P5 Sea $A = \begin{bmatrix} -2 & 2 & 0 \\ 2 & -4 & 2 \\ 0 & 2 & -2 \end{bmatrix}$

Determine una base ortonormal de vectores propios de $A$.

Sol:

$$\det(A - \lambda I) = 0$$

$$\begin{bmatrix} -2-\lambda & 2 & 0 \\ 2 & -4-\lambda & 2 \\ 0 & 2 & -2-\lambda \end{bmatrix} = 0 \Rightarrow -(\lambda+2)\lambda(\lambda+6)$$

$\lambda_1 = -2$ mult alg = 1.
$\lambda_2 = 0$ mult alg = 1.
$\lambda_3 = -6$ mult alg = 1

$\lambda_1 = -2$

$$\begin{bmatrix} 0 & 2 & 0 \\ 2 & -2 & 2 \\ 0 & 2 & 0 \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} V_2 = 0 \\ V_1 - V_2 + V_3 = 0 \\ V_2 = 0 \end{matrix}$$