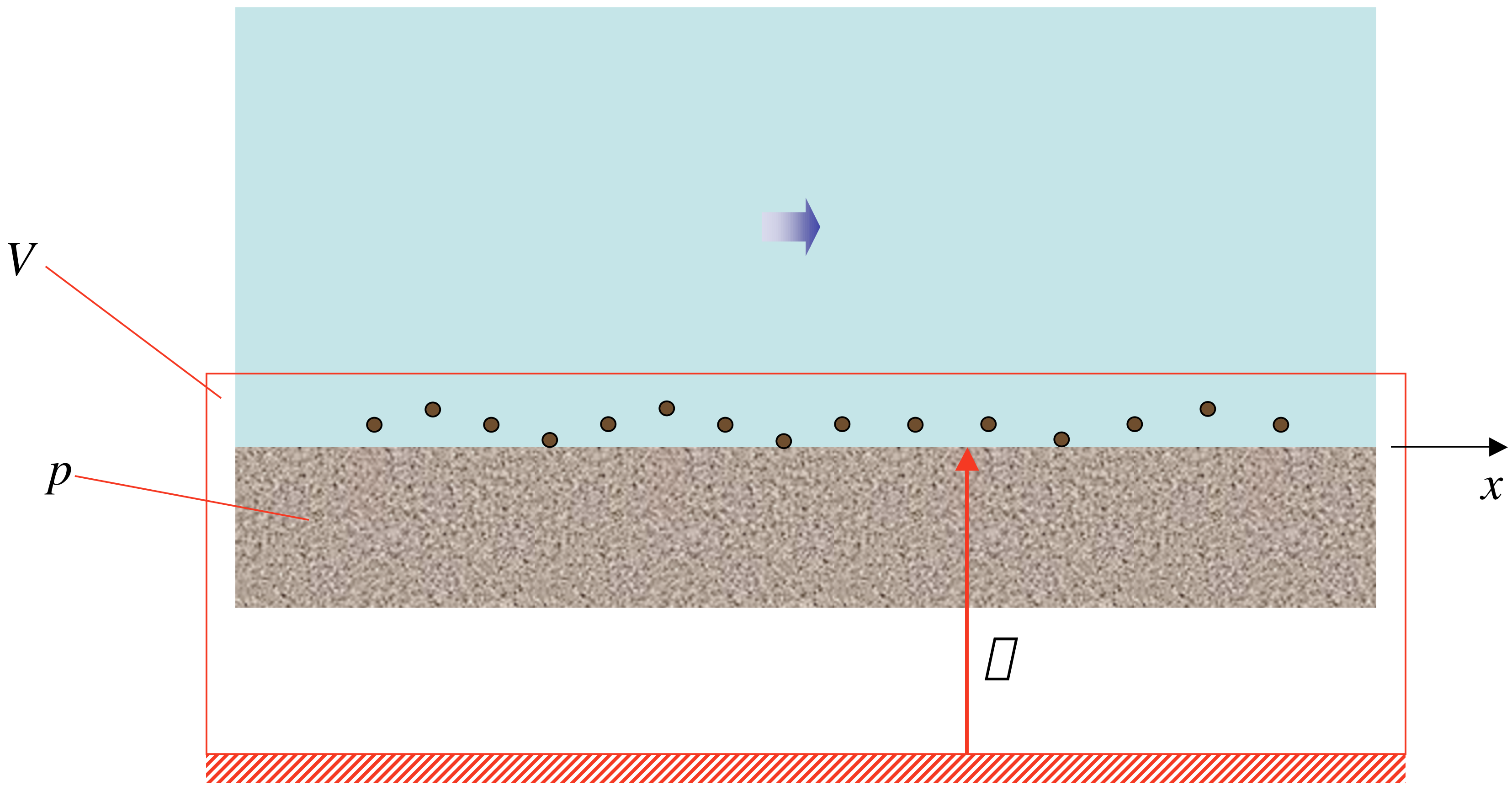
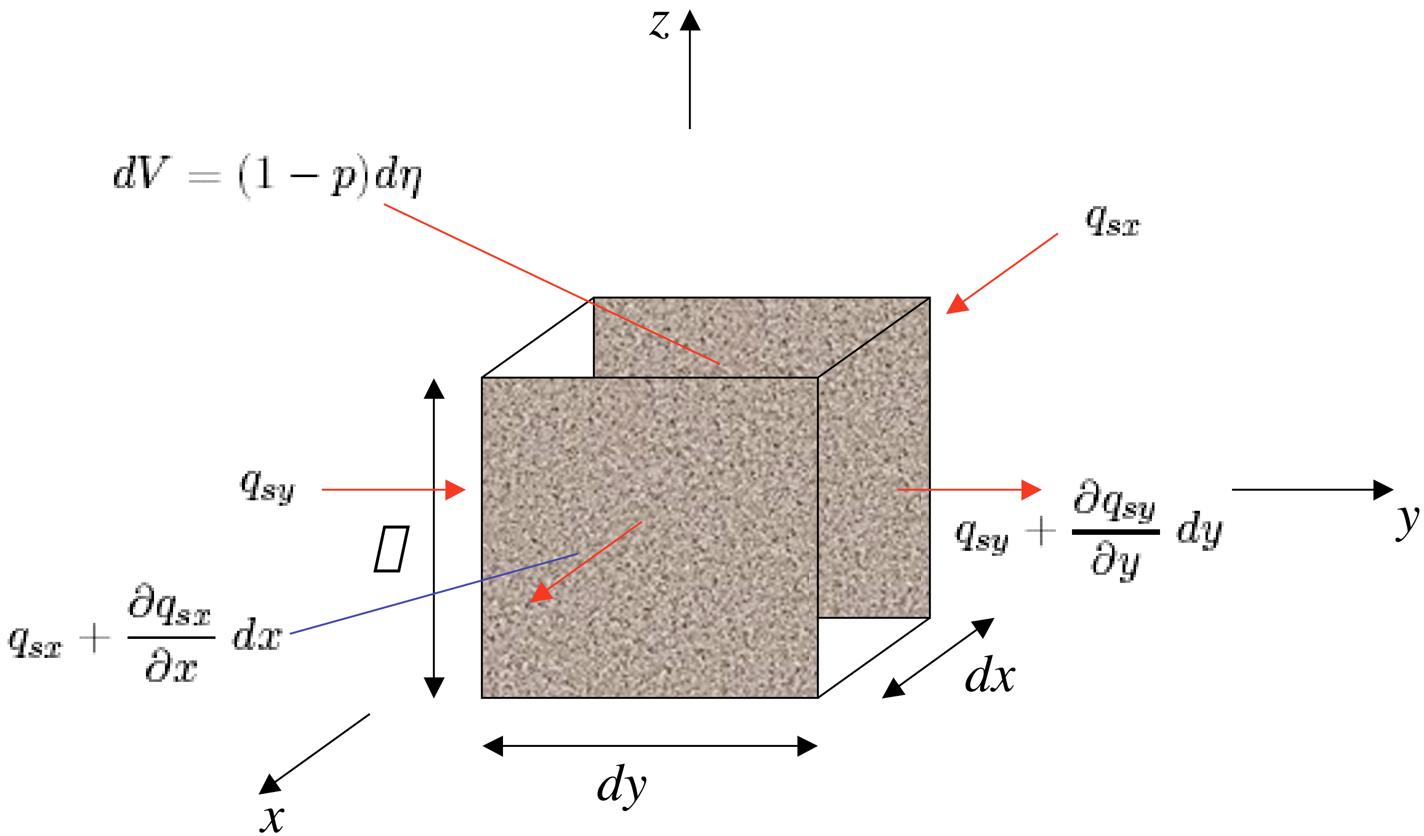
ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

- CASO SEDIMENTO UNIFORME -



ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

- CASO SEDIMENTO UNIFORME -



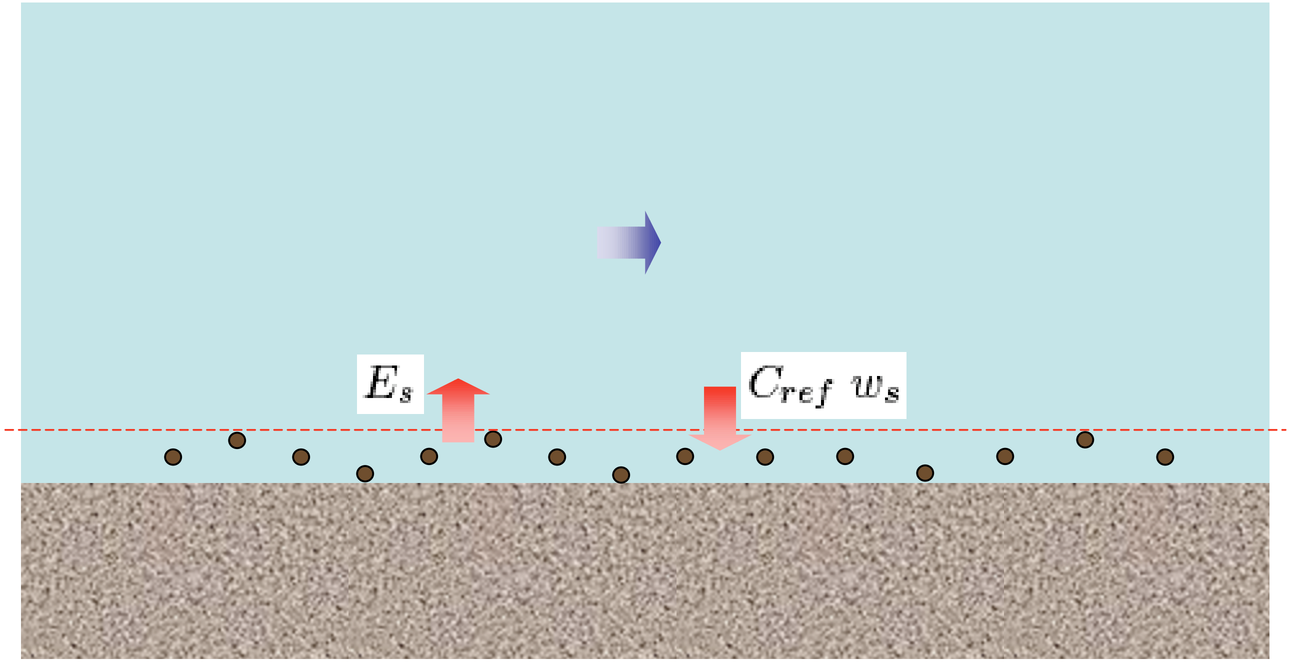
ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

- CASO SEDIMENTO UNIFORME -

$$\frac{\partial \eta}{\partial t} + \frac{1}{1-p}\left(\frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y}\right) = 0$$

ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

- CASO SEDIMENTO UNIFORME -



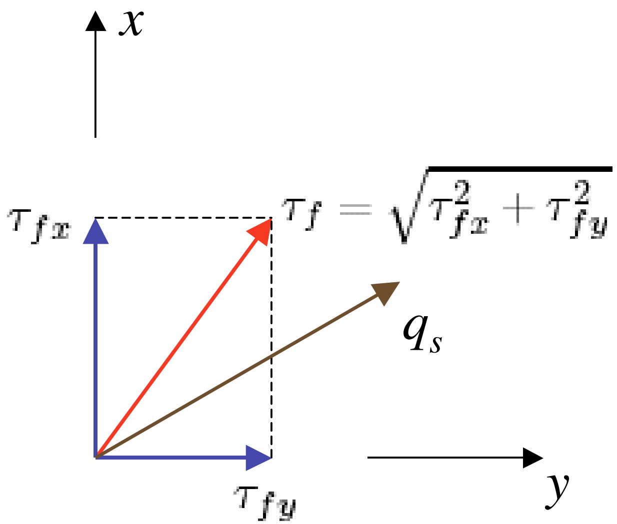
ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

- CASO SEDIMENTO UNIFORME -

$$\frac{\partial \eta}{\partial t} + \frac{1}{1-p}\left(\frac{\partial q_{sx}}{\partial x} + \frac{\partial q_{sy}}{\partial y}\right) = C_{ref}\, w_s - E_s$$

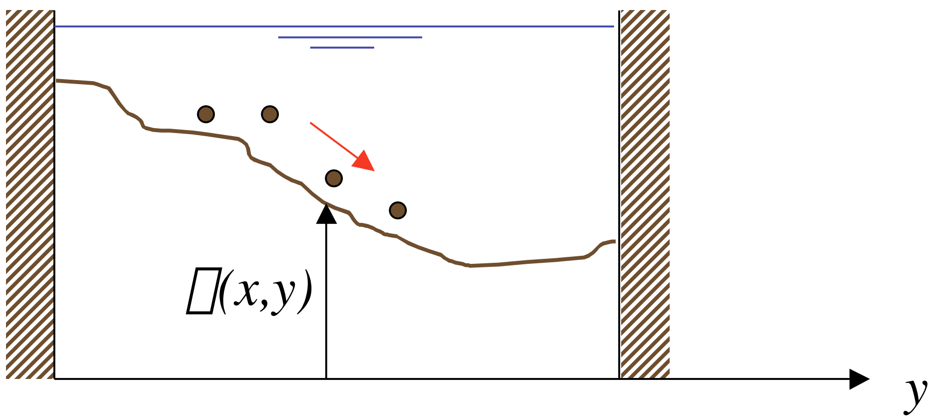
ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

COMPONENTES DEL GASTO SOLIDO DE FONDO



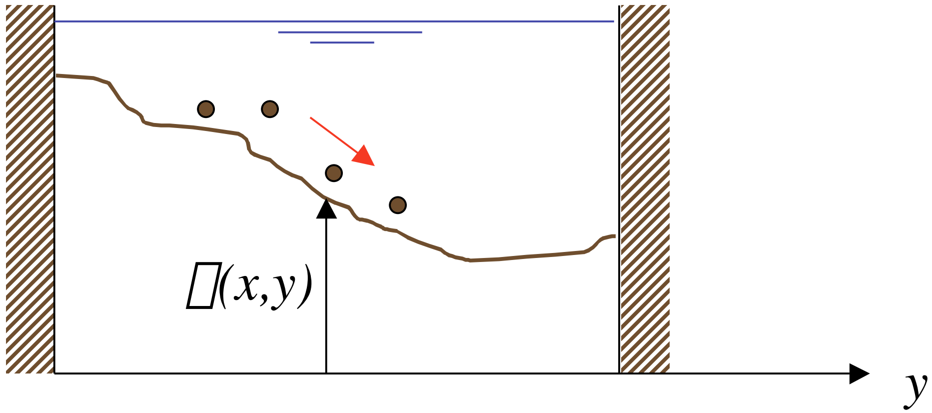
ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

COMPONENTES DEL GASTO SOLIDO DE FONDO



ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

COMPONENTES DEL GASTO SOLIDO DE FONDO



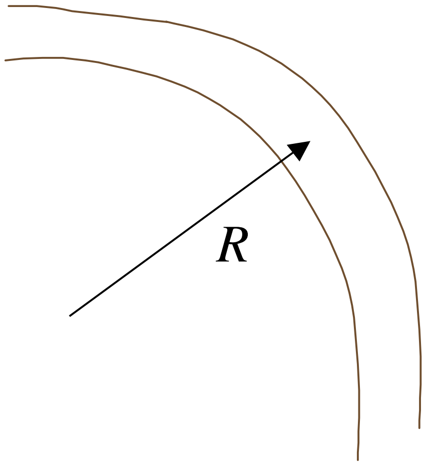
$$\frac{q_{sy}}{q_{sx}} = \frac{\tau_{fy}}{\tau_{fx}} - \frac{r}{\tau_*^{1/2}} \frac{\partial \eta}{\partial y} \qquad \tau_* = \frac{\tau_f}{\rho\, g\, R\, d_s}$$

r = 0.3

ECUACION DE CONTINUIDAD DEL GASTO SOLIDO DE FONDO (ECUACION DE EXNER)

COMPONENTES DEL GASTO SOLIDO DE FONDO
CASO CON CURVATURA

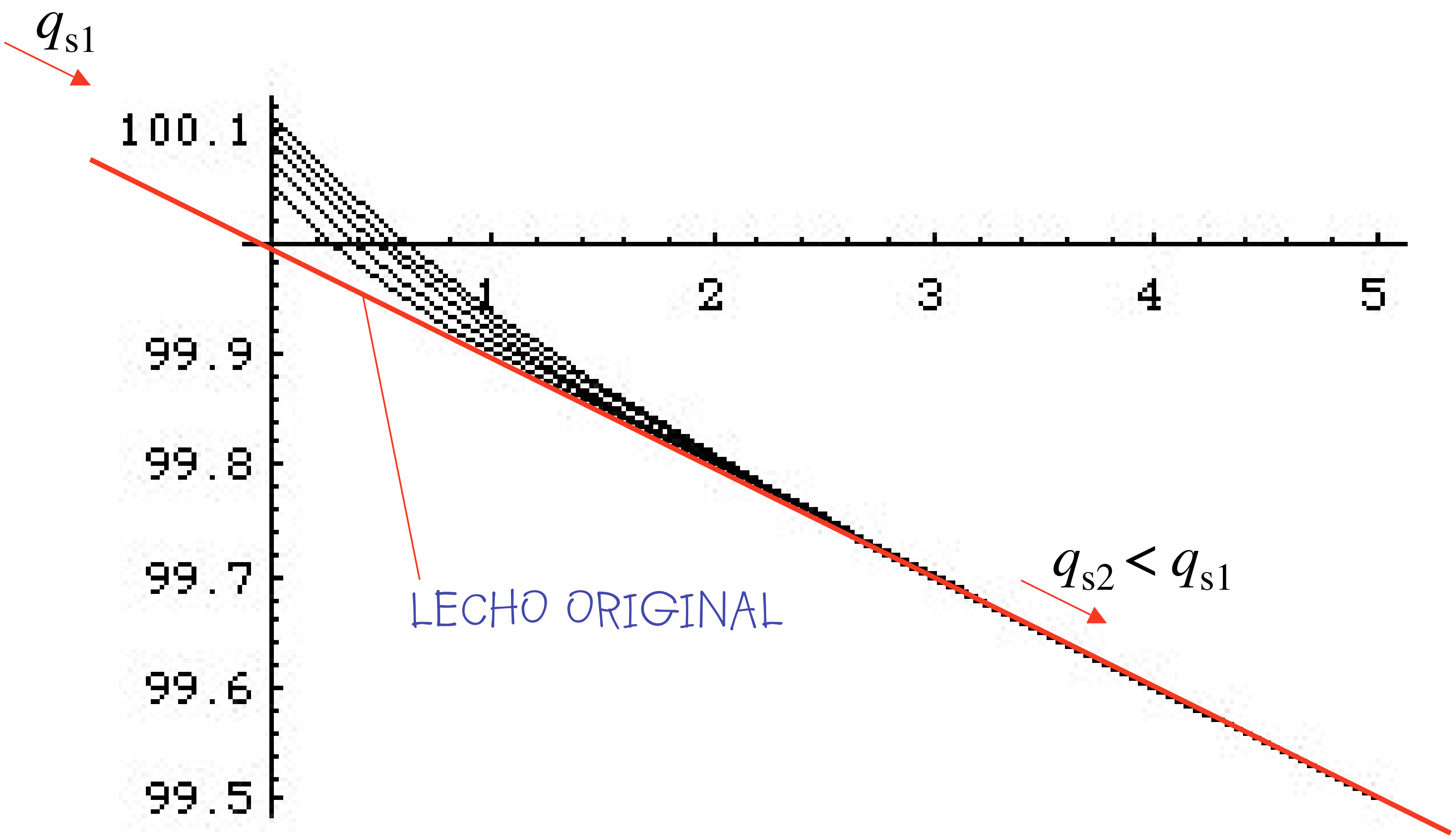
$$\frac{q_n}{q_s} = \frac{\tau_{fn}}{\tau_{fs}} - \frac{r}{\tau_*^{1/2}} \frac{\partial \eta}{\partial n} - k \frac{c}{1 + n\, c}$$



$$c = \frac{1}{R}$$

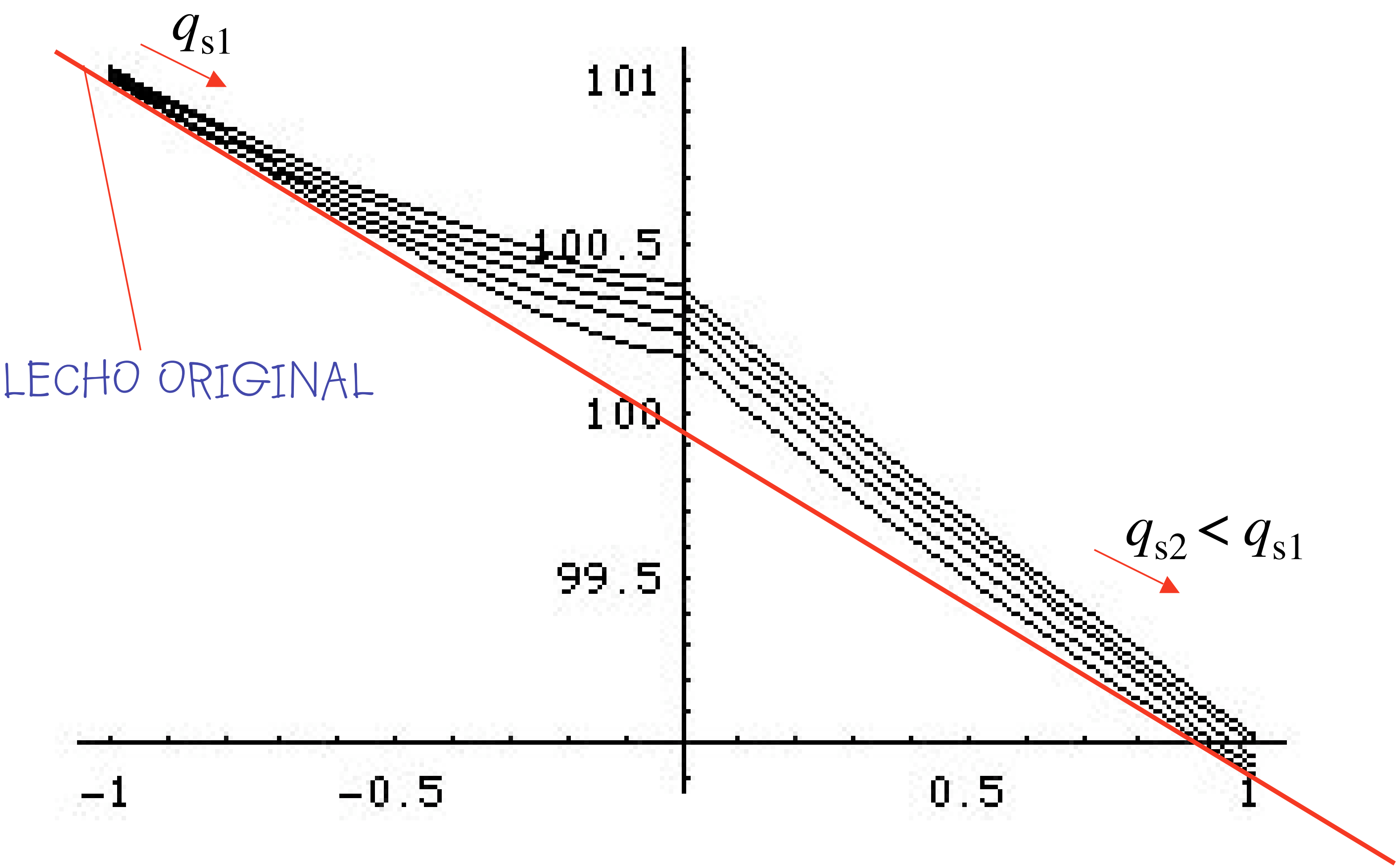
EJEMPLO: EVOLUCION DEL LECHO SEGUN EXNER

AGRADACION

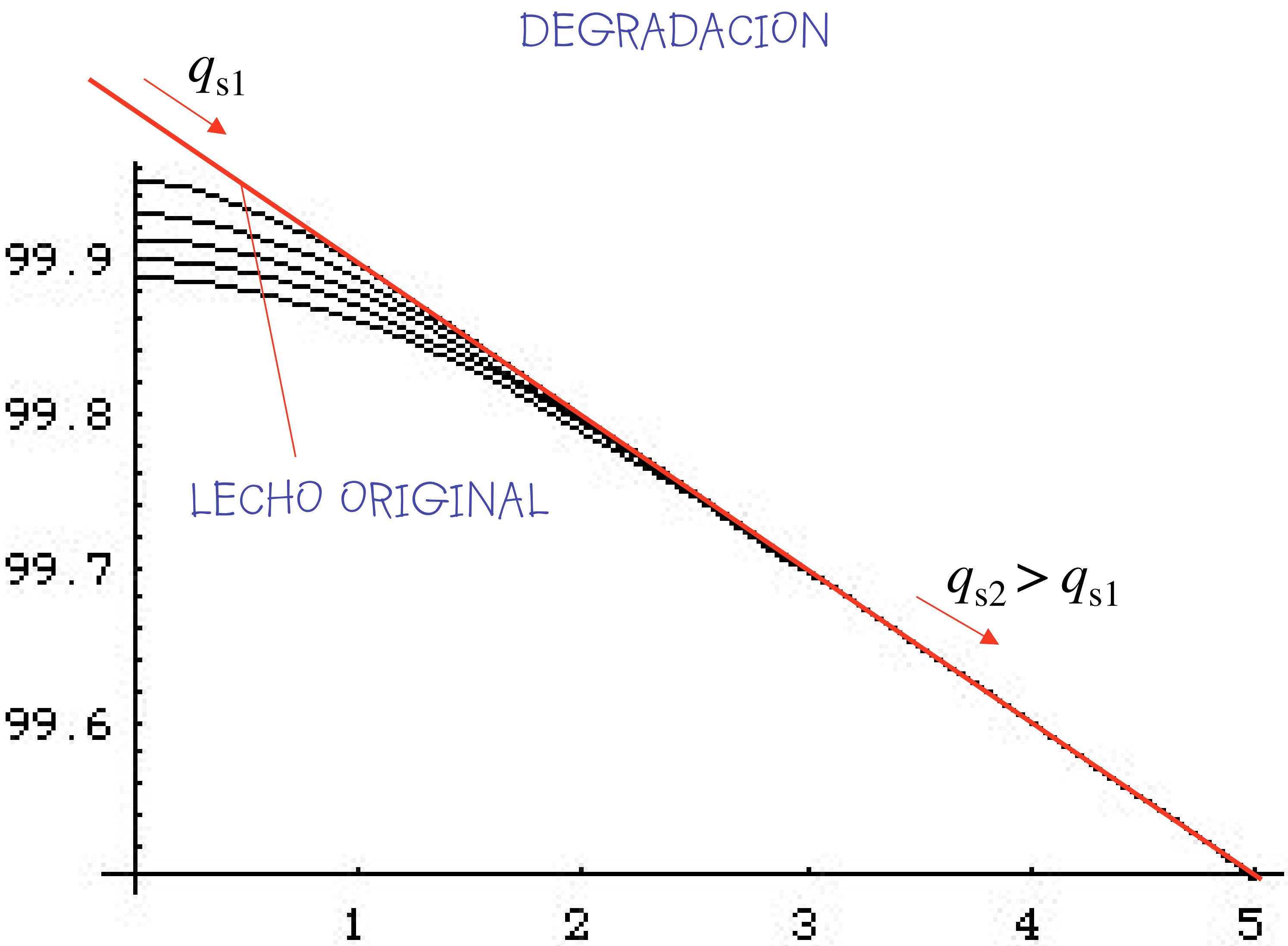


EJEMPLO: EVOLUCION DEL LECHO SEGUN EXNER

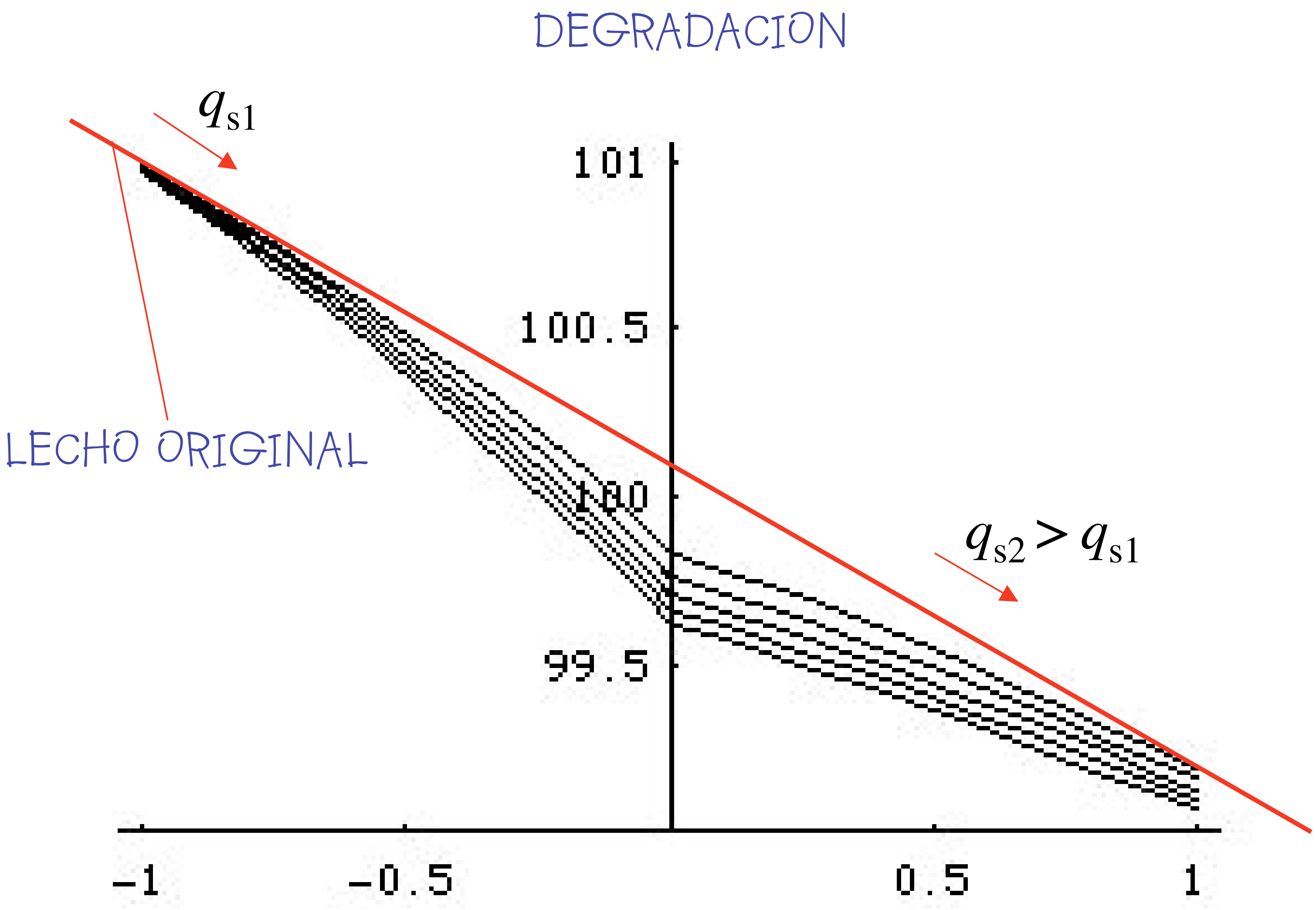
AGRADACION



EJEMPLO: EVOLUCION DEL LECHO SEGUN EXNER



EJEMPLO: EVOLUCION DEL LECHO SEGUN EXNER

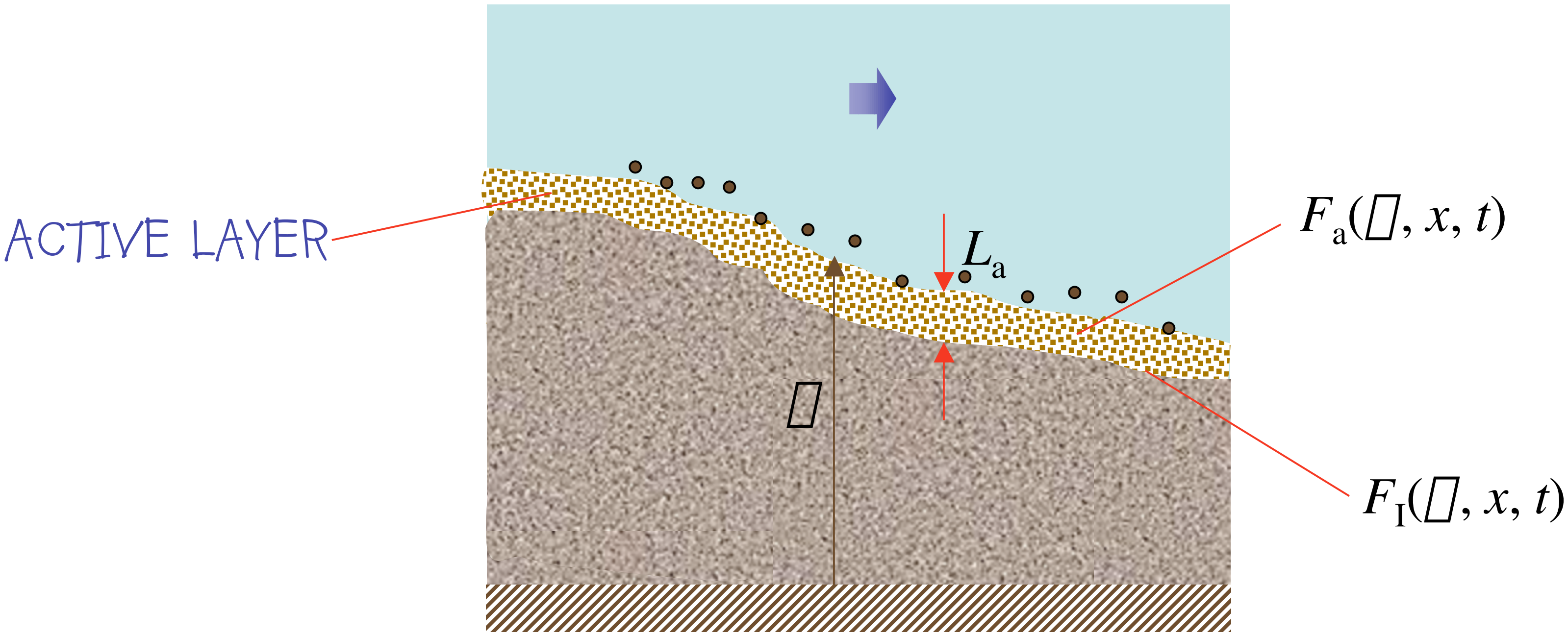


Probabilistic Exner Sediment Continuity Equation for Mixtures with No Active Layer

By Gary Parker,[1] Member, ASCE, Chris Paola,[2] and Suzanne Leclair[3]

ACTIVE LAYER CONCEPT

HIRANO (1971)



“□ - SCALE”

(a veces “□ - SCALE”)

$$\psi = \ln_2(D)$$

F_a(□, x, t) FRACCION MASICA DE GRANOS DE TAMAÑO EN EL RANGO (□, □ + d□) EN LA ACTIVE LAYER

F_I(□, x, t) ID EN LA INTERFAZ

$$\int_{-\infty}^{\infty} F_a \, d\psi = \int_{-\infty}^{\infty} F_I \, d\psi = 1$$

EXNER SEDIMENTO UNIFORME

$$(1 - \lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q}{\partial x}$$

POROSIDAD

EXTENSION A MEZCLAS

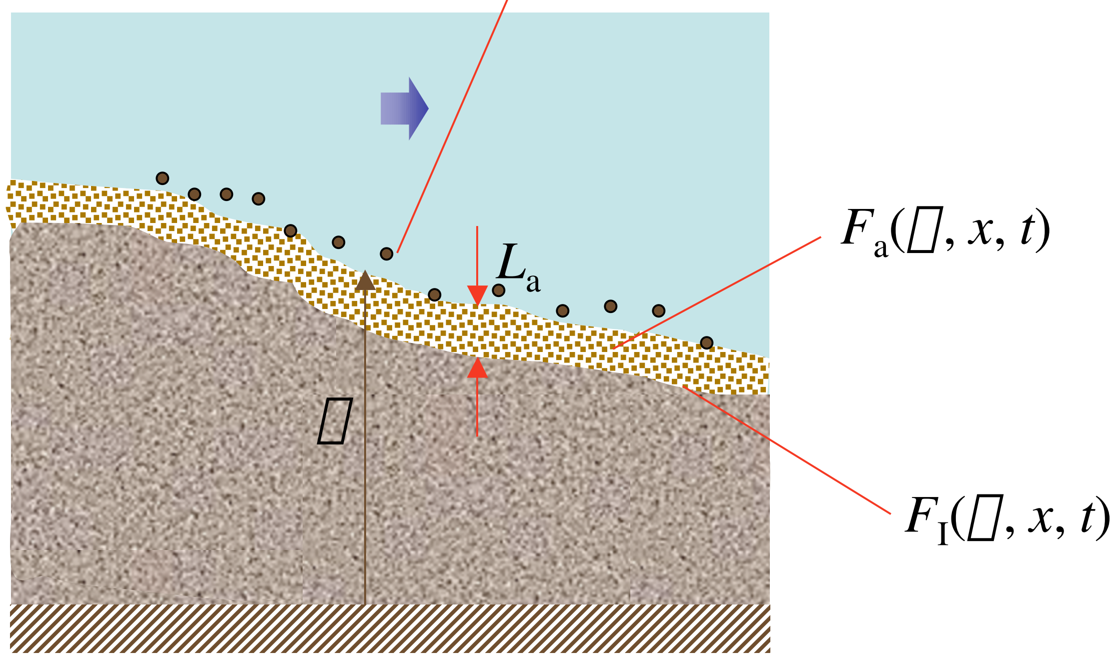
$$(1 - \lambda_p)\left(F_I \frac{\partial \eta}{\partial t} + L_a \frac{\partial F_a}{\partial t}\right) = -\frac{\partial q_\psi}{\partial x}$$

GASTO VOLUMETRICO GRANOS EN EL RANGO (ψ, ψ + dψ)

$$q = \int_{-\infty}^{\infty} q_\psi \, d\psi$$

FRACCION MASICA DEL
GASTO SOLIDO

$$F_t = \frac{q_\psi}{q}$$



EROSION $\partial\eta/\partial t < 0$ F_I = F SUSTRATO

DEPOSITACION $\partial\eta/\partial t > 0$ F_I = VALOR INTERMEDIO ENTRE F_a Y F_t

$$(1-\lambda_p)\left(F_I\frac{\partial\eta}{\partial t}+L_a\frac{\partial F_a}{\partial t}\right)=-\frac{\partial q_\psi}{\partial x}$$

$$(1-\lambda_p)\frac{\partial\eta}{\partial t}=-\frac{\partial q}{\partial x}$$

$$\Rightarrow\quad (1-\lambda_b)L_a\frac{\partial F_a}{\partial t}=-\frac{\partial q_\psi}{\partial x}+F_I\frac{\partial q}{\partial x}$$

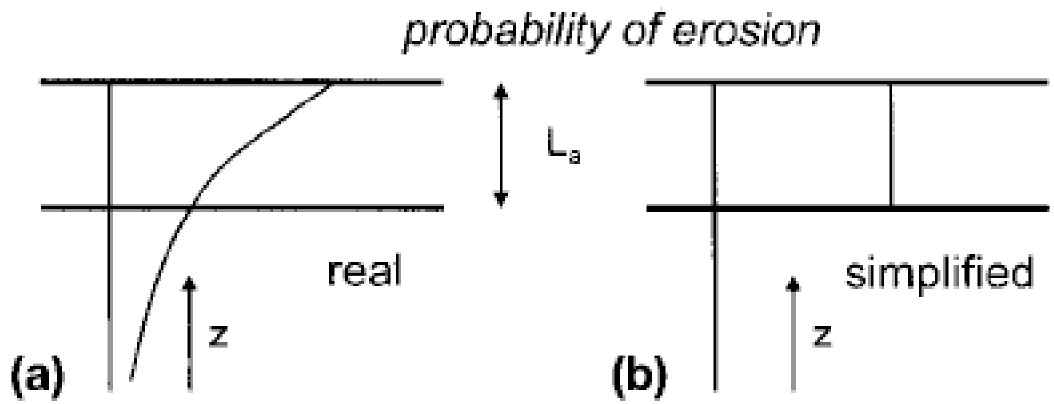


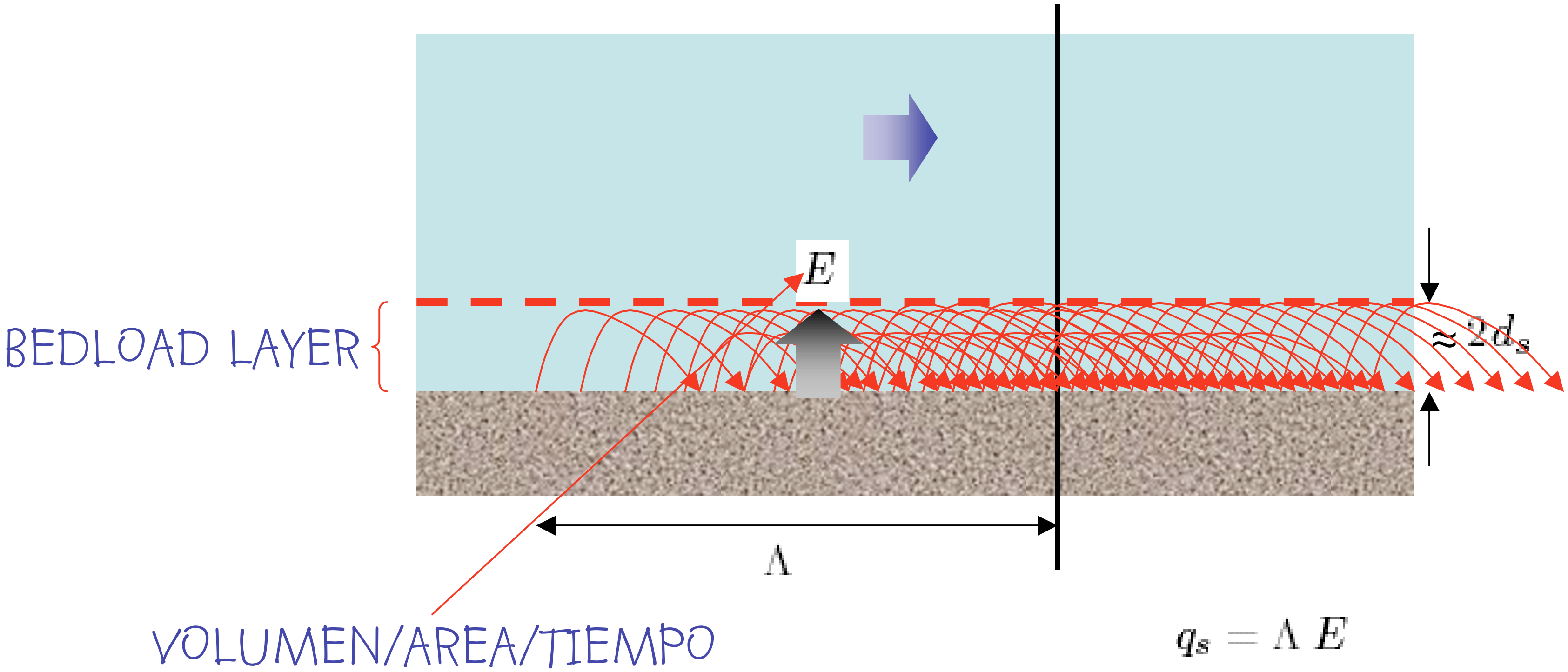
FIG. 2. Variation in Probability of Entrainment: (a) Schematization of Actual Variation in Probability of Entrainment of Particle within Given Time as Function of Depth (Variation is Seen To Be Continuous, Although Active Layer Has Been Drawn In for Reference); (b) Approximation of Variation in Probability of Entrainment within Given Time as Step Function of Depth (Constant and Greater Than Zero in Active Layer and Equal to Zero below Active Layer) Offered by Active Layer Formulation

REFORMULACION DE EC. DE EXNER

$$(1 - \lambda_p)\frac{\partial \eta}{\partial t} = D - E$$

TASA EROSION

TASA DEPOSITACION



E
BEDLOAD LAYER
$\approx 2d_s$
Λ
VOLUMEN/AREA/TIEMPO
$q_s = \Lambda\, E$

$$D(x) = \int_{-\infty}^{x} E(y) f_p(x - y)\ dy$$

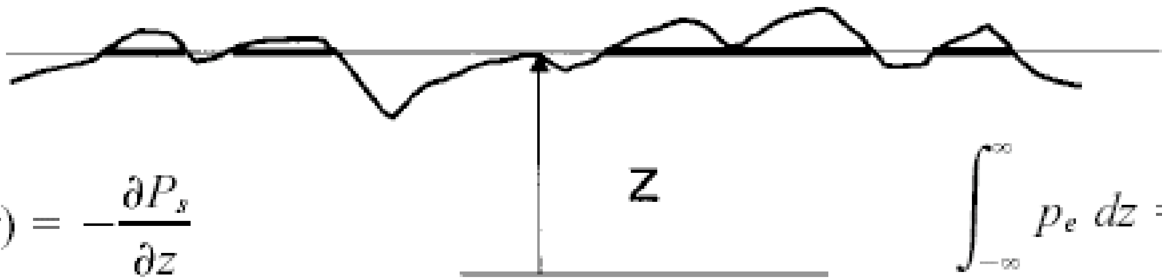
$$q(x) = \int_{-\infty}^{x} E(y) \int_{x-y}^{\infty} f_p(y')\ dy'\ dy$$

$$\frac{\partial q}{\partial x} = E - D$$

$$(1 - \lambda_p)\frac{\partial \eta}{\partial t} = D - E$$

PROBABILISTIC FORMULATION FOR UNIFORM SEDIMENT

$p_e(z)\,dz$ denotes the probability that the instantaneous bed elevation is in the range $(z, z + dz)$.



$$p_e(z) = -\frac{\partial P_s}{\partial z}$$

$$\int_{-\infty}^{\infty} p_e\,dz = 1$$

$$\eta = \int_{-\infty}^{\infty} z p_e(z)\,dz$$

PROBABILISTIC FORMULATION FOR UNIFORM SEDIMENT

$$\frac{\partial}{\partial t}(c_b P_s \, dx \, dz) = (D_e - E_e) \, dx \, dz$$

$$c_b = 1 - \lambda_p$$

PROBABILISTIC FORMULATION FOR UNIFORM SEDIMENT

$$\frac{\partial}{\partial t}(c_b P_s\ dx\ dz) = (D_e - E_e)\ dx\ dz$$

$$D = \int_{-\infty}^{\infty} D_e\ dz; \quad E = \int_{-\infty}^{\infty} E_e\ dz$$

$$c_b \frac{\partial P_s}{\partial t} = D_e - E_e$$

$$P_s = P_s(y); \quad y = z - \eta$$

$$\frac{\partial P_s}{\partial t} = -\frac{\partial P_s}{\partial y}\frac{\partial \eta}{\partial t} = p_e \frac{\partial \eta}{\partial t}$$

$$c_b p_e \frac{\partial \eta}{\partial t} = D_e - E_e$$

PROBABILISTIC FORMULATION FOR UNIFORM SEDIMENT

$$c_b p_e \frac{\partial \eta}{\partial t} = D_e - E_e$$

$$D_e = D\beta_D(y)p_e(y); \quad E_e = E\beta_E(y)p_e(y)$$

BIAS FUNCTIONS

$$\int_{-\infty}^{\infty} \beta_D p_e \, dy = 1; \quad \int_{-\infty}^{\infty} \beta_E p_e \, dy = 1$$

$$D - E = \beta_D D - \beta_E E$$

$$\beta_D = \beta_E = 1$$

PROBABILISTIC FORMULATION FOR SEDIMENT MIXTURES

$$\int_{-\infty}^{\infty} F \, d\psi = 1;$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D_{e\psi} \, dy \, d\psi = D \qquad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_{e\psi} \, dy \, d\psi = E$$

$$\frac{\partial}{\partial t} c_b F P_s = D_{e\psi} - E_{e\psi}$$

$$(1 - \lambda_p) \frac{\partial \eta}{\partial t} = D - E \quad \Rightarrow \quad c_b \left(p_e F \frac{\partial \eta}{\partial t} + P_s \frac{\partial F}{\partial t} \right) = p_e (D \beta_{\psi D} F_t - E \beta_{\psi E} F)$$

PROBABILISTIC FORMULATION FOR SEDIMENT MIXTURES

$$D_{e\psi} = D\beta_{\psi D}(y, \psi)p_e(y)F_t$$

$$E_{e\psi} = E\beta_{\psi E}(y, \psi)p_e(y)F$$

$$\int_{-\infty}^{\infty} \beta_{\psi D} F_t \, d\psi = \beta_D; \quad \int_{-\infty}^{\infty} \beta_{\psi E} F \, d\psi = \beta_E$$

$$\int_{-\infty}^{\infty} \beta_{\psi D} p_e F_t \, dy \, d\psi = 1; \quad \int_{-\infty}^{\infty} \beta_{\psi E} p_e F \, dy \, d\psi = 1$$

PROBABILISTIC FORMULATION FOR SEDIMENT MIXTURES

$$\lambda_p = 1 - \bar{c}_b; \quad \bar{c}_b = \int_{-\infty}^{\infty} c_b p_e \, dy$$

POROSIDAD MEDIA

$$c_b \left(p_e F \frac{\partial \eta}{\partial t} + P_s \frac{\partial F}{\partial t} \right) = p_e (D \beta_{\psi D} F_t - E \beta_{\psi E} F)$$

$$(1 - \lambda_p) \frac{\partial \eta}{\partial t} = D - E$$

$$\Rightarrow \quad c_b P_s \frac{\partial F}{\partial t} = p_e \left[D \left(\beta_{\psi D} F_t - \frac{c_b}{\bar{c}_b} F \right) - E \left(\beta_{\psi E} - \frac{c_b}{\bar{c}_b} \right) F \right]$$

PROBABILISTIC FORMULATION FOR SEDIMENT MIXTURES

$$\frac{c_b}{\bar{c}_b}(D - E) = \beta_D D - \beta_E E$$

$$\beta_D = \beta_E = \frac{c_b}{\bar{c}_b}$$

PROBABILISTIC FORMULATION FOR SEDIMENT MIXTURES

$$(1-\lambda_p)\left(F_I \frac{\partial \eta}{\partial t} + L_a \frac{\partial F_a}{\partial t}\right) = DF_i - EF_a$$

ACTIVE LAYER

$$\lambda_p = 1 - c_{ba}; \quad F_I = F\big|_{y=-L_{na}}; \quad L_a = \int_{-L_{na}}^{\infty} P_s \, dy$$

MENOR ELEVACION DE SOCAVACION

$$L_a = \eta - L_{na}$$

ESPESOR MEDIO DE SEDIMENTO SOBRE EL NIVEL DE MENOR ELEVACION DE SOCAVACION

THE END!