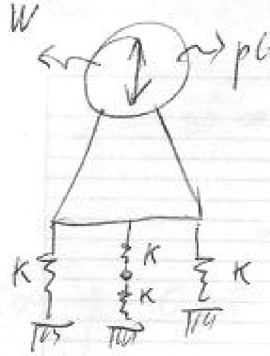
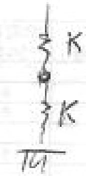
PAUTA EJERCICIO 2 CI4R6

FHP

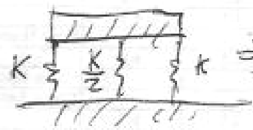


$p(t) = \text{sen}\left(\frac{2\pi}{0{,}9T}\, t\right)$

$\bar{\omega}$



$\Rightarrow \frac{1}{K_{eq}} = \frac{1}{K} + \frac{1}{K} = \frac{2}{K} \Rightarrow K_{eq} = \frac{K}{2}$



$\Rightarrow K_{eq} = K + \frac{K}{2} + K = \frac{5}{2} K = \tilde{K}.$

$\Delta est = \frac{W}{\tilde{K}} = 0{,}01\, m \Rightarrow \tilde{K} = \frac{W}{\Delta est} \Rightarrow K = \frac{2}{5} \cdot \frac{1000\, Kgf}{0{,}01\, m} = 40000\, \frac{Kgf}{m}.$ (1,0)

$y\ \tilde{K} = 100.000\ Kgf/m$

- periodo.

$T = \frac{2\pi}{\omega}, \quad \omega = \sqrt{\frac{K}{m}}, \quad T = 2\pi \cdot \sqrt{\frac{m}{K}} = 2\pi \cdot \sqrt{\frac{1000/9{,}8\ \frac{Kgf \cdot s^2}{m}}{100.000\ Kgf/m}}$

$\Rightarrow T = 0{,}2\ seg.$ (0,5)

- Respuesta permanente:

$$v_p = \frac{p_0}{K} D \cdot \text{sen}(\bar{\omega} t - \theta)$$

$$D = \frac{1}{\sqrt{(1-r^2)^2 + (2\beta r)^2}} \qquad \theta = \text{atan}\left(\frac{2\beta r}{1-r^2}\right)$$

$$\frac{\overline{w}}{w} \quad , \quad \overline{w} = \frac{2\pi}{0{,}9 \cdot T} \qquad T = \frac{2\pi}{w}$$

$$\Rightarrow \overline{w} = \frac{2\pi}{0{,}9 \cdot \frac{2\pi}{w}} \Rightarrow \frac{\overline{w}}{w} = \frac{1}{0{,}9} = 1{,}11 = \gamma$$

Con $\beta = 0{,}05$, se puede evaluar el FAD y el desfase del régimen permanente:

$$D = \frac{1}{\sqrt{(1-(1{,}1)^2)^2 + (2 \cdot 0{,}05 \cdot 1{,}1)^2}} = 3{,}86 \quad , \quad \overline{w} = \frac{2\pi}{0{,}9 \times 0{,}2\,sg} = 11{,}11\pi$$

$$\theta = -25{,}4^\circ \qquad \Delta_{est} = \frac{1}{K} = 0{,}01\ mm$$

$$\Rightarrow v_p(t) = 0{,}01\,mm\ 3{,}86 \times \operatorname{sen}\left(11{,}1\pi\, t + 25{,}4^\circ\right)$$

$$\Rightarrow v_p(t) = 0{,}0386\,mm\ \operatorname{sen}\left(11{,}1\pi\, t + 25{,}4^\circ\right)$$

2,0

Respuesta homogenea:

$$v_h(t) = \rho \cdot e^{-\beta w t} \cdot \cos(w_D t - \varphi)$$

$$\underline{w_D = w\sqrt{1-\beta^2}}$$

Luego: $v(t) = v_h(t) + v_p(t)$

$$v(t) = \rho e^{-\beta w t} \cdot \cos(w_D t - \varphi) + \frac{\overbrace{p_0}^{\Delta est}}{\tilde{k}} \cdot D \cdot \operatorname{sen}\left(\bar{w} t - \theta\right)$$

• Se deben imponer las condiciones iniciales $v_0 = \dot{v}_0 = 0$

$$\dot{v}(t) = -\rho e^{-\beta w t} \cdot \beta w \cdot \cos(w_D t - \varphi) - \rho w_D e^{-\beta w t} \cdot \operatorname{sen}(w_D t - \varphi)$$

$$+ \Delta est \, D \bar{w} \cos\left(\bar{w} t - \theta\right)$$

Evaluando en $t=0$,

$$\rho \cdot \cos(-\varphi) + \Delta est \cdot D \operatorname{sen}(-\theta) = v_0 = 0 \qquad (1)$$

$$-\beta w \rho \cos(-\varphi) - \rho w_D \operatorname{sen}(-\varphi) + \Delta est \, D \bar{w} \cos(-\theta) = 0 \qquad (2)$$

Tomando la primera ecuación

$$\cos(-\varphi) = \frac{-\Delta est \, D \operatorname{sen}(-\theta)}{\rho}$$

Reemplazando en la segunda $\cos(-\varphi)$

$$\Rightarrow -\beta w \cancel{\varphi} \cdot \left(\frac{-\Delta est \cdot D\, sen(-\theta)}{\cancel{\varphi}} \right) - \varphi\, w\, sen(-\varphi)$$

$$+ \Delta est\, D\, \bar{w}\, \cos(-\theta) = 0$$

$$\Rightarrow \beta w\, \Delta est\, D\, sen(-\theta) - \varphi\, w_D\, sen(-\varphi) + \Delta est\, D\, \bar{w} \cos(-\theta) = 0$$

$$\Rightarrow \varphi\, w_D\, sen(-\varphi) = \beta w\, \Delta est\, D\, sen(-\theta) + \Delta est\, D\, \bar{w} \cos(-\theta) \quad (3)$$

y de (1) $\quad \varphi \cos(-\varphi) = -\Delta est\, D\, sen(\theta) \quad / \cdot w_D$

$$\varphi\, w_D \cos(-\varphi) = -\Delta est\, w_D\, D\, sen(\theta) \quad (4)$$

$(3)^2 + (4)^2$

$$\varphi^2 w_D^2 = \left(\beta w\, \Delta est\, D\, sen(-\theta) + \Delta est\, D\, \bar{w} \cos(-\theta) \right)^2 + \left(\Delta est\, w\, D\, sen(\theta) \right)^2$$

$$\Rightarrow \varphi^2 w_D^2 = \beta^2 w^2 \Delta est^2 D^2 sen^2(-\theta) + \Delta est^2 D^2 \bar{w}^2 \cos(-\theta)^2 + 2 \beta w \bar{w} D^2 \Delta est^2\, sen(-\theta) \cos(-\theta) + \Delta est^2 D^2 w^2 sen^2(-\theta)$$

$$\varphi^2 w_D^2 = \Delta est^2 D^2 \cdot \left\{ w^2 (1 + \beta^2)\, sen^2(-\theta) + \bar{w}^2 \cos^2(-\theta) + 2 \beta w \bar{w}\, sen(-\theta) \cos(-\theta) \right\}$$

$$\Rightarrow \varphi^2 = \frac{\Delta est^2 D^2}{(1-\beta^2)} \left\{ (1+\beta^2)\, sen^2(-\theta) + \gamma^2 \cos^2(-\theta) + 2 \beta \gamma\, sen(-\theta) \cos(-\theta) \right\}$$

$$\Rightarrow \varphi = \frac{\Delta est\, D}{\sqrt{1-\beta^2}} \cdot \sqrt{(1+\beta^2)\, sen^2(-\theta) + \gamma^2 \cos^2(-\theta) + 2 \beta \gamma\, sen(-\theta) \cos(-\theta)}$$

$\Rightarrow$ pl. sanho

$\Delta_{EST} = 0{,}01\ m$ $\quad D = 3{,}86 \quad \beta = 0{,}05 \quad \gamma = \frac{1}{0{,}9} = 1{,}1$

$\theta = -25{,}4^{o}$

2,0

$\Rightarrow \underline{\rho = 0{,}043\ mm}$

- Tiempo en que la amplitud ρ se reduce a 1%

$$\Rightarrow \frac{\rho e^{-\beta w t_1}}{\rho} = 0{,}01$$

$$\Rightarrow e^{-\beta w t_1} = 0{,}01 \Rightarrow t_1 = \frac{-\ln(0{,}01)}{\beta w}$$

$\beta = 0{,}05 \quad w = \frac{2\pi}{T} = \sqrt{\frac{100000\ Kg/m}{1000/9{,}8\ Kg\,s^2/m}} = 31{,}3\ rad/seg.$

$\Rightarrow \underline{t_1 = 2{,}94\ seg \approx 3\ seg.}$ 0,5

En tres segundos desaparese la parte transiente para una estructura de un periodo igual a 0,2 seg y una razón de amortiguamiento igual a un 5%.