P3 Control 1 - CVV MAZZA. Prof. J. Dávila.

(i) (1,5). En $\mathbb{R}^2\setminus\{(0,0)\}$ es continua por Alg. y composición de fnes continuas. 0,2

$$\lim_{(x,y)\to(0,0)} \frac{\log(1+x^2+y^2)+x^3+y^3}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{\log(1+x^2+y^2)}{x^2+y^2} + \frac{x^3+y^3}{x^2+y^2}$$

$$\overset{t=x^2+y^2}{=} \underbrace{\lim_{t\to 0} \frac{\log(1+t)}{t}}_{1.} + \lim_{(x,y)\to 0} \frac{x^3+y^3}{x^2+y^2}$$

0,6

$$\left|\frac{x^3+y^3}{x^2+y^2}\right| = \frac{|x+y||x^2-xy+y^2|}{|x^2+y^2|} \leq \frac{|x+y||x^2+y^2| + |xy||x+y|}{|x^2+y^2|} = |x+y| + \frac{|xy||x+y|}{|x^2+y^2|}$$

pero

$$\left.\begin{array}{l} x^2+y^2 \geq 2xy \\ x^2+y^2 \geq -2xy \end{array}\right\} \Rightarrow x^2+y^2 \geq 2|xy| \therefore \frac{1}{x^2+y^2} = \frac{1}{|x^2+y^2|} \leq \frac{1}{2|xy|}$$

$$\Rightarrow \frac{|xy||x+y|}{|x^2+y^2|} \leq \frac{|x+y|}{2}$$

$$\therefore \left|\frac{x^3+y^3}{x^2+y^2}\right| \leq |x+y| + \frac{|x+y|}{2} = \frac{3}{2}|x+y| \xrightarrow[(x,y)\to 0]{} 0$$

0,7

$$\therefore \lim_{(x,y)\to(0,0)} f(x,y) = 1. \quad \therefore \boxed{a=1}.$$

(ii)

$$\frac{\partial f}{\partial x}(x,y) = \frac{\left(\frac{2x}{1+x^2+y^2}+3x^2\right)(x^2+y^2) - \left(\log(1+x^2+y^2)+x^3+y^3\right)2x}{(x^2+y^2)^2}$$

$$\frac{\partial f}{\partial y}(x,y) = \frac{\left(\frac{2y}{1+x^2+y^2}+3y^2\right)(x^2+y^2) - \left(\log(1+x^2+y^2)+x^3+y^3\right)2y}{(x^2+y^2)^2}$$

1,5

(iii)

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{\log(1+h^2)+h^3}{h^2} - 1}{h} = \lim_{h\to 0} \frac{\log(1+h^2)+h^3-h^2}{h^3}$$

0,5

$$\underset{\text{l'Hôp}}{=} \lim_{h\to 0} \frac{\frac{2h}{1+h^2}+3h^2-2h}{3h^2} = \lim_{h\to 0} \frac{\cancel{2h}+3h^2+3h^4-\cancel{2h}-2h^3}{3h^2(1+h^2)} = \lim_{h\to 0} \frac{\cancel{3h^2}(1+h^2-\frac{2}{3}h)}{\cancel{3h^2}(1+h^2)} = 1.$$

1,0

$$\frac{\partial f}{\partial y}(0,0) = 1 \quad \underline{\text{Análogo!}}$$

(iii)

Tomemos $(h,0)$, $h\in\mathbb{R}$

En $\mathbb{R}\setminus\{(0,0)\}$ $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ son continuas x álg. y composición de fns continuas (0,8)

$$\lim_{h\to 0} \frac{\partial f}{\partial x}(h,0) = \frac{\left(\frac{2h}{1+h^2}+3h^2\right)h^2 - \left(\log(1+h^2)+h^3\right)2h}{h^4}$$

$$= 3 + \frac{2\cancel{h}\left(\frac{h^2}{1+h^2} - \left(\log(1+h^2)+h^3\right)\right)}{h^{\cancel{4}\,3}}$$

$$= 3 + \frac{2}{h}\left(\underbrace{\frac{1}{1+h^2}}_{\downarrow\, 1} - \underbrace{\frac{\log(1+h^2)+h^3}{h^2}}_{\downarrow\, 1}\right)$$

vamos l'Hôp (*)

$$\overset{\downarrow}{=} 3 + (-2) = 1$$

y si tomamos $(0,h)$, $h\in\mathbb{R}$

$$\frac{\partial f}{\partial x}(0,h) = 0 \quad \forall h\in\mathbb{R} \Rightarrow \lim_{h\to 0}\frac{\partial f}{\partial x}(0,h) = 0$$

(0,7)

$\Rightarrow$ No es continua en $(0,0)$!

$\frac{\partial f}{\partial x}$ y $\frac{\partial f}{\partial y}$ ~~son~~ son continuas en $\mathbb{R}\setminus\{(0,0)\}$.