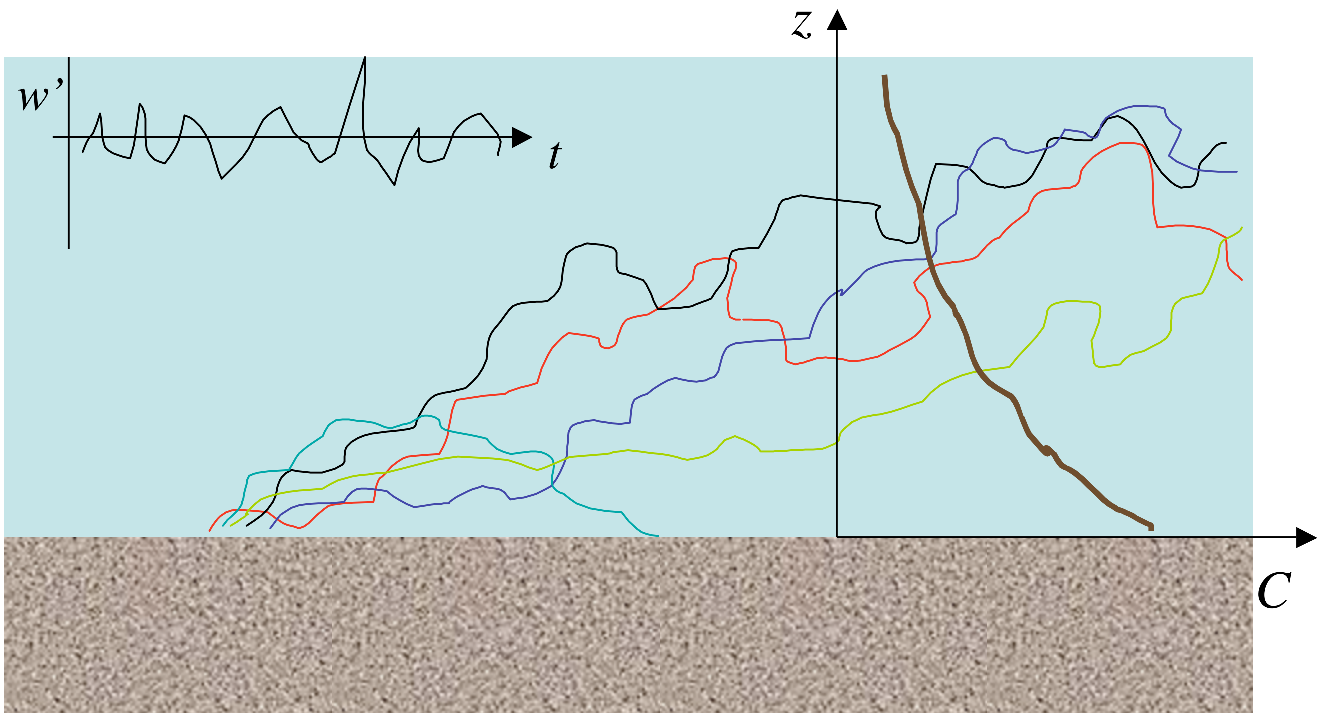
6.3 Gasto sólido en suspensión



Sedimentology (2003) **50**, 247–263

Threshold for particle entrainment into suspension

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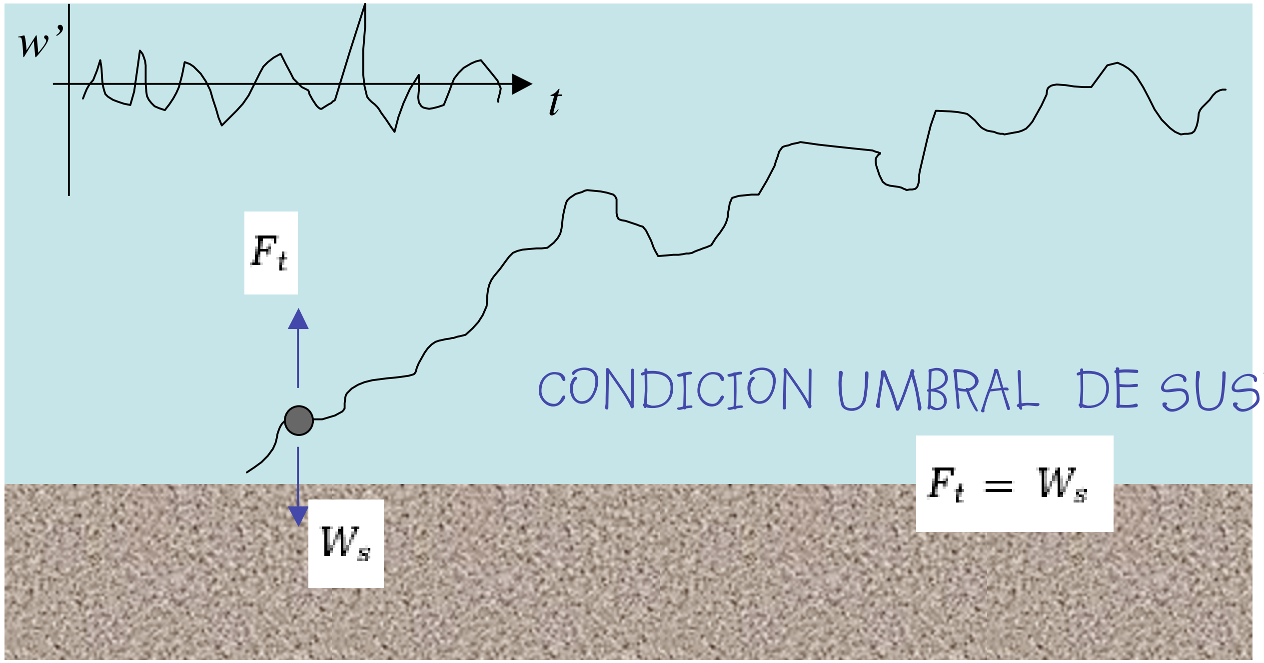


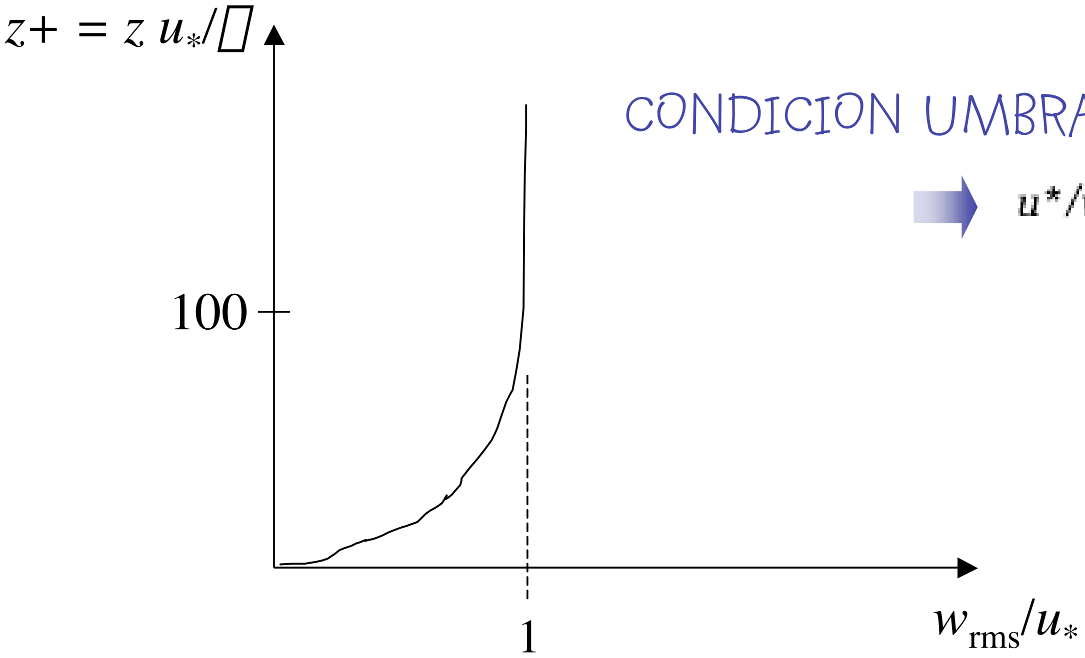
CONDICION UMBRAL DE SUSPENSION:

$$F_t = 1/2 C_D \rho u_e^2 \pi d_p^2/4 \qquad W_s = (\rho_s - \rho) g \pi d_p^3/6$$

$$\Rightarrow \quad u_e/w_s = 1$$

METODO DE BAGNOLD

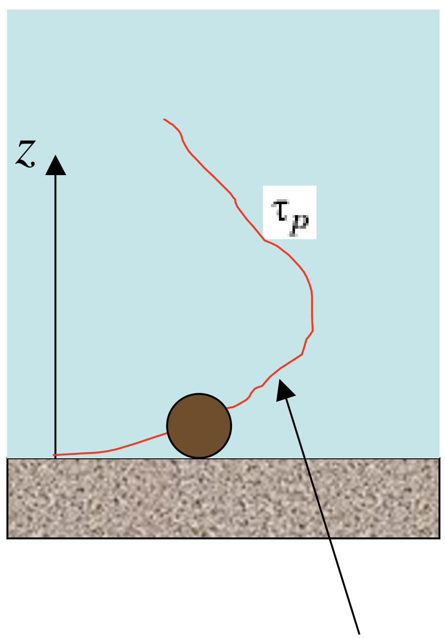
u_e □ w_{rms} □ u_*



CONDICION UMBRAL DE SUSPENSION:

$u^*/w_s = 1$

NIÑO ET AL. (2003)



$$u_e = \xi(\tau_p/\rho)^{1/2}$$

$$\phi = \tau_p/(\rho\, u^{*2})$$

$$u^*/w_s = (1/\xi)\phi^{-1/2}$$

$$\tau(y) = \rho v_t u^{*2}(1 - y/h)/(v + v_t)$$

$$\phi = (v_t/v)/(1 + v_t/v)\ (1 - (Re_p^*/2)/Re_h^*)$$

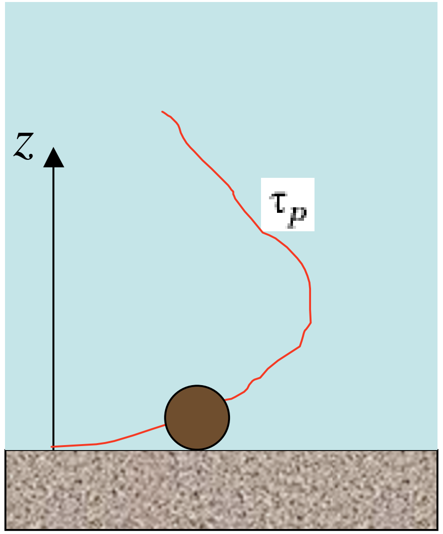
$$v_t/v = \kappa(Re_p^*/2)^3/[\Gamma_0^3 + Re_p^*/2)^2]$$

O'CONNOR (1995)

0·4

7·4

NIÑO ET AL. (2003)



$$u_e = \xi(\tau_p/\rho)^{1/2}$$

$$\phi = \tau_p/(\rho\ u^{*2})$$

$$u^*/w_s = (1/\xi)\phi^{-1/2}$$

$$u^*/w_s = (1/\xi)(8/\kappa)^{1/2} Re_p^{*-3/2}(\Gamma_0^3 + Re_p^{*2}/4 + \kappa Re_p^{*3}/8)^{1/2}/(1-(Re_p^*/2)/Re_h^*)^{1/2}$$

0·4

7·4

0·4

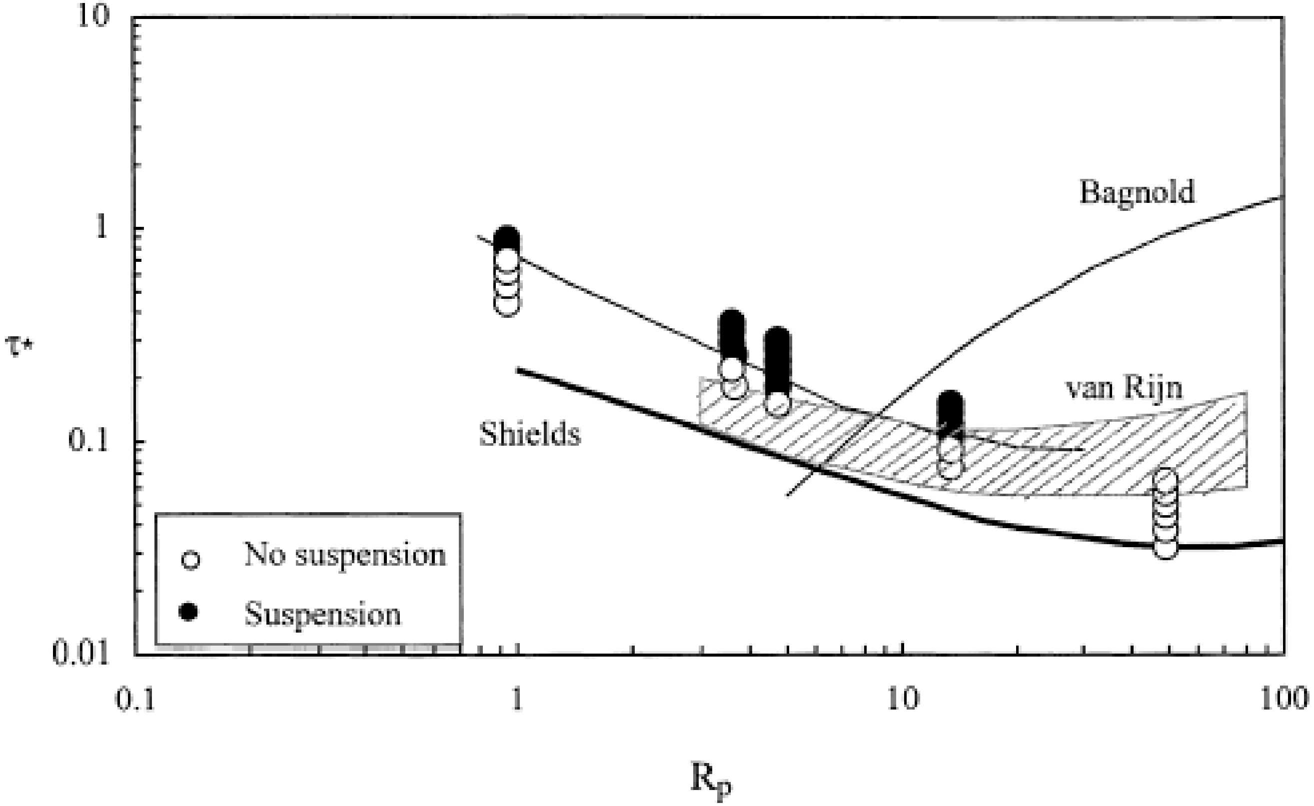
VAN RIJN (1984)

$$u^*/w_s = \begin{cases} 4{\cdot}0\, R_p^{-2/3} & 1 \leq R_P \leq 32 \\ 0{\cdot}4 & R_p \geq 32 \end{cases}$$

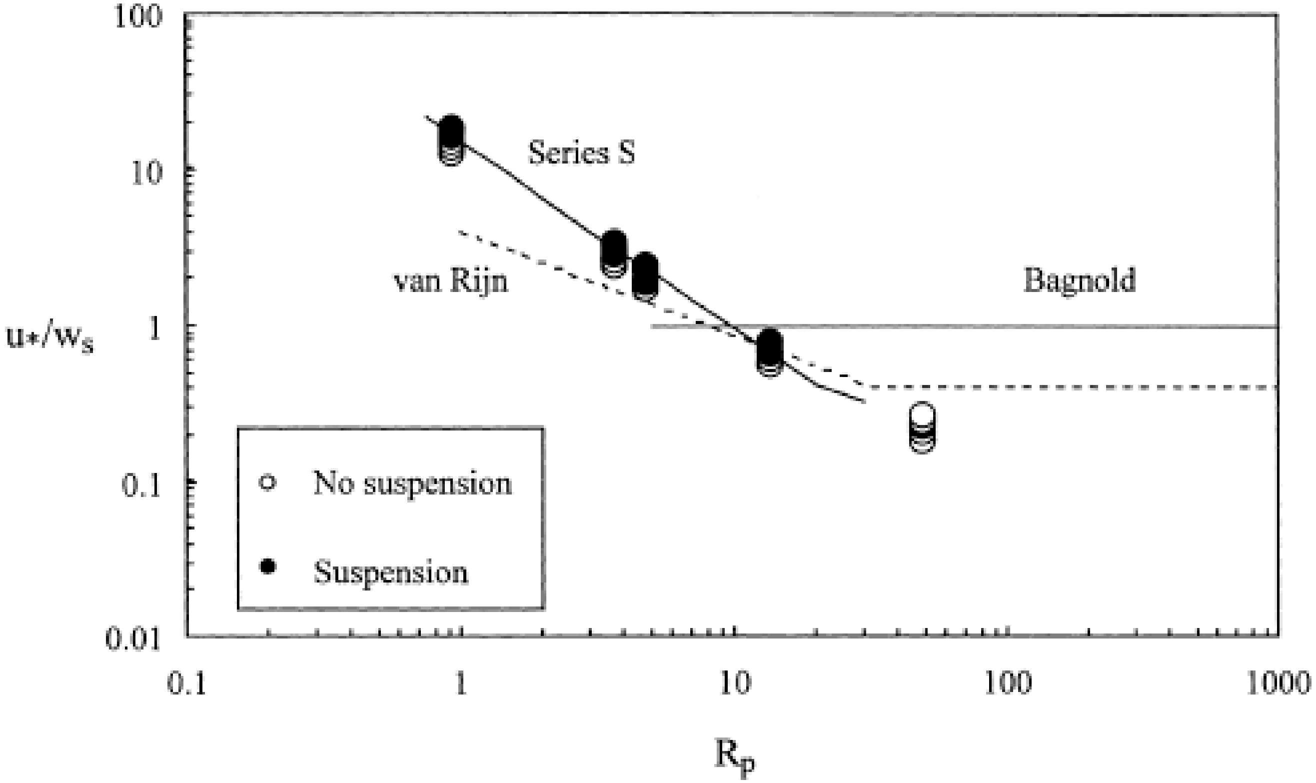
NIÑO ET AL. (2003)
ESTUDIO EXPERIMENTAL

d_p (μm)	w_s (cm s^{-1})	R_p	d_p/d_b
38	0·13	0·9	0·072
94	0·70	3·7	0·177
112	0·96	4·8	0·211
224	3·00	13·5	0·423
530	8·91	49·1	1·000

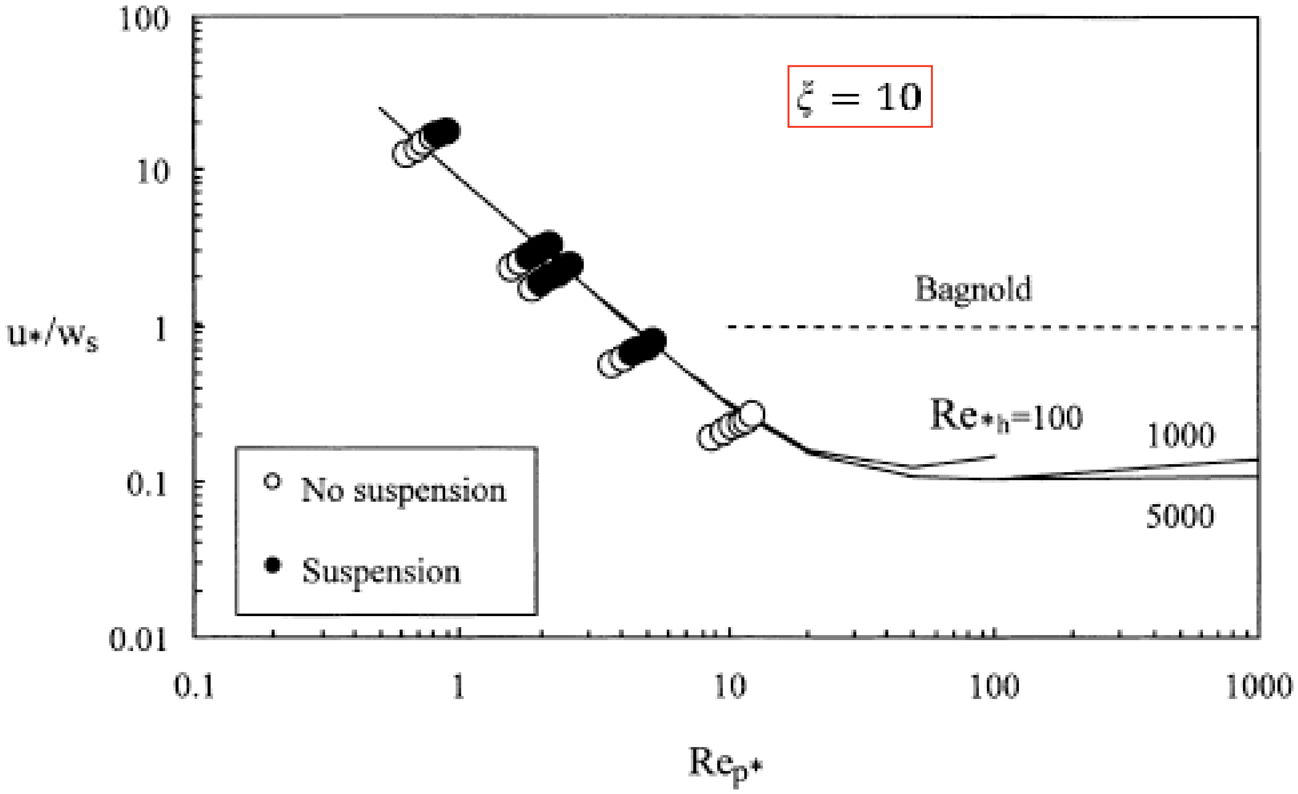
NIÑO ET AL. (2003)
ESTUDIO EXPERIMENTAL



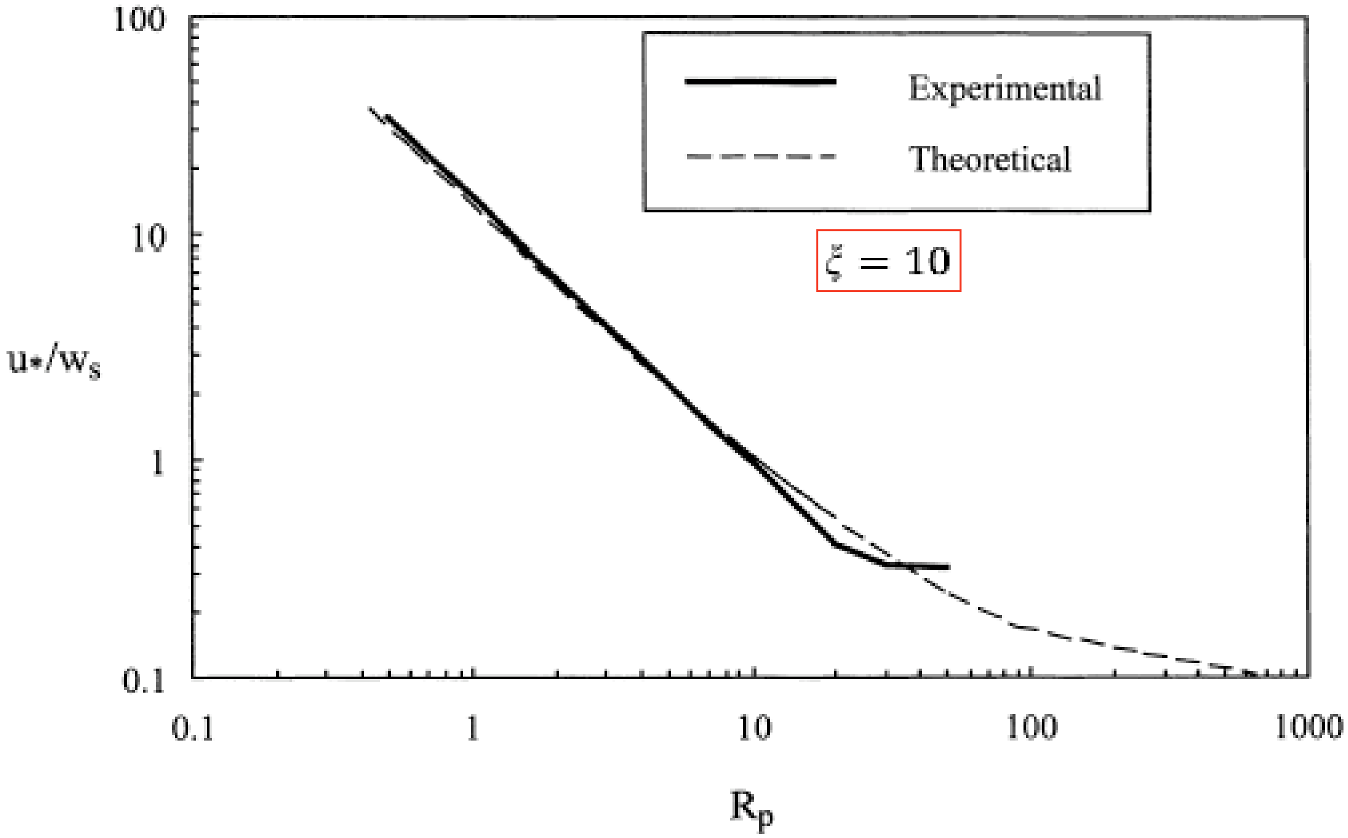
NIÑO ET AL. (2003)
ESTUDIO EXPERIMENTAL



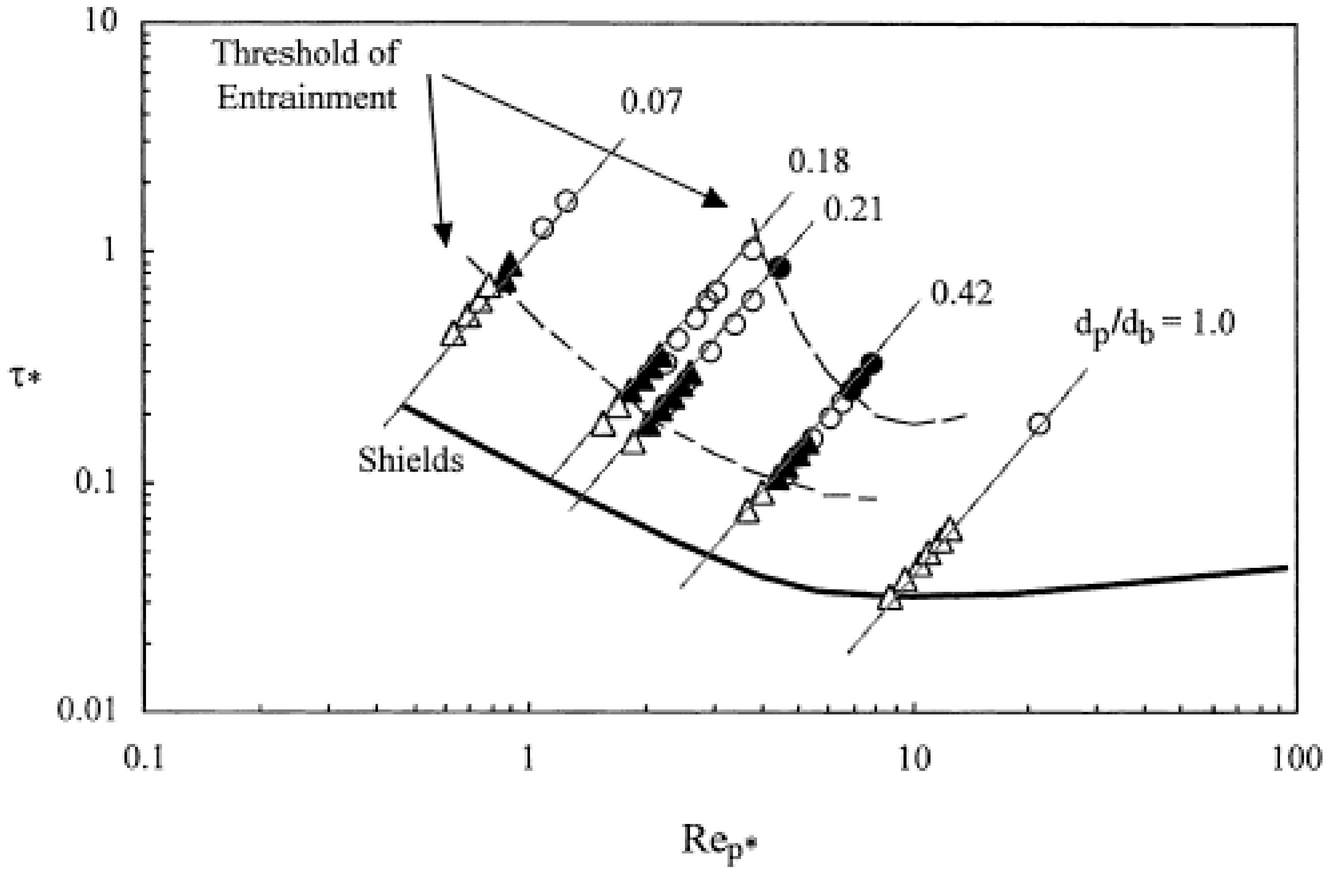
NIÑO ET AL. (2003)
COMPARACION TEORIA V/S ESTUDIO EXPERIMENTAL



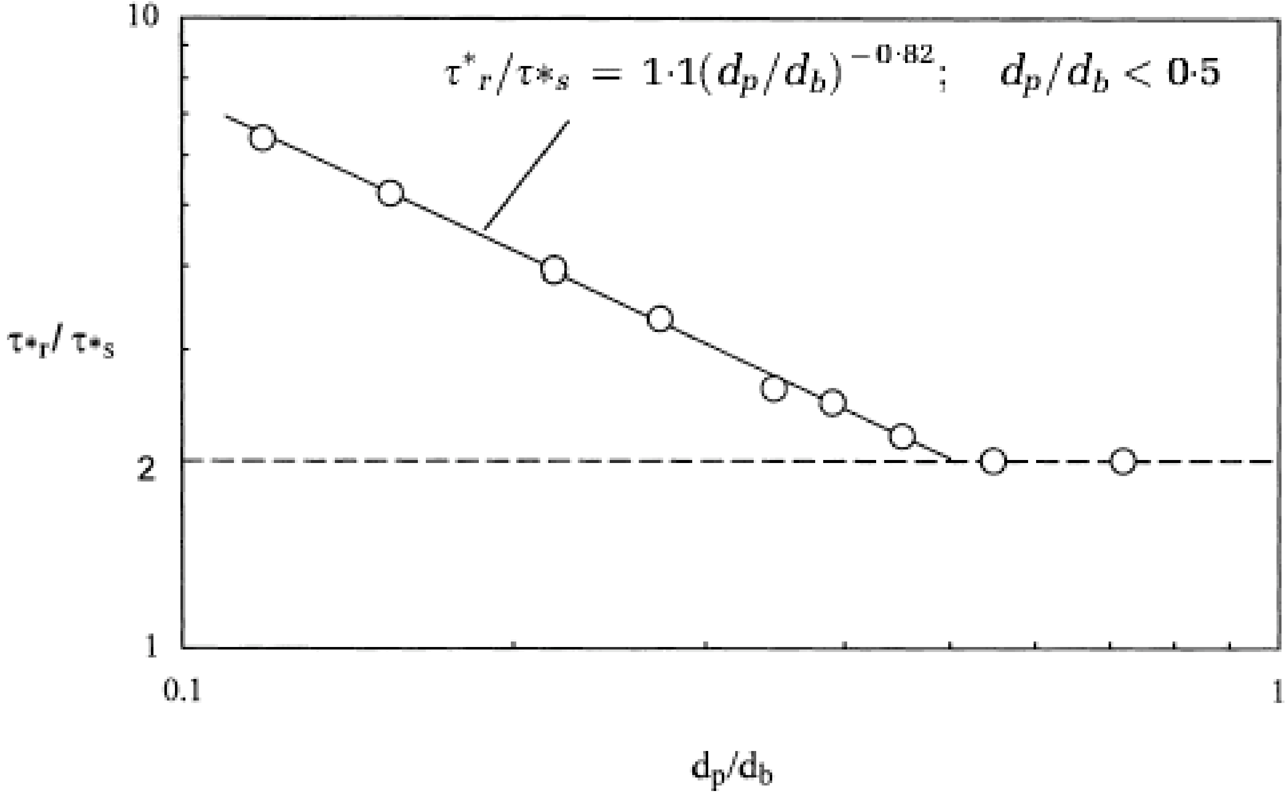
NIÑO ET AL. (2003)
COMPARACION TEORIA V/S ESTUDIO EXPERIMENTAL



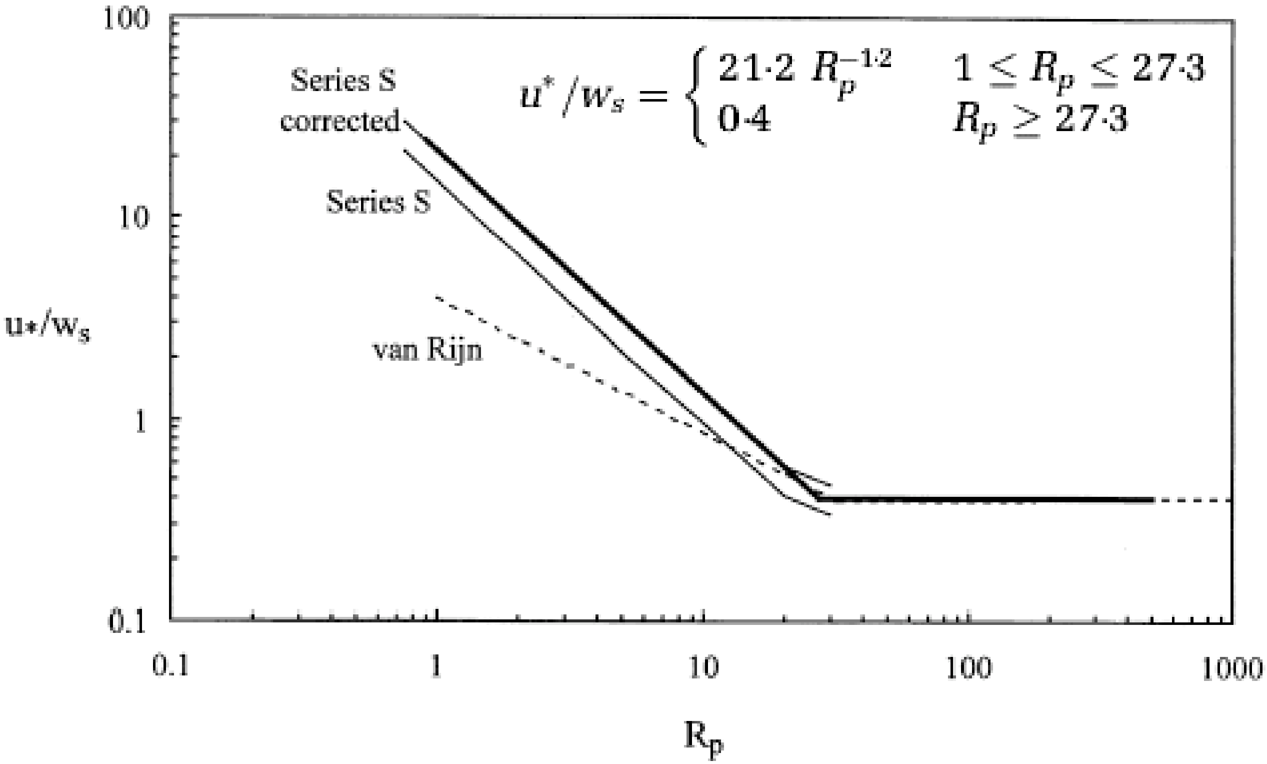
NIÑO ET AL. (2003)
ESTUDIO EXPERIMENTAL



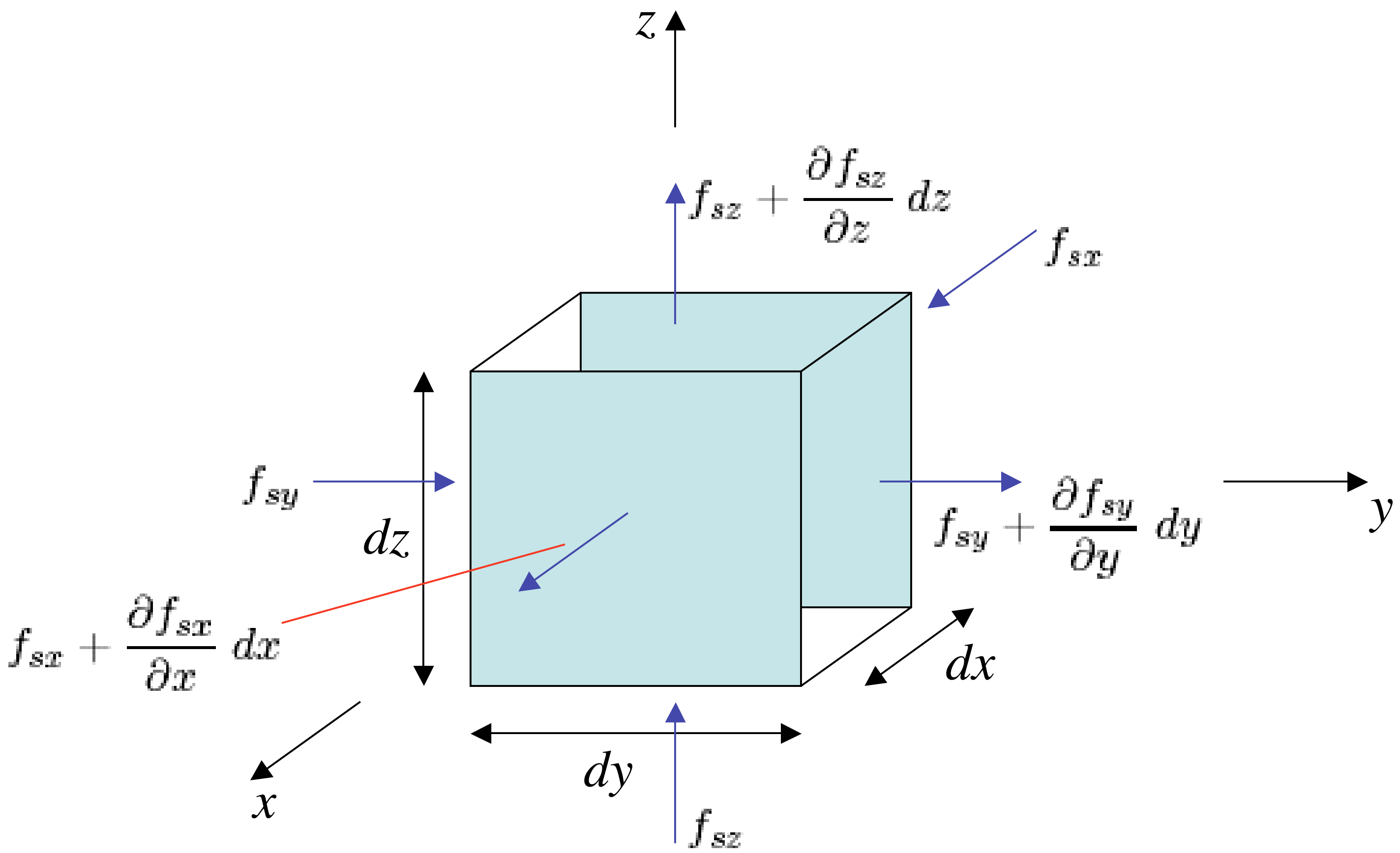
NIÑO ET AL. (2003)
ESTUDIO EXPERIMENTAL



NIÑO ET AL. (2003)
CRITERIO DE UMBRAL DE SUSPENSION



ECUACION DE CONTINUIDAD DEL SEDIMENTO EN SUSPENSION



CONSERVACION DE MASA

$$\frac{\partial C}{\partial t} + \frac{\partial f_{sx}}{\partial x} + \frac{\partial f_{sy}}{\partial y} + \frac{\partial f_{sz}}{\partial z} = 0$$

$$\frac{\partial C}{\partial t} + \frac{\partial f_{si}}{\partial x_i} = 0$$

$$f_{si} = f_{ai} + f_{di}$$

$$f_{ai} = u_{pi}\, C$$ FLUJO ADVECTIVO

$$f_{di} = -D\, \frac{\partial C}{\partial x_i}$$ FLUJO DIFUSIVO

$$\frac{\partial C}{\partial t} + \frac{\partial (u_{pi}\, C)}{\partial x_i} = \frac{\partial}{\partial x_i}(D\, \frac{\partial C}{\partial x_i})$$

VELOCIDADES DE LAS PARTICULAS

$$u_{pi} = u_{fi} - w_s \, \delta_{i3}$$

VELOC. SEDIMENTACION

$$\vec{u}_f = u \, \hat{\imath} + w \, \hat{k}$$

VELOC. FLUJO

$$u_{px} = u \, , \, u_{pz} = w - w_s$$

TURBULENCIA

$$u = \bar{u} + u'$$

$$w = w'$$

$$C = \bar{C} + C'$$

VELOCIDADES DE LAS PARTICULAS

$$u_{px} = \bar{u} + u'$$

$$u_{pz} = w' - w_s$$

PROMEDIANDO LA EC. DE CONSERVACION

$$\frac{\partial \bar{C}}{\partial t} + \frac{\partial \bar{f}_{si}}{\partial x_i} = 0$$

$$\frac{\partial \bar{C}}{\partial t} + \frac{\partial (\overline{u_{pi}\, C})}{\partial x_i} = \frac{\partial}{\partial x_i}(D\, \frac{\partial \bar{C}}{\partial x_i})$$

EC. CONSERVACION SEDIMENTO EN SUSPENSION
PROMEDIADA SOBRE TURBULENCIA

$$\frac{\partial \bar{C}}{\partial t} + \frac{\partial(\bar{u}\,\bar{C})}{\partial x} - \frac{\partial(w_s\,\bar{C})}{\partial z} = \frac{\partial}{\partial x}(D\,\frac{\partial \bar{C}}{\partial x}) + \frac{\partial}{\partial z}(D\,\frac{\partial \bar{C}}{\partial z}) - \frac{\partial(\overline{u'C'})}{\partial x} - \frac{\partial(\overline{w'C'})}{\partial z}$$

$\overline{u'C'}$, $\overline{w'C'}$ FLUJOS MASICOS TURBULENTOS

$$D\,(\frac{\partial^2 \bar{C}}{\partial x^2} + \frac{\partial^2 \bar{C}}{\partial z^2}) << -\frac{\partial(\overline{u'C'})}{\partial x} - \frac{\partial(\overline{w'C'})}{\partial z}$$

$$\frac{\partial(\overline{u'C'})}{\partial x} << \frac{\partial(\overline{w'C'})}{\partial z}$$ APROX. CAPA LIMITE

EC. CONSERVACION SEDIMENTO EN SUSPENSION PROMEDIADA SOBRE TURBULENCIA

$$\frac{\partial \bar{C}}{\partial t} + \frac{\partial (\bar{u}\,\bar{C})}{\partial x} = -\frac{\partial (\overline{w'C'})}{\partial z} + w_s\,\frac{\partial \bar{C}}{\partial z}$$

CAMBIO IMPERMANENTE

ADVECCION X FLUJO MEDIO

DIFUSION VERTICAL TURBULENTA

FLUJO DEPOSICIONAL

CIERRE PARA EL FLUJO TURBULENTO

$$\overline{w'C'} = -\epsilon \frac{\partial \bar{C}}{\partial z}$$

DIFUSIVIDAD TURBULENTA
O DE "REMOLINOS"

$$\frac{\partial \bar{C}}{\partial t} + \frac{\partial (\bar{u}\,\bar{C})}{\partial x} = \frac{\partial}{\partial z}(\epsilon \frac{\partial \bar{C}}{\partial z}) + w_s \frac{\partial \bar{C}}{\partial z}$$

$$\frac{\partial \bar{C}}{\partial t} + \frac{\partial (\bar{u}\,\bar{C})}{\partial x} = \frac{\partial}{\partial z}(\epsilon \frac{\partial \bar{C}}{\partial z} + w_s \bar{C})$$

CASO REGIMEN PERMANENTE Y UNIFORME

$$\frac{\partial}{\partial t} = 0 \quad \frac{\partial}{\partial x} = 0$$

$$\frac{\partial}{\partial z}(\epsilon\,\frac{\partial \bar{C}}{\partial z} + w_s\,\bar{C}) = 0$$

$$\epsilon\,\frac{\partial \bar{C}}{\partial z} + w_s\,\bar{C} = constante\ a\ lo\ largo\ de\ z = -flujo\ neto\ vertical\ hacia\ arriba$$

$$\epsilon\,\frac{\partial \bar{C}}{\partial z} + w_s\,\bar{C} = 0$$

CASO REGIMEN PERMANENTE Y UNIFORME

CONDICIONES DE BORDE:

SUP. LIBRE: $z = h$ $\qquad \epsilon \dfrac{\partial \bar{C}}{\partial z}(h) + w_s \, \bar{C}(h) = 0$

CERCA FONDO: $z = z_{ref}$ $\qquad \epsilon \dfrac{\partial \bar{C}}{\partial z}(z_{ref}) + w_s \, \bar{C}(z_{ref}) = 0$

SEA

$$\begin{cases} \bar{C}(z_{ref}) = C_{ref} & \text{CONC. DE REFERENCIA} \\ -\epsilon \dfrac{\partial \bar{C}}{\partial z}(z_{ref}) = E_s & \text{FLUJO INCORPORACION SED.} \end{cases}$$

$$C_{ref} = \frac{E_s}{w_s}$$

CASO COEFICIENTE DIFUSION TURBULENTA CONSTANTE

$$\epsilon \frac{\partial \bar{C}}{\partial z} + w_s \bar{C} = 0$$

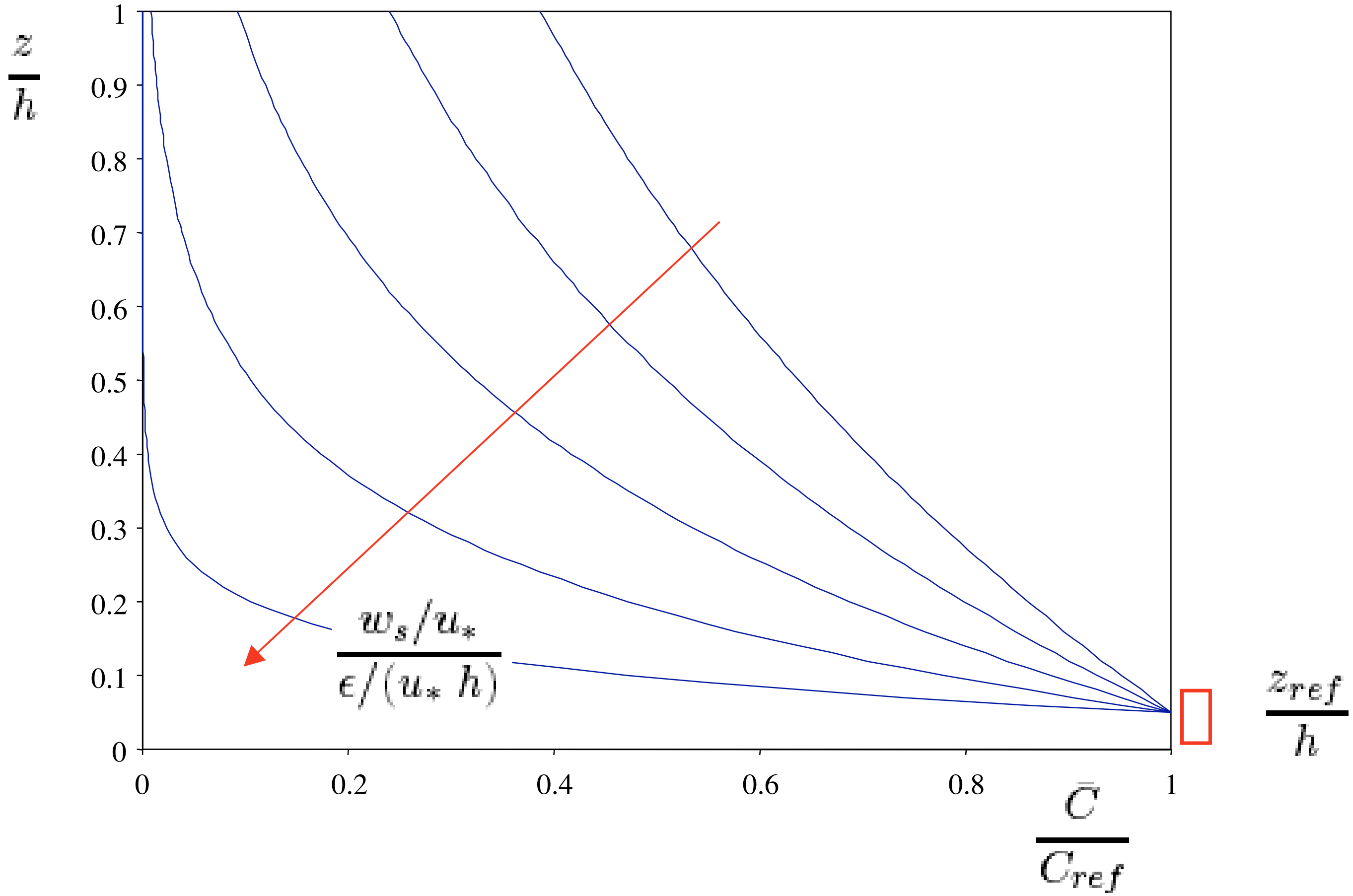
CONSTANTE

$$\frac{d\bar{C}}{\bar{C}} = -\frac{w_s}{\epsilon} dz$$

$$\bar{C} = C_{ref} \exp\left(-\frac{w_s}{\epsilon}(z - z_{ref})\right)$$

CASO COEFICIENTE DIFUSION TURBULENTA CONSTANTE

$$\frac{\bar{C}}{C_{ref}} = \exp\left(-\frac{w_s/u_*}{\epsilon/(u_*\,h)}\left(\frac{z}{h} - \frac{z_{ref}}{h}\right)\right)$$



MODELO DE ROUSE PARA COEFICIENTE DIFUSION TURBULENTA

$$\epsilon = \frac{\nu_T}{Sc_s}$$

VISCOSIDAD DE REMOLINOS (ν_T)

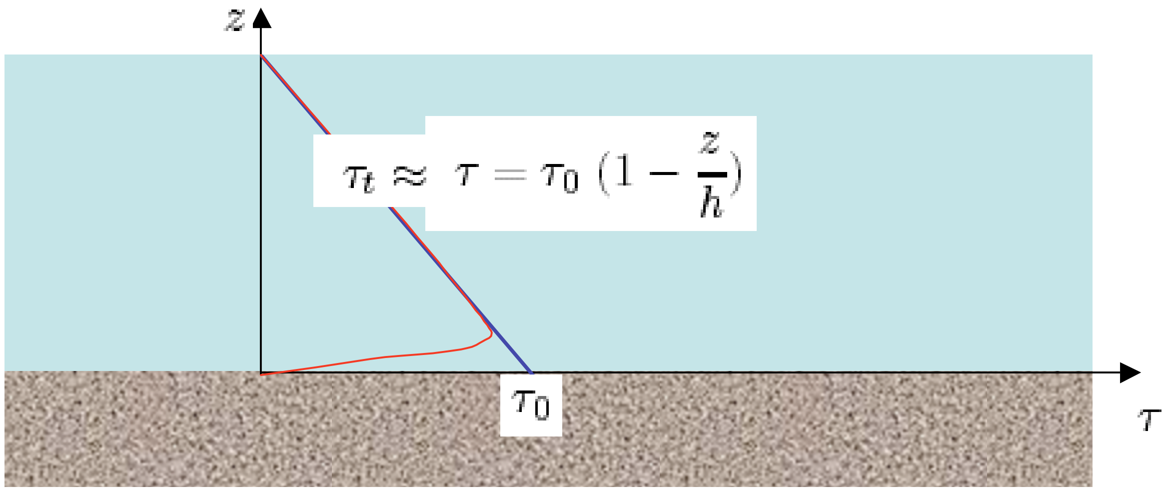
NUMERO DE SCHMIDT (Sc_s)

$$Sc_s \approx 1 \quad \Rightarrow \quad \epsilon = \nu_T$$

MODELO DE ROUSE PARA COEFICIENTE DIFUSION TURBULENTA

$$\tau_t = -\rho\,\overline{u'w'} = \rho\,\nu_T\,\frac{d\bar{u}}{dz}$$

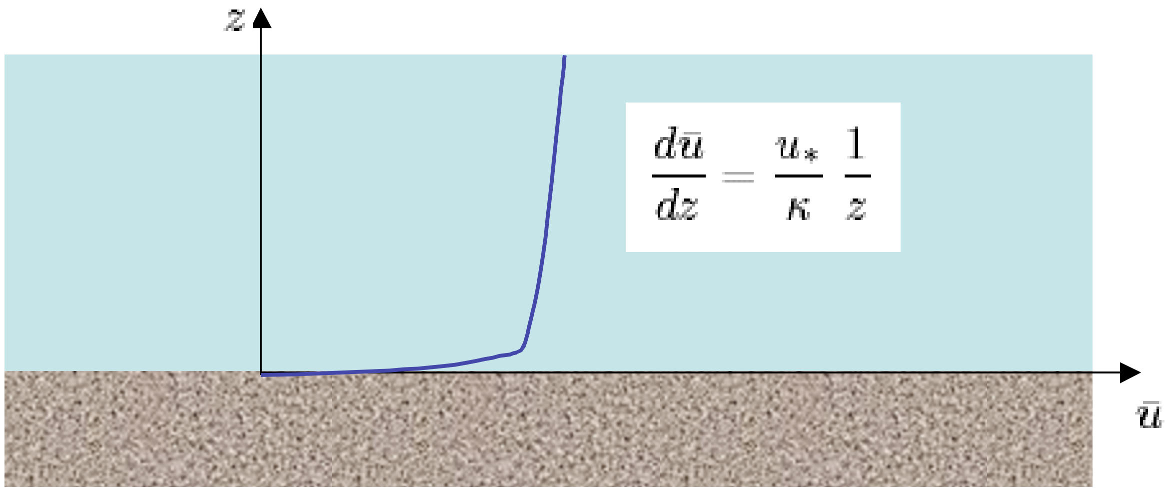
$$\tau = \tau_0\,(1 - \frac{z}{h}) = \rho\,u_*^2\,(1 - \frac{z}{h})$$



MODELO DE ROUSE PARA COEFICIENTE DIFUSION TURBULENTA

$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa} \ln \frac{z\ u_*}{\nu} + C_1$$

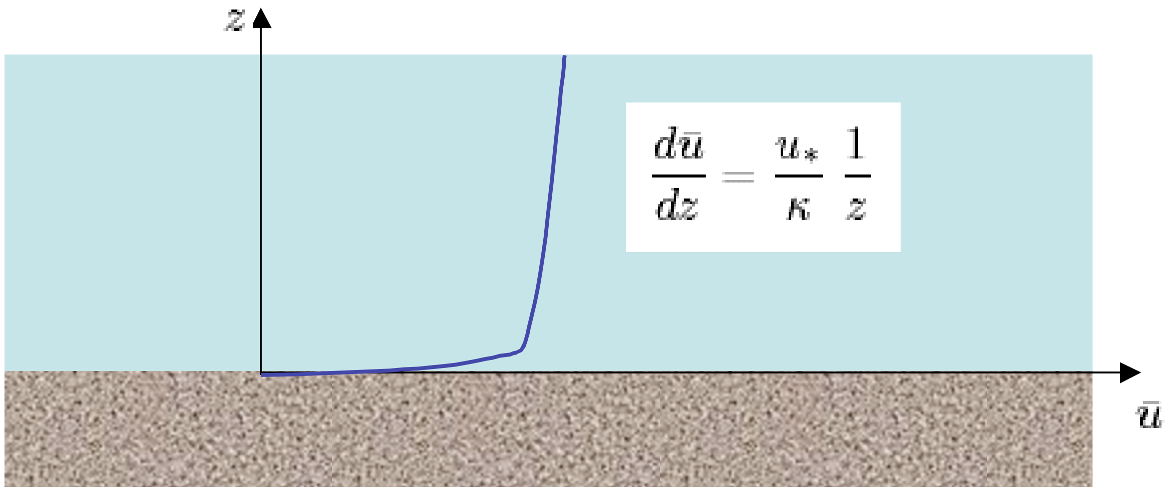
$$\frac{\bar{u}}{u_*} = \frac{1}{\kappa} \ln \frac{z}{k_s} + C_2$$



MODELO DE ROUSE PARA COEFICIENTE DIFUSION TURBULENTA

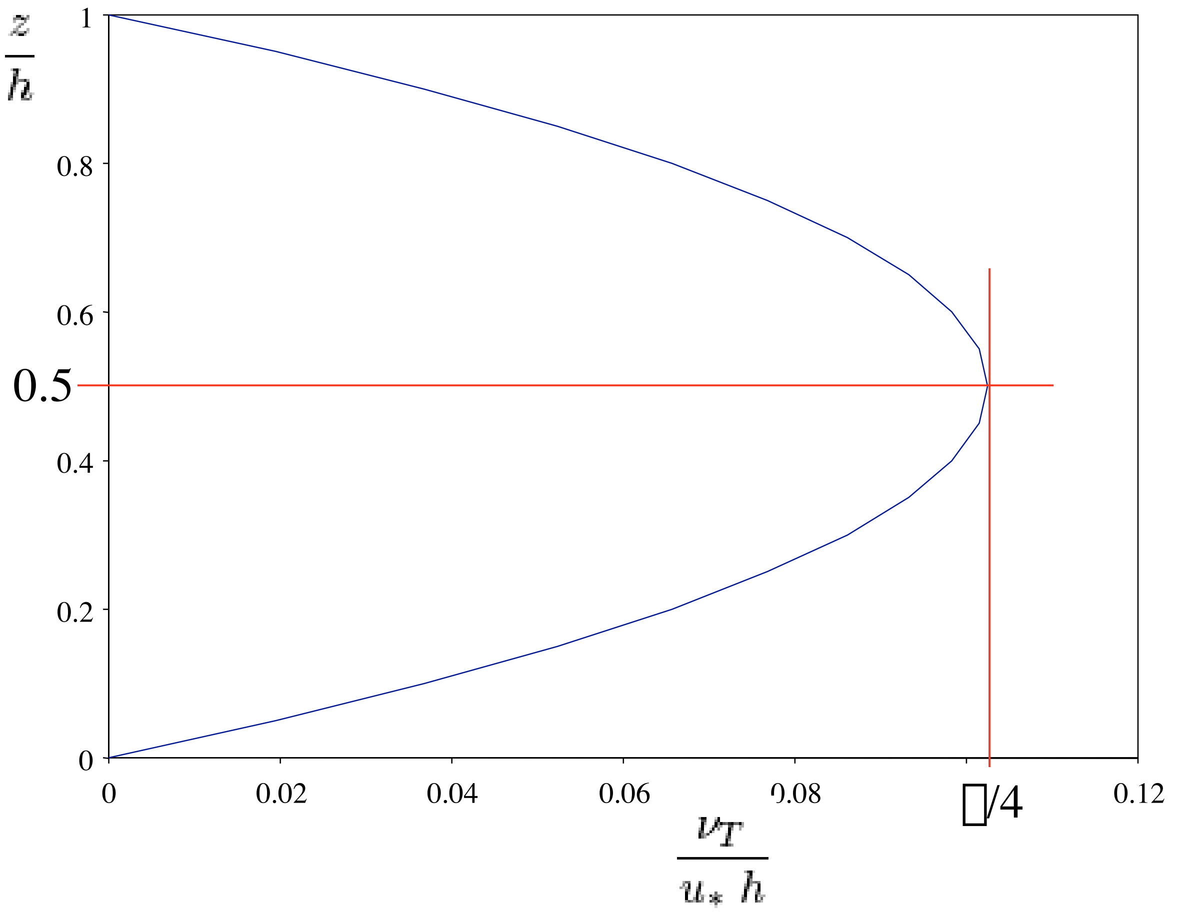
$$\tau_t = -\rho\, \nu_T\, \frac{u_*}{\kappa}\, \frac{1}{z} \approx \rho\, u_*^2\, (1 - \frac{z}{h})$$

$$\nu_T = \kappa\, u_*\, z\, (1 - \frac{z}{h})$$



MODELO DE ROUSE PARA COEFICIENTE DIFUSION TURBULENTA

$$\frac{\nu_T}{u_* \, h} = \kappa \, \frac{z}{h} \, (1 - \frac{z}{h}) \quad \Rightarrow \quad \frac{\epsilon}{u_* \, h} = \kappa \, \frac{z}{h} \, (1 - \frac{z}{h})$$



MODELO DE ROUSE PARA CONCENTRACION SEDIMENTO SUSPENSION

$$\kappa\, u_*\, z\, (1 - \frac{z}{h})\, \frac{d\bar{C}}{dz} + w_s\, \bar{C} = 0$$

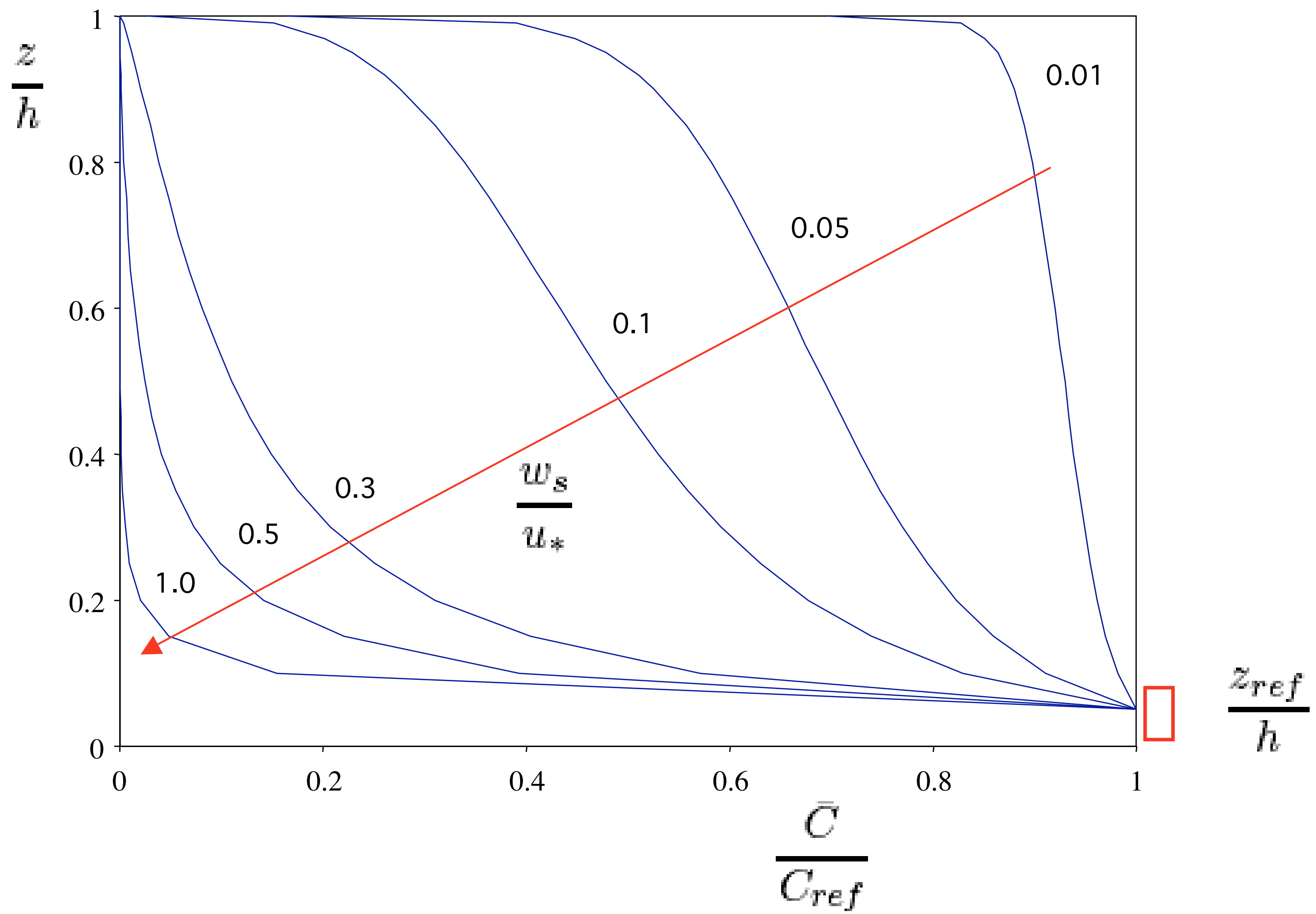
$$\frac{d\bar{C}}{\bar{C}} = -\frac{w_s}{\kappa\, u_*}\, \frac{dz}{z\,(1 - z/h)}$$

$$\int_{C_{ref}}^{\bar{C}} \frac{d\bar{C}}{\bar{C}} = -\frac{w_s}{\kappa\, u_*} \int_{z_{ref}}^{z} \frac{dz}{z\,(1 - z/h)}$$

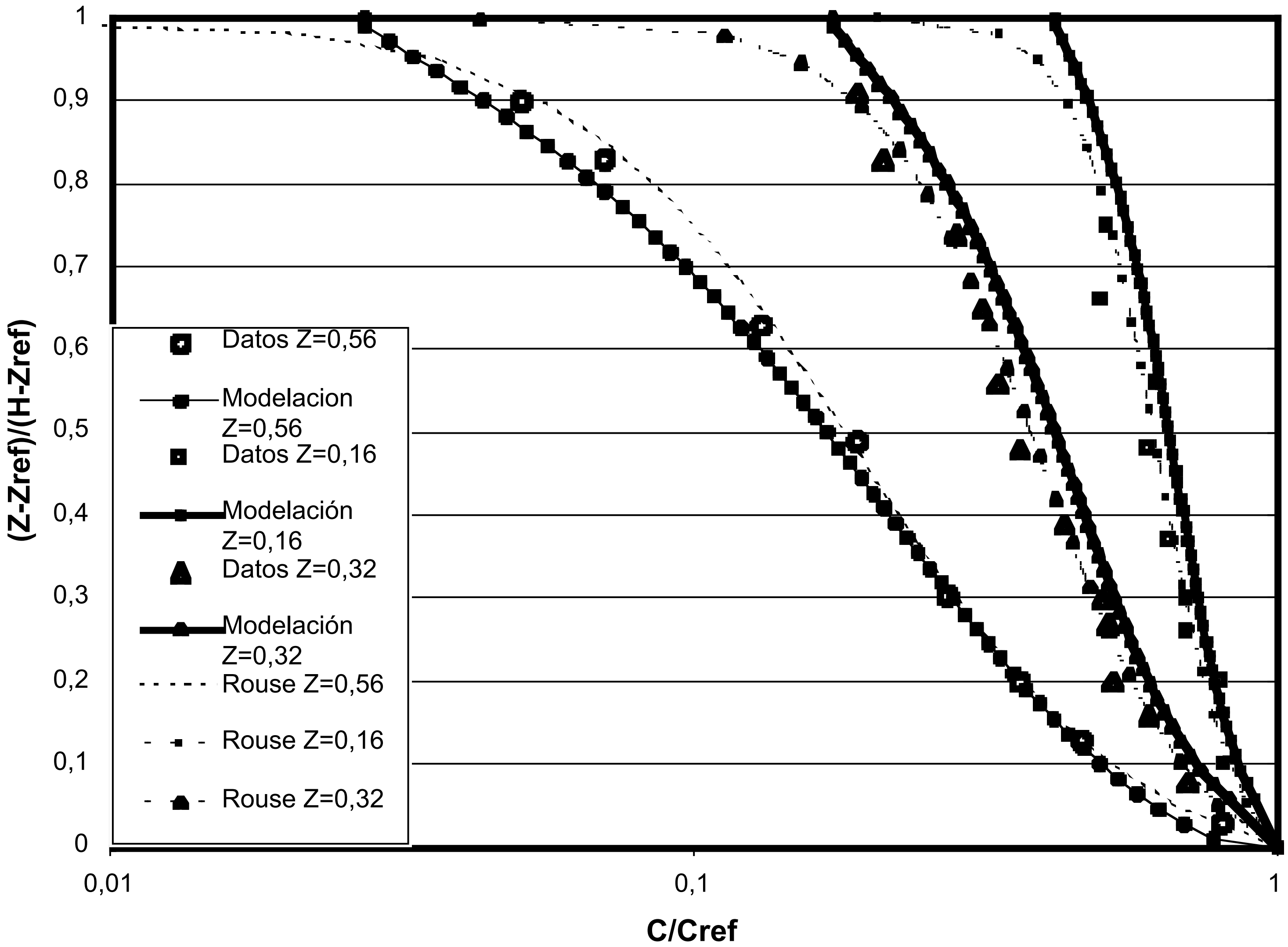
SEA: $\xi = z/h$, $\xi_{ref} = z_{ref}/h$ ➡ $$\frac{\bar{C}}{C_{ref}} = \{\frac{1-\xi}{\xi}\, \frac{\xi_{ref}}{1-\xi_{ref}}\}^{Z}$$

$$Z = \frac{w_s}{\kappa\, u_*} \approx 2.5\, \frac{w_s}{u_*}$$

MODELO DE ROUSE PARA CONCENTRACION SEDIMENTO SUSPENSION



MODELO DE ROUSE PARA CONCENTRACION SEDIMENTO SUSPENSION



MODELO DE ROUSE PARA CONCENTRACION SEDIMENTO SUSPENSION

TIPICAMENTE SE USA: $z_{ref}/h = 0.05$

× DEFINIR: C_{ref}

CALCULO DE GASTO SOLIDO EN SUSPENSION

CONCENTRACION MEDIA EN LA VERTICAL:

$$C_s = \frac{1}{h} \int_{z_ref}^{h} \bar{C}(z)\, dz$$

$$C_s = C_{ref}\, \Im_1$$

$$\Im_1 = \int_{\xi_{ref}}^{1} \{\frac{(1-\xi)}{\xi}\, \frac{\xi_{ref}}{(1-\xi_{ref})}\}^Z\, d\xi$$

CALCULO DE GASTO SOLIDO EN SUSPENSION

GASTO SOLIDO:

$$q_{ss} = \int_{z_{ref}}^{h} \bar{u}(z)\, \bar{C}(z)\, dz$$

PERFIL VELOCIDAD MEDIA

$$\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\, \ln(30\, \frac{z}{k_s})$$

PARED HID. RUGOSA

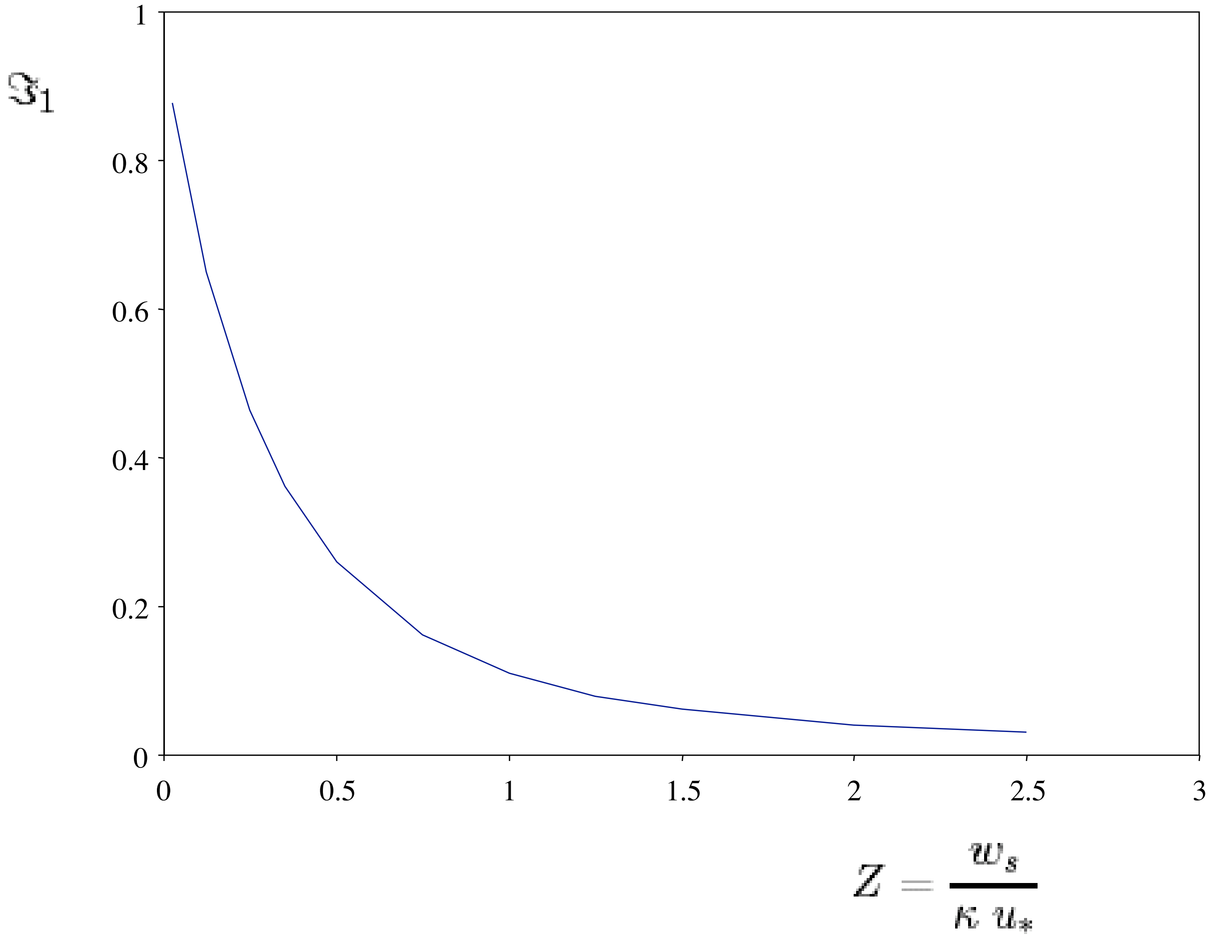
$$q_{ss} = \frac{1}{\kappa}\, C_{ref}\, u_*\, h\, (\Im_1\, \ln(30\, \frac{h}{k_s}) + \Im_2)$$

CALCULO DE GASTO SOLIDO EN SUSPENSION

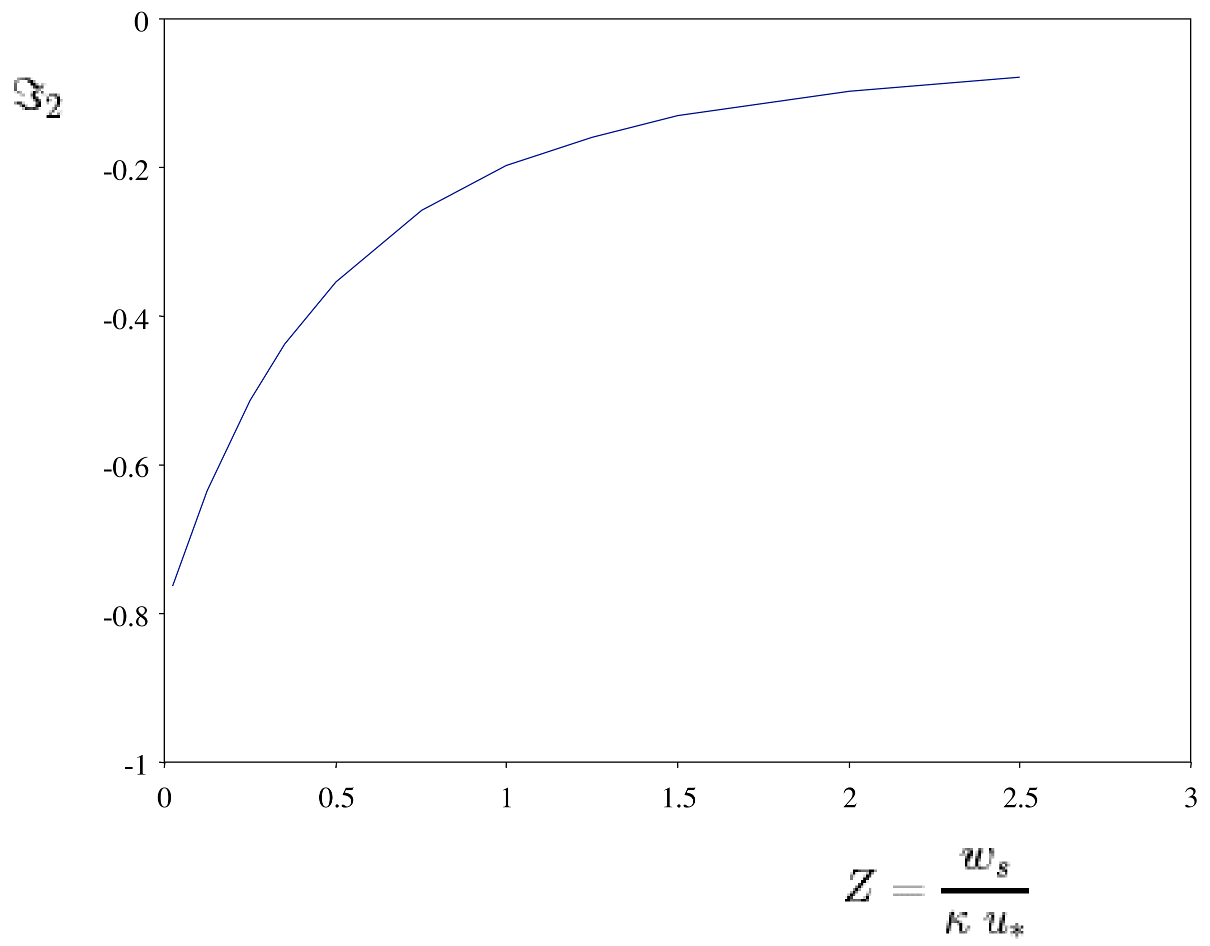
$$\Im_1 = \int_{\xi_{ref}}^{1} \{\frac{(1-\xi)}{\xi} \; \frac{\xi_{ref}}{(1-\xi_{ref})}\}^Z \; d\xi$$

$$\Im_2 = \int_{\xi_{ref}}^{1} \{\frac{(1-\xi)}{\xi} \; \frac{\xi_{ref}}{(1-\xi_{ref})}\}^Z \; \ln(\xi) \; d\xi$$

CALCULO DE GASTO SOLIDO EN SUSPENSION



CALCULO DE GASTO SOLIDO EN SUSPENSION



INTEGRALES DE EINSTEIN

$$\Im_1 = \frac{\xi_{ref}}{0.216} I_1 \qquad \Im_2 = \frac{\xi_{ref}}{0.216} I_2$$

$$I_1 \;,\; I_2 = f(\xi_{ref} \;,\; Z)$$

CONCENTRACION DE REFERENCIA

$$C_{ref}\, w_s = E_s$$

TASA DE INCORPORACION
DE SED. EN SUSPENSION

$$C_{ref} = \frac{E_s}{w_s}$$

CONCENTRACION DE REFERENCIA

GARCIA & PARKER (1991)

$$\xi_{ref} = 0.05$$

$$\frac{E_s}{w_s} = \frac{A\, Z_u^5}{1 + \frac{A}{0.3}\, Z_u^5}$$

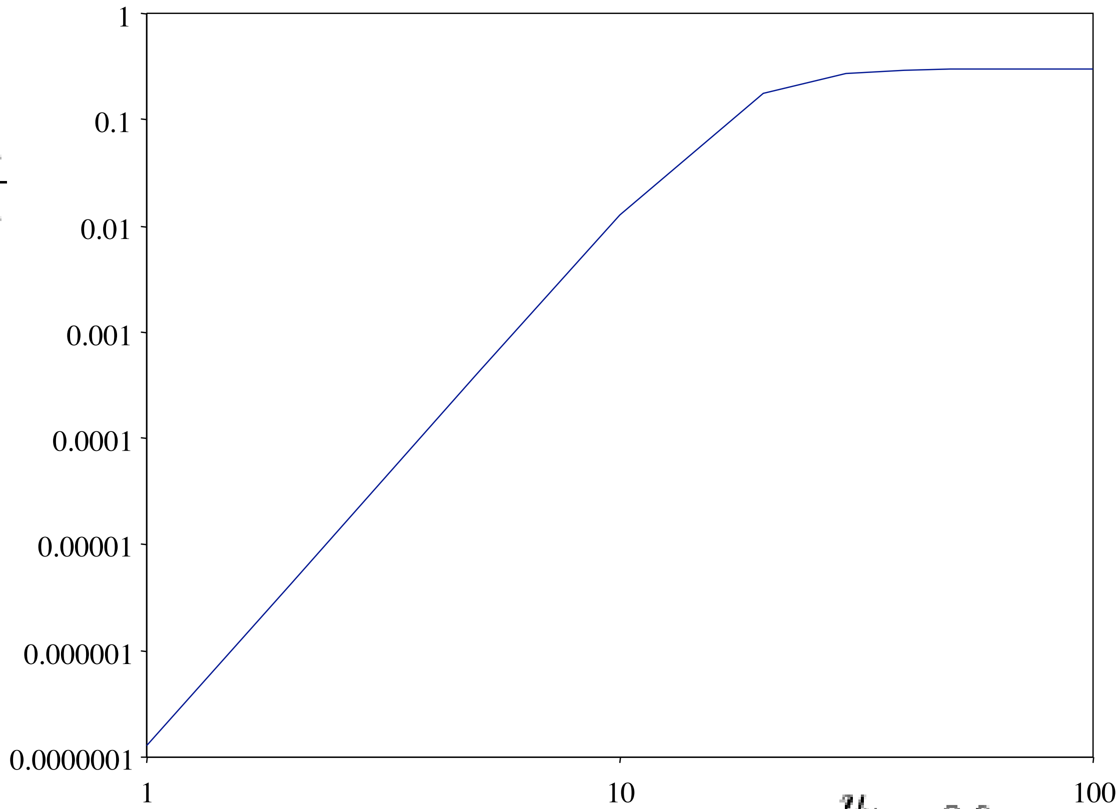
$$A = 1.3 \times 10^{-7}$$

$$Z_u = \frac{u_*}{w_s}\, R_p^{0.6} \qquad R_p = \frac{\sqrt{g\, R\, d_s^3}}{\nu}$$

CONCENTRACION DE REFERENCIA

GARCIA & PARKER (1991)

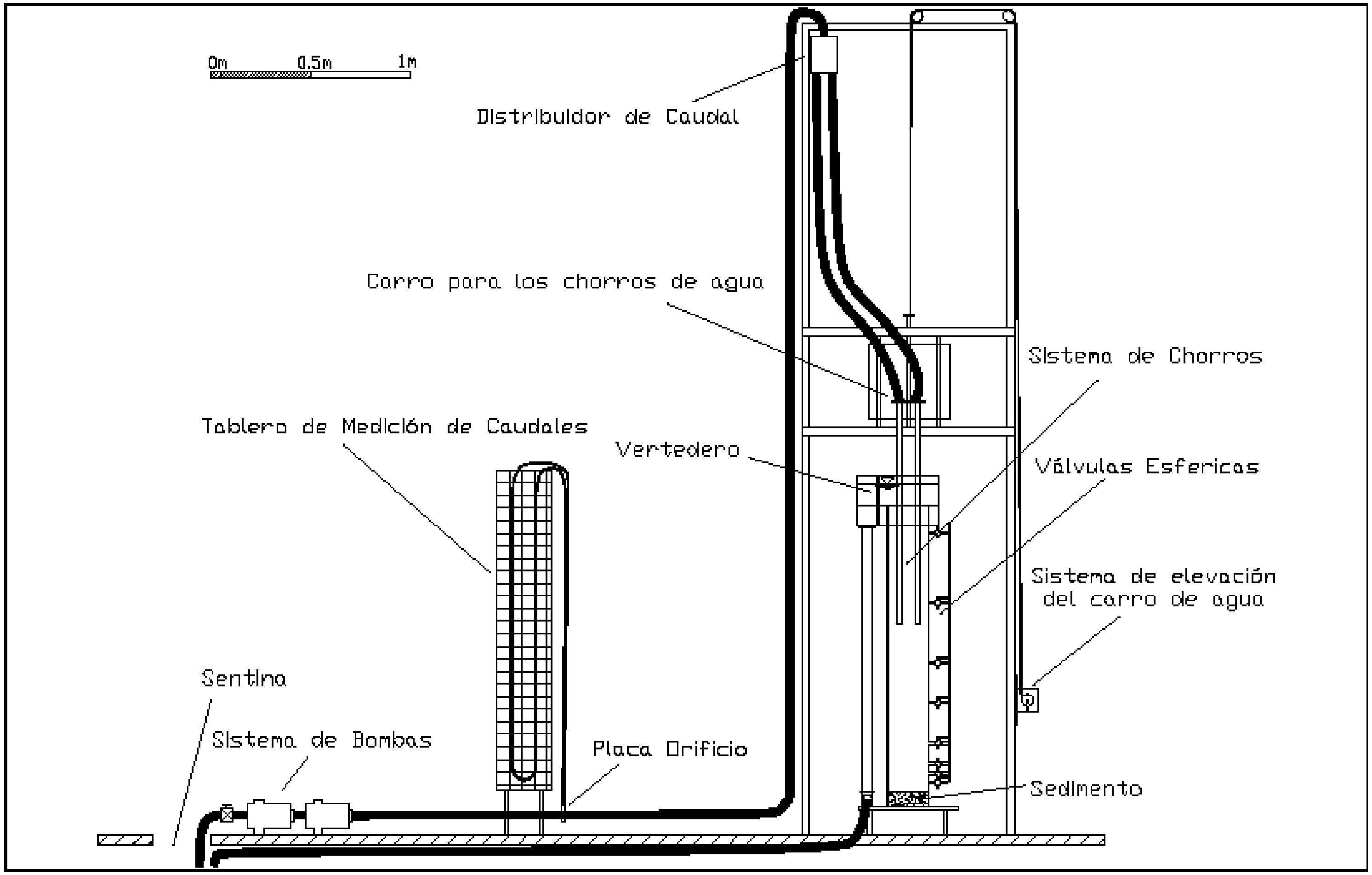
$$C_{ref} = \frac{E_s}{w_s}$$



$$Z_u = \frac{u_*}{w_s} R_p^{0.6}$$

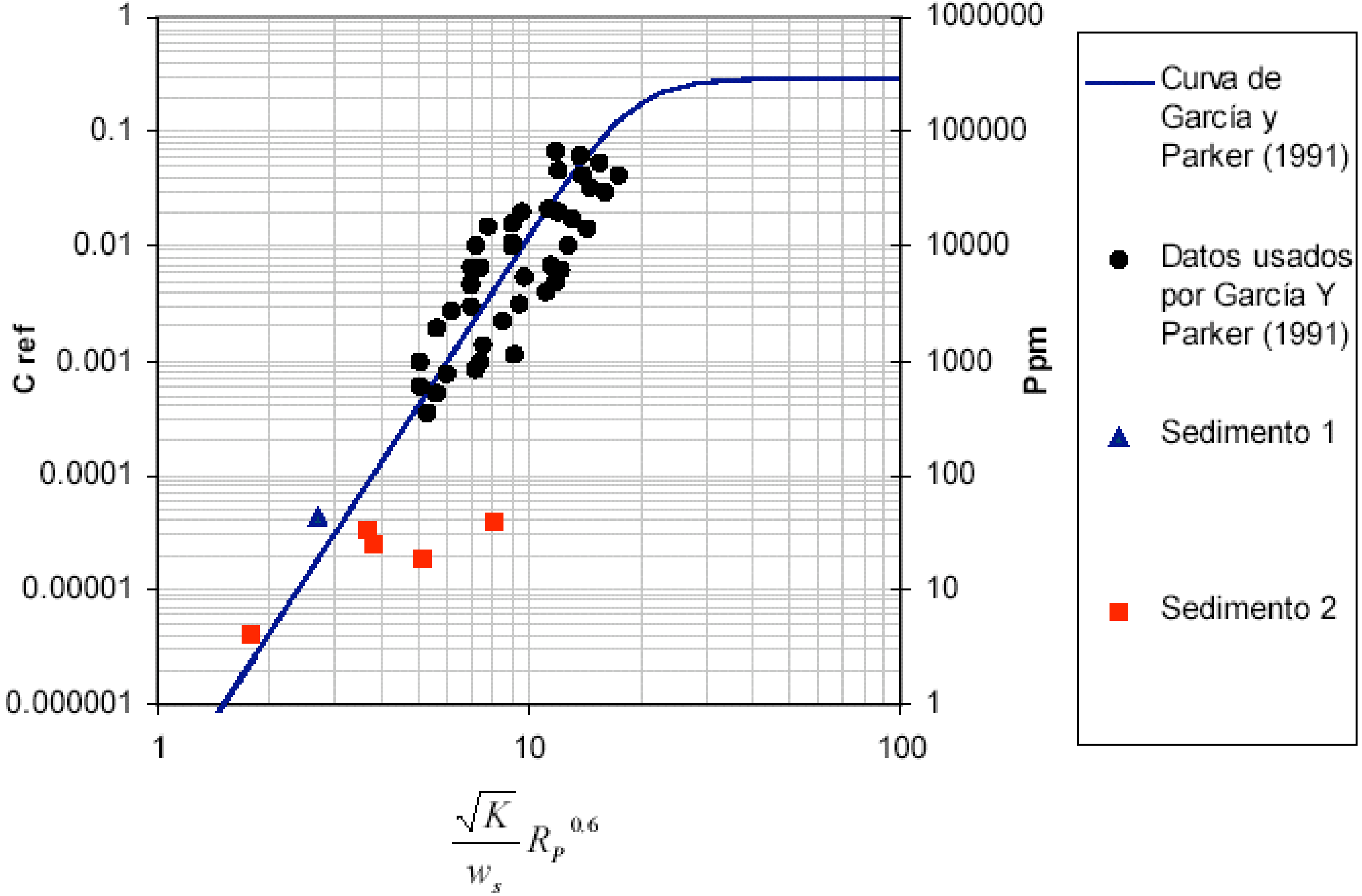
CONCENTRACION DE REFERENCIA

MUÑOZ (2002)



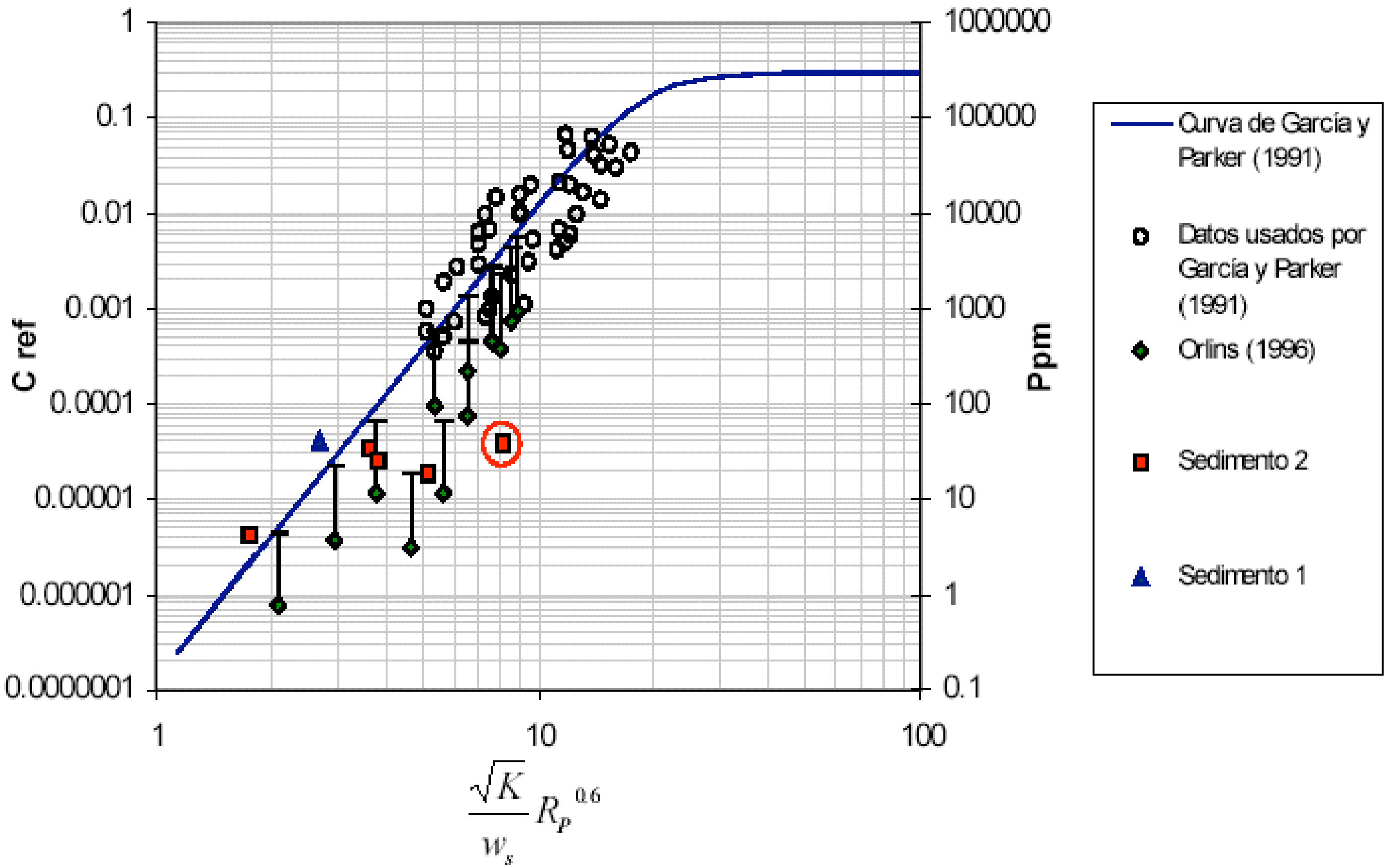
CONCENTRACION DE REFERENCIA

MUÑOZ (2002)



CONCENTRACION DE REFERENCIA

MUÑOZ (2002)



CONCENTRACION DE REFERENCIA

GARCIA & PARKER (1991)

GRANULOMETRIA EXTENDIDA

$$Z_{m,k} = \frac{u'_*}{v_{s,k}} R_{p,k}^{0.6} \left(\frac{D_k}{D_{50}} \right)^m$$

k: FRACCION GRANULOMETRICA

CONCENTRACION DE REFERENCIA

SMITH & MCLEAN (1977)

$$C_{ref} = \frac{0.65\gamma_o S_o}{1+\gamma_o S_o}$$

$$\gamma_o = 2.4x10^{-3} \qquad S_o = \frac{\tau_S^* - \tau_C^*}{\tau_C^*}$$

$$Z_{ref} = \alpha_o\left(\tau_S^* - \tau_C^*\right)D_S + k_S$$

$$\alpha_o = 26.3$$

CONCENTRACION DE REFERENCIA

VAN RIJN (1984)

$$C_{ref} = \frac{0.015 D_S S_o^{1.5}}{Z_{ref} D_*^{0.3}}$$

$$Z_{ref} = 0.5 k_s \qquad Z_{ref\,\mathbf{min}} = 0.01 D_S$$