The General Theory of Second Best[1]

There is an important basic similarity underlying a number of recent works in apparently widely separated fields of economic theory. Upon examination, it would appear that the authors have been rediscovering, in some of the many guises given it by various specific problems, a single general theorem. This theorem forms the core of what may be called *The General Theory of Second Best.* Although the main principles of the theory of second best have undoubtedly gained wide acceptance, no general statement of them seems to exist. Furthermore, the principles often seem to be forgotten in the context of specific problems and, when they are rediscovered and stated in the form pertinent to some problem, this seems to evoke expressions of surprise and doubt rather than of immediate agreement and satisfaction at the discovery of yet another application of the already accepted generalizations.

In this paper, an attempt is made to develop a *general* theory of second best. In Section I there is given, by way of introduction, a verbal statement of the theory's main general theorem, together with two important negative corollaries. Section II outlines the scope of the general theory of second best. Next, a brief survey is given of some of the recent literature on the subject. This survey brings together a number of cases in which the general theory has been applied to various problems in theoretical economics. The implications of the general theory of second best for piecemeal policy recommendations, especially in welfare economics, are considered in Section IV. This general discussion is followed by two sections giving examples of the application of the theory in specific models. These examples lead up to the general statement and rigorous proof of the central theorem given in Section VII. A brief consideration of the existence of second best solutions is followed by a classificatory discussion of the nature of these solutions. This taxonomy serves to illustrate some of the important negative corollaries of the theorem. The paper is concluded with a brief discussion of the difficult problem of multiple-layer second best optima.

I A GENERAL THEOREM IN THE THEORY OF SECOND BEST[2]

It is well known that the attainment of a Paretian optimum requires the simultaneous fulfillment of all the optimum conditions. The general theorem for the second best optimum states that if there is introduced into a general equilibrium system a constraint which prevents the attainment of one of the Paretian conditions, the other Paretian conditions, although still attainable, are, in general, no longer desirable. In other words, given that one of the Paretian optimum conditions cannot be fulfilled, then an optimum situation can be achieved only by departing from all the other Paretian conditions. The optimum situation finally attained may be termed a second best optimum because it is achieved subject to a constraint which, by definition, prevents the attainment of a Paretian optimum.

From this theorem there follows the important negative corollary that there is no *a priori* way to judge as between various situations in which some of the Paretian optimum

[1] The authors are indebted to Professor Harry G. Johnson for a number of helpful suggestions relating to this paper. The appelation, " Theory of Second Best," is derived from the writings of Professor Meade ; See Meade, J. E., *Trade and Welfare*, London, Oxford University Press, 1955. Meade has given, *in Trade and Welfare*, what seems to be the only attempt to date to deal systematically with a number of problems in the theory of second best. His treatment, however, is concerned with the detailed case study of several problems, rather than with the development of a general theory of second best.

[2] See section VII for formal proofs of the statements made in this section.

conditions are fulfilled while others are not. Specifically, it is *not* true that a situation in which more, but not all, of the optimum conditions are fulfilled is necessarily, or is even likely to be, superior to a situation in which fewer are fulfilled. It follows, therefore, that in a situation in which there exist many constraints which prevent the fulfillment of the Parteian optimum conditions, the removal of any one constraint may affect welfare or efficiency either by raising it, by lowering it, or by leaving it unchanged.

The general theorem of the second best states that if one of the Paretian optimum conditions cannot be fulfilled a second best optimum situation is achieved only by departing from all other optimum conditions. It is important to note that in general, nothing can be said about the direction or the magnitude of the secondary departures from optimum conditions made necessary by the original non-fulfillment of one condition. Consider, for example, a case in which the central authority levies a tax on the purchase of one commodity and returns the revenue to the purchasers in the form of a gift so that the sole effect of the tax is to distort relative prices. Then all that can be said in general is that given the existence and invariability of this tax, a second best optimum can be achieved by levying some system of taxes and subsidies on all other commodities. The required tax on some commodities may exceed the given tax, on other commodities it may be less than the given tax, while on still others a subsidy, rather than a tax, may be required.[1]

It follows from the above that there is no *a priori* way to judge as between various situations in which none of the Paretian optimum conditions are fulfilled. In particular, it is *not* true that a situation in which all departures from the optimum conditions are of the same direction and magnitude is necessarily superior to one in which the deviations vary in direction and magnitude. For example, there is no reason to believe that a situation in which there is the same degree of monopoly in all industries will necessarily be in any sense superior to a situation in which the degree of monopoly varies as between industries.

II THE SCOPE OF THE THEORY OF SECOND BEST

Perhaps the best way to approach the problem of defining the scope of the theory of second best is to consider the role of constraints in economic theory. In the general economic problem of maximization a function is maximised subject to at least one constraint. For example, in the simplest welfare theory a welfare function is maximized subject to the constraint exercised by a transformation function. The theory of the Paretian optimum is concerned with the conditions that must be fulfilled in order to maximize some function subject to a set of constraints which are generally considered to be " in the nature of things ". There are, of course, a whole host of possible constraints beyond those assumed to operate in the Paretian optimization problem. These further constraints vary from the " nature-dictated " ones, such as indivisibilities and boundaries to production functions, to the obviously " policy created " ones such as taxes and subsidies. In general, there would seem to be no logical division between those constraints which occur in the Paretian optimum theory and those which occur only in the theory of second best. All that can be said is that, in the theory of the Paretian optimum, certain constraints are assumed to be operative and the conditions necessary for the maximization of some function subject to these constraints are examined. In the theory of second best there is admitted at least one constraint additional to the ones existing in Paretian optimum theory and it is in the nature of this constraint that it prevents the satisfaction of at least one of the Paretian optimum conditions. Consideration is then given to the nature of the conditions that must be satisfied in order to maximize some function subject to this new set of constraints.[2]

[1] See Section V.

[2] The general theory of second best is, thus, concerned with all maximization problems not just with welfare theory. See Section III for examples of non-welfare applications.

It is important to note that even in a single general equilibrium system where there is only one Paretian optimum, there will be a multiplicity of second best optimum positions. This is so because there are many possible combinations of constraints with a second best solution for each combination.[1] For this reason one may speak of the existence of *the* Paretian optimum but should, strictly speaking, refer to *a* second best optimum.

It is possible to approach problems in the theory of second best from two quite different directions. On the one hand, the approach used in this paper is to assume the existence of one constraint additional to those in the Paretian optimum problem (e.g., one tax, one tariff, one subsidy, or one monopoly) and then to investigate the nature of the conditions that must be satisfied in order to achieve a second best optimum and, where possible, to compare these conditions with those necessary for the attainment of a Paretian optimum. On the other hand, the approach used by Professor Meade is to assume the existence of a large number of taxes, tariffs, monopolies, *etcetera*, and then to inquire into the effect of changing any one of them. Meade, therefore, deals with a system containing many constraints and investigates the optimum (second best) level for one of them, assuming the invariability of all the others.[2] It would be futile to argue that one of these approaches was superior to the other. Meade's is probably the appropriate one when considering problems of actual policy in a world where many imperfections exist and only a few can be removed at any one time. On the other hand, the approach used in the present paper would seem to be the more appropriate one for a systematic study of the general principles of the theory of second best.

III THE THEORY OF SECOND BEST IN THE LITERATURE OF ECONOMICS

The Theory of second best has been, in one form or another, a constantly recurring theme in the post-war literature on the discriminatory reduction of trade barriers. There can be no doubt that the theory of customs unions provides an important case study in the application of the general theory of second best. Until customs union theory was subjected to searching analysis, the 'free trader'[3] often seemed ready to argue that any reduction in tariffs would necessarily lead to an improvement in world productive efficiency and welfare. In his path-breaking work on the theory of customs unions[4] Professor Viner has shown that the removal of tariffs from some imports may cause a decrease in the efficiency of world production.

One important reason for the shifts in the location of production which would follow the creation of a customs union was described by Viner as follows :[5]

> There will be commodities which one of the members of the customs union will now newly import from the other, whereas before the customs union it imported them from a third country, because that was the cheapest possible source of supply even after payment of the duty. The shift in the locus of production is now not as between the two member countries but as between a low-cost third country and the other, high-cost, member country.

[1] There may be more than one second best optimum for any given set of constraints. See Section VIII.
[2] Meade, J. E., *Trade and Welfare, op. cit.*, especially p. 96.
[3] i.e., one who believes that trade carried on in the absence of any restraints necessarily leads to an optimum situation. Of course, as soon as there exist restrictions preventing the satisfaction of at least one of the Paretian optimum conditions in the domestic market of any of the trading countries, there is no longer a case for perfectly free trade "... the general case for free trade rests on the contention that in a world of *utopian* domestic policies (i.e., where domestic economic policies ensure the satisfaction of all the Paretian optimum conditions) it sets internationally the proper *marginal* conditions for economic *efficiency*". Meade, J. E., *Trade and Welfare, op. cit.*, p. 139.
[4] Viner, Jacob, *The Customs Union Issue*, New York, Carnegie Endowment for International Peace, 1950.
[5] *Ibid.*, p. 43.

Viner used the term trade diversion to describe production shifts of this sort and he took it as self-evident that they would reduce the efficiency of world production. Since it is quite possible to conceive of a customs union having only trade diverting production effects, it follows, in Viner's analysis, that the discriminatory reduction of tariffs may reduce, rather than raise, the efficiency of world production.

Viner emphasised the production effects of customs unions,[1] directing his attention to changes in the location, and hence the cost, of world production. Recently Professor Meade has shown that a customs union has exactly parallel effects on the location, and hence the " utility " of world consumption.[2] Meade isolates the " consumption effects " of customs unions by considering an example in which world production is fixed. In this case Viner's problem of the effects of a union on the cost of world production cannot arise. Meade argues that, under these circumstances, a customs union will tend to raise welfare by encouraging trade between the member countries but that, at the same time, it will tend to lower welfare by discouraging the already hampered trade between the union area and the rest of the world. In the final analysis a customs union will raise welfare, lower it, or leave it unchanged, depending on the relative strength of these two opposing tendencies.[3] The Viner-Meade conclusions provide an application of the general theorem's negative corollary that nothing can be said *a priori* about the welfare and efficiency effects of a change which permits the satisfaction of some but not all of the Paretian optimum conditions.

Another application of second best theory to the theory of tariffs has been provided by S. A. Ozga who has shown that a non-preferential reduction of tariffs by a single country may lead " away from the free trade position "[4] In other words, the adoption of a free trade policy by one country, in a multi-country tariff ridden world, may actually lower the real income of that country and of the world. Ozga demonstrates the existence of this possibility by assuming that all commodities are, in consumption, rigidly complementary, so that their production either increases or decreases simultaneously. He then shows that in a three country world with tariffs all around, one country may adopt a policy of free trade and, as a result, the world production of all commodities may decrease. This is one way of demonstrating a result which follows directly from the general theory of second best.

In the field of Public Finance, the problems of second best seem to have found a particularly perplexing guise in the long controversy on the relative merits of direct *versus* indirect taxation. It would be tedious to review all the literature on the subject at this time. In his 1951 article, I. M. D. Little[5] has shown that because of the existence of the " commodity " leisure, the price of which cannot be directly taxed, both direct and indirect taxes must prevent the satisfaction of some of the conditions necessary for the attainment of a Paretian optimum. An indirect tax on one good disturbs rates of substitution between that good and all others while an income tax[6] disturbs rates of substitution between leisure and all other goods. Little then argues that there is no *a priori* way to judge as between

[1] His neglect of the demand side of the problem allowed him to reach the erroneous conclusion that trade diversion necessarily led to a decrease in welfare. It is quite possible for an increase in welfare to follow from the formation of a customs union whose sole effect is to divert trade from lower- to higher-cost sources of supply. Furthermore, this welfare gain may be enjoyed by the country whose import trade is diverted to the higher-cost source, by the customs union area considered as a unit and by the world as a whole.

See : Lipsey, R. G., " The Theory of Customs Unions : Trade Diversion and Welfare " in a forthcoming issue of *Economica.*

[2] Meade, J. E., *The Theory of Customs Unions*, Amsterdam, the North Holland Publishing Co., 1955.

[3] *Ibid.*, Chapter III.

[4] Ozga, S. A., " An Essay in the Theory of Tariffs ", *Journal of Political Economy*, December, 1955, p. 489.

[5] Little, I. M. D., " Direct *versus* Indirect Taxes ", *The Economic Journal*, September, 1951.

[6] In this analysis an income tax may be treated as a uniform *ad valorem* rate of tax on all commodities except leisure.

these two positions where some Paretian optimum conditions are satisfied while others are not. This is undoubtedly correct. However, Little might have gone on to suggest that there is an *a priori* case in favour of raising a given amount of revenue by some system of *unequal indirect taxes* rather than by either an income tax or an indirect tax on only one commodity. This interesting conclusion was first stated by W. J. Corlett and D. C. Hague[1]. These authors have demonstrated that the optimum way to raise any given amount of revenue is by a system of unequal indirect taxes in which commodities " most complementary " to leisure have the highest tax rates while commodities " most competitive " with leisure have the lowest rates.[2] The reason for this general arrangement of tax rates should be intuitively obvious. When an equal *ad valorem* rate of tax is placed on all goods the consumption of leisure will be too high while the consumption of all other goods will be too low.[3] The consumption of untaxed leisure may be discouraged by placing especially high rates of tax on commodities which are complementary in consumption to leisure and by placing especially low rates of tax on commodities which are competitive in consumption with leisure.

Professor Meade has recently given an alternate analysis of the same problem.[4] His conclusions, however, support those of Corlett and Hague. In theory at least, the tables have been completely turned and the indirect tax is proved to be superior to the income tax, provided that the optimum system of indirect taxes is levied.[5] This conclusion is but another example of an application of the general theorem that if one of the Paretian optimum conditions cannot be fulfilled then a second best optimum situation can be obtained by departing from all the other optimum conditions.

What is perhaps not so obvious is that the problem of direct *versus* indirect taxes and that of the " consumption effects " of customs unions are analytically identical. The Little analysis deals with a problem in which some commodities can be taxed at various rates while others must be taxed at a fixed rate. (It is not necessary that the fixed rate of tax should be zero). In the theory of customs unions one is concerned with the welfare and efficiency effects of varying some tariff rates while leaving others unchanged. In Little's analysis there are three commodities, X, Y and Z ; commodity Z being leisure. By renaming Z home goods and X and Y imports from two different countries one passes immediately to the theory of customs unions. An income tax in Little's analysis becomes a system of non-discriminatory import duties while a single indirect tax becomes the discriminatory tariff introduced after the formation of a customs union with the producers of the now untaxed import. A model of this sort is considered further in Section V.

An application of the general theory of second best to yet another field of economic theory is provided by A. Smithies in his article, *The Boundaries of the Production and Utility Function.*[6] Smithies considers the case of a multi-input firm seeking to maximize its profits. This will be done when for each factor the firm equates marginal cost with marginal revenue productivity. Smithies then suggests that there may exist boundaries to the production function. These boundaries would take the form of irreducible minimum amounts of

[1] Corlett, W. J. and Hague, D. C., " Complementarity and the Excess Burden of Taxation ", *Review of Economic Studies*, Vol. XXI, No. 54, 1953-54.

[2] *Ibid.*, p. 24.

[3] Too high and too low in the sense that a decrease in the consumption of leisure combined with an increase in the consumption of all other goods would raise the welfare of any consumer.

[4] Meade, J. E., *Trade and Welfare, Mathematical Supplement*, London, Oxford University Press, 1955, Chapter III.

[5] Of course two special cases are always possible. In the first the optimum rates of tax will be equal for all commodities (i.e., the income tax is the optimum tax). This will occur if the supply of effort (the demand for leisure) is perfectly inelastic. In the second case the optimum rates will be zero for all but one commodity (i.e., an indirect tax on one commodity is the optimum tax). This will occur if the demand for one commodity is perfectly inelastic.

[6] Smithies, A., " The Boundaries of the Production and Utility Function ", in : *Exploration in Economics*, London, McGraw-Hill, 1936.

certain inputs, it being possible to employ more but not less than these minimum amounts. It might happen, however, that profit maximization called for the employment of an amount of one factor less than the minimum technically possible amount. In this case production would take place " on the boundary " and the minimum possible amount of the input would be used. However, in the case of this input, marginal cost would no longer be equated with marginal productivity, the boundary conditions forcing its employment beyond the optimum level. Smithies then shows that given the constraint, marginal cost does not equal marginal productivity for this input, profits will be maximised only by departing from the condition marginal cost equals marginal productivity for all other inputs. Furthermore, there is no *a priori* reason for thinking that the nature of the inequality will be the same for all factors. Profit maximization may require that some factors be employed only to a point where marginal productivity exceeds marginal cost while other factors are used up to a point where marginal productivity falls below marginal cost.

Problems of the " mixed economy " provide an application of second best theory frequently encountered in popular discussion. Consider, for example, a case where one section of an economy is rigidly controlled by the central authority while another section is virtually uncontrolled. It is generally agreed that the economy is not functioning efficiently but there is disagreement as to the appropriate remedy. One faction argues that more control over the uncontrolled sector is needed, while another faction pleads for a relaxation of the degree of control exercised in the public sector. The principles of the general theory of second best suggest that *both sides* in the controversy may be advocating a policy appropriate to the desired ends. Given the high degree of control in one sector and the almost complete absence of control in another, it is unlikely that anything like a second best optimum position has been reached. If this is so, then it follows that efficiency would be increased either by increasing the degree of control exercised over the uncontrolled sector or by relaxing the control exercised over the controlled sector. Both of these policies will move the economy in the direction of some second best optimum position.

Finally mention may be made of the problem of " degrees of monopoly ". It is not intended to review the voluminous literature on this controversy. It may be mentioned in passing that, in all but the simplest models, a Paretian optimum requires that marginal costs *equal* marginal revenues throughout the entire economy.[1] If this equality is not established in one firm, then the second best conditions require that the equality be departed from in all other firms. However, as is usual in second best cases there is no presumption in favour of the same degree of inequality in all firms. In general, the second best position may well be one in which marginal revenues greatly exceed marginal costs in some firms, only slightly exceed marginal costs in others, while, in still other firms, marginal revenues actually fall short of marginal costs.

A similar problem is considered by Lionel W. McKenzie in his article " Ideal Output and the Interdependence of Firms."[2] He deals with the problem of increasing the money value of output in situations in which marginal costs do not equal prices in all firms. The analysis is not conducted in a general equilibrium setting and many simplifying assumptions are made such as the one that resources can be shifted between occupations as desired without affecting their supplies. McKenzie shows that even in this partial equilibrium setting if allowance is made for inter-firm sales of intermediate products, the condition that marginal costs should bear the same relation to prices in all firms does not provide a sufficient condition for an increase in the value of output. Given that the optimum condition, marginal costs equals price cannot be achieved, McKenzie shows that a second

[1] For example, if the supply of effort is not perfectly inelastic an equal degree of monopoly throughout the entire economy has the same effect as an income tax on wage earners. Following Little's analysis it is obvious that this tax will prevent the attainment of a Paretian optimum.

[2] McKenzie, Lionel W., " Ideal Output and the Interdependence of Firms ", *Economic Journal*, 1951.

best optimum would require a complex set of relations in which the ratio of marginal cost to price would vary as between firms. Although the analysis is not of a full general equilibrium, the conclusions follow the now familiar pattern : (1) If a Paretian optimum cannot be achieved a second best optimum requires a general departure from all the Paretian optimum conditions and (2) there are unlikely to be any simple sufficient conditions for an *increase* when a *maximum* cannot be obtained.

IV THE THEORY OF SECOND BEST AND " PIECEMEAL " POLICY RECOMMENDATIONS

It should be obvious from the discussion in the preceding sections that the principles of the general theory of second best show the futility of " piecemeal welfare economics ".[1] To apply to only a small part of an economy welfare rules which would lead to a Paretian optimum if they were applied everywhere, may move the economy away from, not toward, a second best optimum position. A nationalized industry conducting its price-output policy according to the Lerner-Lange " Rule " in an imperfectly competitive economy may well diminish both the general productive efficiency of the economy and the welfare of its members.

The problem of sufficient conditions for an increase in welfare, as compared to necessary conditions for a welfare maximum, is obviously important if policy recommendations are to be made in the real world. Piecemeal welfare economics is often based on the belief that a study of the *necessary* conditions for a Paretian welfare optimum may lead to the discovery of *sufficient* conditions for an increase in welfare.[2] In his *Critique of Welfare Economics*, I. M. D. Little discusses the optimum conditions for exchange and production " . . . both as necessary conditions for a maximum, and as sufficient conditions for a desirable economic change ".[3] Later on in his discussion Little says " . . . necessary conditions are not very interesting. It is *sufficient* conditions for improvements that we really want . . . "[4] But the theory of second best leads to the conclusion that there are in general no such sufficient conditions for an increase in welfare. There are necessary conditions for a Paretian optimum. In a simple situation there may exist a condition that is necessary and sufficient. But in a general equilibrium situation there will be no conditions which in general are sufficient for an increase in welfare without also being sufficient for a welfare maximum.[5]

The preceding generalizations may be illustrated by considering the following optimum condition for exchange : " The marginal rate of substitution between any two ' goods ' must be the same for every individual who consumes them both."[6] Little concludes that this condition gives a sufficient condition for an increase in welfare provided only that when it is put into effect, " . . . the distribution of welfare is not thereby made worse."[7] However, the whole discussion of this optimum condition occurs only after Little has postulated " . . . a fixed stock of ' goods ' to be distributed between a number of ' individuals '."[8] The optimum condition that all consumers should be faced with the same set of prices becomes in this case a sufficient condition for an increase in welfare, because the problem at hand is merely how to distribute efficiently a fixed stock of goods. But in this case the

[1] For a description of this type of welfare economics see I. M. D. Little, *A Critique of Welfare Economics*, Oxford, The Clarendon Press, 1950, p. 89.

[2] Indeed any economics that attempts piecemeal policy recommendations must be based on the belief that there can be discovered sufficient conditions for an increase in, as distinct from necessary conditions for a maximum of, whatever it is that is being considered.

[3] Little, *op cit.*, p. 120.

[4] *Ibid.*, p. 129.

[5] This conclusion follows directly from the negative corollary stated in the second paragraph of Section I.

[6] Little, I. M. D., *op. cit.*, p. 121.

[7] *Ibid.*, p. 122.

[8] *Ibid.*, 121.

condition is a necessary and sufficient condition for a Paretian optimum. As soon as variations in output are admitted, the condition is no longer sufficient for a welfare maximum and it is also no longer sufficient for increase in welfare.

The above conclusion may be illustrated by a simple example. Consider a community of two individuals having different taste patterns. The " government " of the community desires to raise a certain sum which it will give away to a foreign country. The community has made its value judgement about the distribution of income by deciding that each individual must contribute half of the required revenue. It has also been decided that the funds are to be raised by means of indirect taxes. It follows from the Corlett and Hague analysis that the best way to raise the revenue is by a system of *unequal* indirect taxes in which commodities " most complementary " to leisure are taxed at the highest rates while commodities " most substitutable " for leisure are taxed at the lowest rates. But the two individuals have different tastes so that commodity X is substitutable for leisure for individual I and complementary to leisure for individual II, while commodity Y and leisure are complements for individual I and substitutes for II. The optimum way to raise the revenue, therefore, is to tax commodity X at a low rate when it is sold to individual I and at a high rate when it is sold to individual II, while Y is taxed at a high rate when sold to I but a low rate when sold to II. A second best optimum thus requires that the two individuals be faced with different sets of relative prices.

Assume that the optimum tax rates are charged. The government then changes the tax system to make it non-discriminatory as between persons while adjusting the rates to keep revenue unchanged. Now the Paretian optimum exchange condition is fulfilled, but welfare has been decreased, for both individuals have been moved to lower indifference curves. Therefore, in the assumed circumstances, this Paretian optimum condition is a sufficient condition for a *decrease* in welfare.

V A PROBLEM IN THE THEORY OF TARIFFS

In this section the simple type of model used in the analysis of direct *versus* indirect taxes is applied to a problem in the theory of tariffs. In the Little-Meade-Corlett & Hague analysis it is assumed that the government raises a fixed amount of revenue which it spends in some specified manner. The optimum way of raising this revenue is then investigated. A somewhat different problem is created by changing this assumption about the disposition of the tax revenue. In the present analysis it is assumed that the government returns the tax revenue to the consumers in the form of a gift so that the only effect of the tax is to change relative prices.[1]

A simple three commodity model is used, there being one domestic commodity and two imports. It is assumed that the domestic commodity is un-taxed and that a fixed rate of tariff is levied on one of the imports. The optimum level for the tariff on the other import is then investigated. This is an obvious problem in the theory of second best. Also it is interesting to note that the conclusions reached have immediate applications to the theory of customs unions. In the second part of this section the conclusions of part A are applied to the problem of the welfare effects of a customs union which causes neither trade creation nor trade diversion, but only the expansion and contraction of the volumes of already existing trade.

A. Second Best Optimum Tariff Systems with Fixed Terms of Trade :

The conditions of the model are as follows : Country A is a small country specializing in the production of one commodity (Z). Some of Z is consumed at home and the

[1] If consumers have different utility functions then each consumer must receive from the government an amount equal to what he pays in taxes. However, if all consumers have identical homogeneous utility functions then all that is required is that the tax revenue be returned to some consumer or consumers.

remainder is exported in return for two imports, X from country B and Y from country C. The prices of X and Y in terms of Z are unaffected by any taxes or tariffs levied in country A. It is further assumed that none of the tariffs actually levied by A are high enough to protect domestic industries producing either X or Y,[1] that country B does not produce commodity Y and that country C does not produce commodity X.[2] The welfare of country A is defined by a community welfare function which is of the same form as the welfare functions of the identical individuals who inhabit A.

It is assumed that A levies some fixed tariff on imports of commodity Y and that commodity Z is not taxed. It is then asked : What tariff ($\leqslant 0$) on imports of commodity X will maximize welfare in country A ? This tariff will be termed the optimum X tariff.[3]

The model may be set out as follows : Let there be three commodities, X, Y and Z. Let p_x and p_y be the prices of X and Y in terms of Z. Let the rate of *ad valorem* tariff charged on X and Y be $t_x - 1$ and $t_y - 1$.[4]

$$u = u(\mathrm{x}, y, z) \tag{5.1}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial z} p_x t_x \tag{5.2–a}$$

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial z} p_y t_y \tag{5.2–b}$$

$$Xp_x + Yp_y + Z = C \tag{5.3}$$

Equation (5.1) expresses country A's community welfare function. Equations (5.2–*a* and –*b*) are the demand equilibrium conditions. Equation (5.3) gives the condition that A's international payments be in balance.[5]

These equations will yield a solution in general for any t_x and t_y, in X, Y and Z. Hence, for given p_x, p_y, C and whatever parameters enter into (5.1) :

$$X = f(t_x, t_y) \tag{5.4–a}$$

$$Y = g(t_x, t_y) \tag{5.4–b}$$

$$Z = h(t_x, t_y) \tag{5.4–c}$$

Attention is directed to the sign of the change in U when t_x changes with $t_y > 1$ kept constant. From equations (5.1) and (5.4) :

$$\frac{\partial u}{\partial t_x} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t_x} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial t_x} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial t_x} \tag{5.5}$$

Substitute (5.2–*a* and –*b*) into (5.5) :

$$\frac{\partial u}{\partial t_x} = p_x t_x \frac{\partial u}{\partial z} \cdot \frac{\partial x}{\partial t_x} + p_y t_y \frac{\partial u}{\partial z} \cdot \frac{\partial y}{\partial t_x} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial t_x} \tag{5.6}$$

$$= \frac{\partial u}{\partial z} \left(p_x t_x \frac{\partial x}{\partial t_x} + p_y t_y \frac{\partial y}{\partial t_x} + \frac{\partial z}{\partial t_x} \right) \tag{5.6}$$

[1] Therefore, there can be no question of a reduction of A's tariffs causing trade creation.

[2] Therefore, there can be no question of a preferential reduction of A's tariffs causing trade diversion.

[3] Obviously, this is a problem in the theory of second best. The initial tariff on Y causes the consumption of Y to be too low relative to both X and Z. If the consumption of Y can be encouraged at the expense of X, welfare will be increased. However, if the consumption of Z is encouraged at the expense of X, welfare will be lowered. A tariff on X is likely to cause both sorts of consumption shift and the optimum X tariff will be that one where, at the margin, the harmful effect of the shift from X to Z just balances the beneficial effect of the shift from X to Y.

[4] We are greatly indebted to Dr. George Morton for suggesting the following mathematical demonstration. It now replaces a much more cumbersome demonstration.

[5] i.e. The value of imports ($Xp_x + Yp_y$) plus the value of domestic production *consumed at home* (Z) equals the total value of domestic production (C).

Next, take the partial derivative of (5.3) with respect to t_x.

$$px \frac{\partial x}{\partial t_x} + py \frac{\partial y}{\partial t_x} + \frac{\partial z}{\partial t_x} = 0$$

or :

$$px \frac{\partial x}{\partial t_x} + py \frac{\partial y}{\partial t_x} = -\frac{\partial z}{\partial t_x} \tag{5.7}$$

Substitute (5.7) into (5.6) :

$$\frac{\partial u}{\partial t_x} = \frac{\partial u}{\partial z}\left(p_x t_x \frac{\partial x}{\partial t_x} + p_y t_y \frac{\partial y}{\partial t_x} - p_x \frac{\partial x}{\partial t_x} - p_y \frac{\partial y}{\varrho t_x}\right)$$

$$= \frac{\partial u}{\partial z}\left[px \frac{\partial x}{\partial t_x}(t_x - 1) + p_y \frac{\partial y}{\partial t_x}(t_y - 1)\right] \tag{5.8}$$

It is assumed, first, that some tariff is levied on Y but that X is imported duty free. Therefore, $t_x = 1$ and $t_y > 1$. Equation (5.8) reduces to :

$$\frac{\partial u}{\partial t_x} = \frac{\partial u}{\partial z}\left[p_y \frac{\partial y}{\partial t_x}(t_y - 1)\right] \tag{5.9}$$

In (5.9) $\frac{\partial u}{\partial t_x}$ takes the same sign as $\frac{\partial y}{\partial t_x}$.[1] It follows that the introduction of a marginal tariff on X will raise welfare if it causes an increase in imports of commodity Y, will leave welfare unchanged if it causes no change in imports of Y and will lower welfare if it causes a decrease in imports of Y. Therefore, the optimum tariff on X is, in fact, a subsidy, if imports of Y fall when a tariff is placed on X, it is zero if the X tariff has no effect on imports of Y and it is positive if imports of Y rise when the tariff is placed on X.

It is now assumed that a uniform rate of tariff is charged on X and Y. Therefore, $t_x = t_y \equiv T$ and equation (5.8) becomes :

$$\frac{\partial u}{\partial t_x} = \frac{\partial u}{\partial z}(T - 1)\left(px \frac{\partial x}{\partial T} + py \frac{\partial y}{\partial T}\right)$$

Substituting from (5.7) :

$$\frac{\partial u}{\partial t_x} = -\left[\frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial t_x}(T - 1)\right] \tag{5.10}$$

In (5.10) the sign of $\frac{\partial u}{\partial t_x}$ will be opposite to the sign of $\frac{\partial z}{\partial t_x}$. It follows that a marginal increase in the tariff on X will increase welfare if it causes a decrease in the consumption of Z, will leave welfare unchanged if it causes no change in the consumption of Z and will lower welfare if it causes an increase in the consumption of Z. It may be concluded, therefore, that the optimum tariff on X exceeds the given tariff on Y if an increase in the X tariff reduces the consumption of Z, that the optimum X tariff equals the given Y tariff if there is no relation between the X tariff and the consumption of Z and that the optimum X tariff is less than the given Y tariff if an increase in the X tariff causes an increase in consumption of Z.

In the case where an increase in the tariff on X causes an increase in the consumption of Y and of Z the optimum X tariff is greater than zero but less than the given tariff on Y.

[1] These relationships are not as simple as they might appear. If worked out $\frac{\partial y}{\partial t_x}$ and $\frac{\partial z}{\partial t_x}$ would be found to be of the same order of complexity as are the Q_i's in Section IX.

B. Welfare Effects of a Customs Union Causing only Trade Expansion and Trade Contraction :

It is assumed that country *A* initially charges a uniform *ad valorem* rate of tariff on imports of *X* and *Y*. *A* then forms a customs union with country *B*. Now *X* is imported duty free while the pre-union tariff still applies to *Y*. What is the effect on *A*'s welfare of such a customs union ? Some answers[1] follow immediately from the previous analysis :

Case 1 : Any increase in the tariff on *X* causes a fall in the consumption of *Y*. The optimum tariff on *X* is, in fact, a subsidy. Therefore, the customs union must raise *A*'s welfare.

Case 2 : Variations in the tariff on *X* have no effect on consumption of *Y*. The optimum tariff on *X* is now zero. The customs union raises welfare in *A*. Furthermore, it raises it to a second best optimum level (assuming that only the *X* tariff can be varied).

Case 3 : Variations in the tariff on *X* have no effect on the purchases of *Z*. The optimum tariff on *X* is equal to the *Y* tariff. The customs union lowers *A*'s welfare. Furthermore, the union disturbs an already achieved second best optimum.

Case 4 : An increase in the tariff on *X* causes a fall in the consumption of *Z*. In this case the optimum tariff on *X* exceeds the given *Y* tariff. Therefore, the customs union lowers *A*'s welfare.

Case 5 : An increase in the tariff on *X* causes an increase in the consumption of both *Y* and *Z*. The optimum *X* tariff is greater than zero but less than the given *Y* tariff. The effect of the customs union on welfare is not known. Assume, however, that the *X* tariff is removed by a series of stages. It follows that the initial stages of tariff reduction must raise welfare and that the final stages must lower it. Although nothing can be said about the welfare effect of a complete removal of the *X* tariff, another important conclusion is suggested. A small reduction in tariffs must raise welfare. A large reduction may raise or lower it. It follows, therefore, that a partial preferential reduction of tariffs is more likely to raise welfare than is a complete preferential elimination of tariffs. Of course, this conclusion depends upon the specific assumptions made in the present model but it does provide an interesting and suggestive hypothesis for further investigation.[2] [3]

VI NATIONALISED INDUSTRY IN AN ECONOMY WITH MONOPOLY : A SIMPLE MODEL

An interesting, and not unlikely, situation in which a " second best " type of policy may have to be pursued is that of a mixed economy which includes both nationalised industries and industries which are subject to monopoly control.

The monopoly is assumed to be one of the data : for one reason or another this monopoly cannot be removed, and the task of the nationalised industry is to determine that pricing policy which is most in " the public interest ".

When there is full employment of resources then, if the monopoly is exercising its power, it will be producing less of the monopolised product than is required to give an optimum (in the Paretian sense) allocation of resources. Since there is less than the optimum production of the monopolised good, there will be more than the optimum production of the non-monopolised goods as a group.

[1] Only the most obvious applications of the conclusions reached in part *A* are given in this part. This is not the place for a detailed report on original work in the theory of customs unions.

[2] This conclusion is also reached by Professor Meade, See : *The Theory of Customs Unions*, *op. cit.*, p. 51.

[3] Another conclusion suggested by the analysis is that the higher are the tariffs reduced by the union relative to all other tariffs, the more likely is it that the union will raise welfare, c.f. Meade, *op. cit.*, pp. 108-9.

Suppose that one of the non-monopolised industries is now nationalised. What should be its price/output policy ? If it behaves competitively then it will tend to produce more of its product, relative to the monopolised good, than the Paretian optimum would require. If, on the other hand, it behaves monopolistically itself, then it will cut down the excess of its own production relative to that of the monopoly but will increase the excess of the remaining goods relative to both its own product and that of the monopolised industry. This is a typical " second best " situation : any policy will make some things worse and some better.

It is clear that no policy on the part of the nationalised industry can restore the Paretian optimum, for the existence of the monopoly prevents this. The nationalised industry must aim at a second best policy, designed to achieve the best that still remains open to the economy. In purely general terms it is impossible to be more definite than this, as will be shown in section IX. Intuitively, however, one might expect that, in some situations at least, the best policy for the nationalised industry would be to behave something like the monopoly, but to a lesser extent. In the case of the simple model to be presented in this section, one's intuitions would be correct.

There are assumed to be, in the present model, three industries producing goods x, y, z. Labour is the only input, costs are constant, and the total supply of labour is fixed. These assumptions define a unique linear transformation function relating the quantities of the three goods :

$$ax + by + cz = L \tag{6.1}$$

The production functions from which this is derived are :

$$x = \frac{1}{a} l_x, \; y = \frac{1}{b} l_y, \; z = \frac{1}{c} l_z \; ; \quad l_x + l_y + l_z = L. \tag{5.2}$$

The marginal costs are constant and proportional to a, b, c.

The " public interest " is assumed to be defined by a community preference function, which is of the same form as the preference functions of the identical individuals who make up the society. For simplicity, this preference function is assumed to take the logarithmic form :

$$U = x^{\alpha} y^{\beta} z^{\gamma}, \; \alpha, \beta, \gamma > 0 \tag{6.3}$$

The partial derivatives of this are :

$$\frac{\partial U}{\partial x} = \alpha \frac{U}{x}, \; \frac{\partial U}{\partial y} = \beta \frac{U}{y}, \; \frac{\partial U}{\partial z} = \gamma \frac{U}{z}$$

so that the marginal utilities of x, y, z are proportional, respectively, to $\frac{\alpha}{x}$, $\frac{\beta}{y}$, $\frac{\gamma}{z}$. For a utility function of this type, all goods are substitutes in both the Edgeworth-Pareto and Hicksian senses.

If there were no constraints in the economy (other than the transformation function itself), the Paretian optimum would be that found by maximising the expression $U - \lambda(ax + by + cz - L)$, where λ is the Lagrangian multiplier. This would lead to the three equations :

$$\left.\begin{aligned} \frac{\partial U}{\partial x} - \lambda a &= 0 \\ \frac{\partial U}{\partial y} - \lambda b &= 0 \\ \frac{\partial U}{\partial z} - \lambda c &= 0 \end{aligned}\right\} \tag{6.4}$$

which can be expressed in the proportional form :

$$\frac{a}{\alpha}x = \frac{b}{\beta}y = \frac{x}{\gamma}z \tag{6.5}$$

These conditions are of the familiar Paretian type, namely that the marginal utilities (or prices which, assuming the ordinary consumer behaviour equations, are proportional to them) are proportional to the marginal costs. There being no monetary conditions, and the supply of labour being fixed, equality between prices and marginal costs is not necessarily implied.

Suppose now that the industry producing x is a monopoly. The monopoly will set the price of x higher (in terms of some numeraire, which will be taken to be z) in relation to marginal cost than in the conditions of the Paretian optimum. A numeraire is necessary since money, and money prices, are not being considered.

For the present purposes, the exact margin between marginal cost and price in the monopolised industry (relative to the numéraire) does not matter. It is necessary only for the problem that the monopolist set the prices of x higher, relative to the price of z, than the ratio of the marginal cost of prodcuing x to the marginal cost of producing z.

In other words, the monopolist's behaviour can be expressed by :

$$\frac{p_x}{p_z} > \frac{mc_x}{mc_z}$$

Substituting for $\frac{p_x}{p_z}\left(= \frac{\partial U}{\partial x}\bigg/\frac{\partial U}{\partial z} = \frac{\alpha z}{\gamma x}\right)$ and $\frac{mc_x}{mc_z}\left(= \frac{a}{c}\right)$, this gives :

$$\frac{\alpha z}{\gamma x} > \frac{a}{c}$$

$$c\alpha z > a\gamma x$$

$$= ka\gamma x \quad \text{where } k > 1 \tag{6.6}$$

The actual value of k (provided it is > 1) does not matter for the analysis. It is not necessary for the argument that k is constant as the monopolist faces the changes brought about by the policies of the nationalised industries, but it simplifies the algebra to assume this.

The behaviour of the monopolist, assumed unalterable, becomes an additional constraint on the system. The best that can be done in the economy is to maximise U subject to two constraints, the transformation function (6.1) and the monopoly behaviour condition (6.6). The conditions for attaining the second best optimum (the Paretian optimum being no longer attainable) are found, therefore, as the conditions for the maximum of the function $U - \mu(c\alpha z - ka\gamma x) - \lambda'(ax + by + cz - L)$, where there are now two Lagrangean multipliers μ, λ'. Neither of these multipliers can be identified with the multiplier λ in the equations (6.4).

The conditions for attaining the second best are, therefore :

$$\alpha\frac{U}{x} - \mu ka\gamma - \lambda' a = 0 \tag{6.7}$$

$$\beta\frac{U}{y} - \lambda' b = 0 \tag{6.8}$$

$$\gamma\frac{U}{z} + \mu c\alpha - \lambda' c = 0 \tag{6.9}$$

To appreciate these conditions, it is necessary to compute the ratio $\frac{p_y}{p_z}$, compare it with the ratio $\frac{mc_y}{mc_z}$, and relate the result to both the Paretian optimum conditions and the mode of behaviour of the monopolist.

Although there are three equations (6.7), (6.8), (6.9) above, these involve the two Lagrangean multipliers, so that there is actually only one degree of freedom. Hence, the policy of the nationalised industry (that which produces y) is sufficient for attaining the second best. If the nationalised industry sets its price, relative to its marginal cost, so as to satisfy the above conditions, it will have done all that is within its power to further the public interest.

To complete the solution it is necessary to determine μ and λ'.

From (6.7) $$\mu ka\gamma x = \alpha U - \lambda' ax \tag{6.10}$$

and from (6.9) $$-\mu c\alpha z = \gamma U - \lambda' cz \tag{6.11}$$

Hence, $$\mu(ka\gamma x - c\alpha z) = (\alpha + \gamma)U - \lambda'(ax + cz)$$

but, from (6.6), $ka\gamma x - c\alpha z = 0$

so that, $$(a + \gamma)U - \lambda'(ax + cz) = 0$$

$$\lambda' = \frac{(a + \gamma)U}{ax + cz} \tag{6.12}$$

Substituting for λ' in (6.10)

$$\begin{aligned} \mu ka\gamma x &= \alpha U - \frac{(\alpha + \gamma)U}{ax + cz} \\ &= \frac{c\alpha z - \gamma ax}{ax + cz} U \\ &= (k - 1) \frac{\gamma ax}{ax + cz} U \quad [c\alpha z = k\gamma ax, \text{ from (6.6)}] \end{aligned}$$

$$\mu = \frac{k - 1}{k} \cdot \frac{U}{ax + cz} \tag{6.13}$$

The correct pricing policy for the nationalised industry is given from the ratio $\frac{p_y}{p_z}$ which is implicit in the equations (6.7), (6.8), (6.9).

$$\begin{aligned} \frac{p_y}{p_z} &= \frac{\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial z}} \\ &= \frac{\beta \frac{U}{y}}{\gamma \frac{U}{z}} \\ &= \frac{\lambda' b}{-mc\alpha + \lambda' c} \quad \text{[From (6.8), (6.9)]} \\ &= \frac{b}{c - \frac{\partial}{\lambda'} c\alpha} \quad \text{[From (6.12), (6.13)]} \end{aligned}$$

$$= \frac{\frac{b}{c}}{1 - \frac{k-1}{k} \cdot \frac{\alpha}{\alpha+\gamma}} \tag{6.14}$$

Now $\frac{b}{c} = \frac{MC_y}{MC_z}$, from (6.2), so that :

$$\frac{p_y}{p_z} = \frac{MC_y}{MC_z} \cdot \left(\frac{1}{1 - \frac{k-1}{k} \cdot \frac{\alpha}{\alpha+\gamma}} \right) \tag{6.15}$$

Consider the expression $\left(\frac{k-1}{k} \cdot \frac{\alpha}{\alpha+\gamma} \right)$. Since $k > 1$, $0 < \frac{k-1}{k} < 1$, and $\frac{\alpha}{\alpha+\gamma} < 1$ since $\gamma > 0$. Thus the bracketed expression on the right hand side of (6.15) is greater than unity.

In other words, $\frac{p_y}{p_z} > \frac{MC_y}{MC_z}$, so that, relative to the numéraire, the nationalised industry should set its price higher than its marginal cost and, to that extent, behave like the monopoly.

But now consider the relationship between the nationalised industry and the monopoly.

$$\frac{p_y}{p_x} = \frac{\beta \frac{U}{y}}{\alpha \frac{U}{x}}$$

$$= \frac{\frac{b}{a}}{\frac{\mu}{\lambda'} k\gamma + 1}$$

$$= \frac{\frac{b}{a}}{\frac{k-1}{\alpha+\gamma} \cdot \gamma + 1}$$

$$= \frac{b}{a} \cdot \frac{\alpha+\gamma}{\alpha + k\gamma} \tag{6.16}$$

In this case, since $k > 1$, $\alpha, \gamma > 0$, $\frac{\alpha+\gamma}{\alpha + k\gamma} < 1$. Since $\frac{b}{a} = \frac{MC_y}{MC_x}$, the nationalised industry should set its price less high, in relation to marginal cost, than the monopoly.

In short, in the particular model analysed, the correct policy for the nationalised industry, with monopoly entrenched in one of the other industries, would be to take an intermediate path. On the one hand, it should set its price higher than marginal cost (relative to the numéraire) but, on the other hand, it should not set its price so far above marginal cost as is the case in the monopolised industry.

These conclusions refer, it should be emphasised, to the particular model which has been analysed above. This model has many simplifying (and therefore special) features, including the existence of only one input, constant marginal costs and a special type of utility function. As is demonstrated later, in Section IX, there can be no *a priori* expectations about the nature of a second best solution in circumstances where a generalised utility function is all that can be specified.

VII A GENERAL THEOREM OF THE SECOND BEST

Let there be some function $F(x_1 \ldots x_n)$ of the n variables $x_1 \ldots . x_n$, which is to be maximised (minimised) subject to a constraint on the variables $\Phi(x_1 \ldots . x_n) = 0$. This is a formalisation of the typical choice situation in economic analysis.

Let the solution of this problem—the Paretian optimum—be the n—1 condition $\Omega^i(x_1 \ldots . x_n) = 0$, $i = 1 \ldots . n-1$. Then the following theorem, the theorem of the second best, can be given :

If there is an additional constraint imposed of the type $\Omega^i \neq 0$ for $i = j$, then the maximum (minimum) of F subject to both the constraint Φ and the constraint $\Omega^i \neq 0$ will, in general, be such that none of the still attainable Paretian conditions $\Omega^i = 0$, $i \neq j$, will be satisfied.

PROOF :

In the absence of the second constraint, the solution of the original maximum (minimum) problem is both simple and familiar. Using the Lagrange method, the Paretian conditions are given by the n equations :

$$F_i - \lambda \Phi_i = 0 \qquad i = 1 \ldots . n \tag{7.1}$$

Eliminating the multiplier, these reduce to the n—1 proportionality conditions :

$$\frac{F_i}{F_n} = \frac{\Phi_i}{\Phi_n} \qquad i = 1 \ldots . n-1 \tag{7.2}$$

where the n'th commodity is chosen as numéraire.

The equations (7.2) are the first order conditions for the attainment of the Paretian optimum. Now let there be a constraint imposed which prevents the attainment of one of the conditions (7.2). Such a constraint will be of the form (the numbering of the commodities is, of course, arbitrary) :

$$\frac{F_1}{F_n} = k \frac{\Phi_1}{\Phi_n} \qquad k \neq 1 \tag{7.3}$$

It is not necessary that k be constant, but it is assumed to be so in the present analysis. There is now an additional constraint in the system so that, using the Lagrangean method, the function to be maximised (minimised) will be :

$$F - \lambda' \Phi - \mu \left(\frac{F_1}{F_n} - k \frac{\Phi_1}{\Phi_n} \right) \tag{7.4}$$

The multipliers λ', μ will both be different, in general, from the multiplier λ in (7.1).

The conditions that the expression (7.4) shall be at a maximum (minimum) are as follows :

$$F_i - \lambda' \Phi_i - \mu \left\{ \frac{F_n F_{1i} - F_1 F_{ni}}{F_n^2} - k \frac{\Phi_n \Phi_{1i} - \Phi_1 \Phi_{ni}}{\Phi_n^2} \right\} = 0 \qquad i = 1 \ldots n \tag{7.5}$$

If the expression $\frac{F_n F_{1i} - F_1 F_{ni}}{F_n^2}$ is denoted by Q_i and the equivalent expression for the Φ's by R_i, then the conditions (7.5) can be re-written in the following form :

$$\frac{F_i}{F_n} = \frac{\Phi_i}{\Phi_n} \frac{\left[1 + \frac{\mu}{\lambda'} (Q_i - kR_i)\right]}{\left[1 + \frac{\mu}{\lambda'} (Q_n - kR_n)\right]} \tag{7.6}$$

These are the conditions for the attainment of the second best position, given the constraint (7.3), expressed in a form comparable with the Paretian conditions as set out in (7.2).

Clearly, any one of the conditions for the second best will be the same as the equivalent Paretian condition only if the expression :

$$\frac{1 + \frac{\mu}{\lambda'}(Q_i - kR_i)}{1 + \frac{\mu}{\lambda'}(Q_n - kR_n)} \quad \text{is unity.}$$

Now this will only be the case if :

(i) $\mu = 0$

or (ii) $\mu \neq 0$, but $Q_i - kR_i = Q_n - kR_n$

The first of these cannot be true for, if it were, then, when $i = 1$, $\frac{F_1}{F_n}$ would be equal to $\frac{\Phi_1}{\Phi_n}$, in contradiction with the constriant condition (7.3).

It is clear from the nature of the expressions Q_i, Q_n, R_i, R_n that nothing is known, in general, about their signs, let alone their magnitudes, and even the signs would not be sufficient to determine whether (ii) was satisfied or not.

Consider $Q_n = \dfrac{F_nF_{1n} - F_1F_{nn}}{F_n^2}$. If F were a utility function then it would be known that F_1, F_n were positive and F_{nn} negative, but the sign of F_{1n} may be either positive or negative.[1] Even if the sign of F_{1n} were known to be negative, the sign of Q_n would still be indeterminate, since it would depend on whether the negative or the positive term in the expression was numerically the greater. In the case of Q_i, where $i \neq n$, the indeterminacy is even greater, since there are two expressions F_{i1} and F_{ni} for which the signs may be either positive or negative.

The same considerations as apply for the Q's also apply for the R's of course. In general, therefore, the conditions for the second best optimum, given the constraint (7.3), will all differ from the corresponding conditions for the attainment of the Paretian optimum. Conversely, given the constraint (7.3), the application of these rules of behaviour of the Paretian type which are still attainable will not lead, in general, to the best position in the circumstances.

The general conditions for the achievement of the second best optimum in the type of case with which this analysis is concerned will be of the type $\frac{F_i}{F_n} = k_i \frac{\Phi_i}{\Phi_n}$, where $k_i \neq k_j \neq 1$, so that $\frac{F_i}{F_j} = \frac{\Phi_i}{\Phi_j}$, $\frac{F_i}{F_j} \neq \frac{F_k}{F_j}$, $\frac{\Phi_i}{\Phi_j} \neq \frac{\Phi_k}{\Phi_j}$, and the usual Paretian rules will be broken all round.

VIII THE EXISTENCE OF A SECOND BEST SOLUTION

The essential condition that a true second best solution to a given constrained situation should exist is that, if there is a Paretian optimum in which F has a maximum (minimum) when the constraint is removed, then the expression (7.4) must also have a true maximum (minimum). There is no reason why this should, in general, be the case.

For one thing, whereas well-behaved functions F and Φ will always have a solution which satisfies the comparatively simple first order conditions for a Paretian optimum,

[1] The Hicksian definitions of complementarity and substitution give no information about the signs of the individual F_{ij}'s, where F is a utility function. The Edgeworth-Pareto definitions do, and section IX considers the extent to which the knowledge of these signs enables *a priori* statements to be made about the nature of second best solutions.

it is by no means certain that the much more complex first order conditions (7.5) for a second best solution will be satisfied, since these conditions involve second order derivatives whose behaviour (subject only to convexity-concavity conditions of the functions) is unknown.

If the first order conditions for the existence of second best solutions present difficulties, the difficulties are quite insurmountable in the case of the second order conditions. Let it be supposed, for concreteness, that the nature of the case is such that F is to be maximised. Then the existence of a second best solution requires that the first order conditions (7.5) shall give a maximum, not a minimum or a turning point. This requires that the second differential of the expression (7.4) shall be negative. But the second differential of (7.4) involves the *third* order derivatives of F and Φ. Absolutely nothing is known about these in the general case, and their properties cannot be derived from the second order condition that the Paretian optimum represents a true maximum for F.

IX THE NATURE OF SECOND BEST SOLUTIONS

The extraordinary difficulty of making *a priori* judgments about the types of policy likely to be required in situations where the Paretian optimum is unattainable, and the second best must be aimed at, is well illustrated by examining the conditions (7.6) in the light of possible knowledge about the signs of some of the expressions involved.

In order to simplify the problem, and to render it less abstract, the function F will be supposed to be a utility function and Φ, which will be supposed to be a transformation function, will be assumed to be linear. The second derivatives of Φ disappear, so that $R_i = 0$ for all i, and attention can be concentrated on the expressions Q.

With the problem in this form, the derivatives F_i are proportional to the prices p_i, and the derivatives Φ_i are proportional to the marginal costs MC_i. As an additional simplification which assists verbal discussion but which does not affect the essentials of the model, it will be supposed that price equals marginal cost for the n'th commodity, which will be referred to as the numéraire.

From (7.6), with these additional assumptions, therefore :

$$\frac{\dfrac{F_i}{F_n}}{\dfrac{\Phi_i}{\Phi_n}} = \frac{\dfrac{p_i}{p_n}}{\dfrac{MC_i}{MC_n}} = \frac{p_i}{MC_i} = \frac{1 + \dfrac{\mu}{\lambda'} Q_i}{1 + \dfrac{\mu}{\lambda'} Q_n} = \frac{1 + \theta Q_i}{1 + \theta Q_n} \quad \left(\theta = \frac{\mu}{\lambda'}\right) \tag{9.1}$$

Thus, for the i'th commodity, price is $\begin{pmatrix} \text{above} \\ \text{equal to} \\ \text{below} \end{pmatrix}$ marginal cost, when the second best optimum is attained, according as :

$$P = \frac{1 + \theta Q_i}{1 + \theta Q_n} \gtreqless 1$$

The problem is reduced to that of discovering what can be said, *a priori*, about the magnitude of this expression.

Now $Q_i = \dfrac{F_n F_{1i} - F_1 F_{ni}}{F_n^2}$. At most, it may be possible to deduce the *sign* of Q_i but the order of its magnitude will remain unknown unless a specific utility function is given.

With knowledge of signs, and no more, the most that can be said can be summarised very simply :

$$
\begin{aligned}
&\text{(i) If } \theta > 0,\ P > 1 \text{ if } Q_i > 0,\ Q_n < 0. \\
&\qquad\qquad\quad P < 1 \text{ if } Q_i < 0,\ Q_n > 0. \\
&\text{(ii) If } \theta < 0,\ P > 1 \text{ if } Q_i < 0,\ Q_n > 0. \\
&\qquad\qquad\quad P < 1 \text{ if } Q_i > 0,\ Q_n < 0.
\end{aligned}
\tag{9.2}
$$

Nothing can be said about P if Q_i, Q_n are of the same signs.

Now consider Q_i. The denominator is always positive, and F_1, F_n are both positive, so that the determining factors are the signs of the mixed partial derivatives F_{1i} and F_{ni}. It is assumed that goods are known to be substitutes ($F_{ij} < 0$) or complements ($F_{ij} > 0$) in the Edgeworth-Pareto sense. There are four possible cases :

(*a*) If $F_{1i} > 0$, $F_{ni} > 0$, then $Q_i \gtrless 0$
(*b*) If $F_{1i} < 0$, $F_{ni} < 0$, then $Q- \gtrless 0$
(*c*) If $F_{1i} > 0$, $F_{ni} < 0$, then $Q_i > 0$
(*d*) If $F_{1i} < 0$, $F_{ni} > 0$, then $Q_i < 0$.

In cases (*c*) and (*d*), but not in cases (*a*) and (*b*), therefore, the sign of Q_i is determinate.

To complete the picture the sign of θ is also needed. Where the sign of this can be found at all, it is found by putting $i = 1$ and substituting in the constraint condition (7.3). For concreteness, let k be >1 (the first good will be referred to as the monopolised good). Then, since $\dfrac{1 + \theta Q_1}{1 + \theta Q_n} = k > 1$, it can be deduced that, if $Q_1 < 0$, $Q_n > 0$, then $\theta < 0$, and if $Q_1 > 0$, $Q_n < 0$, then $\theta < 0$. In all other cases the sign of θ is indeterminate.

For Q_1, Q_n, it is known that F_{11}, $F_{nn} < 0$, and $F_{n1} = F_{1n}$ so that there are only two cases, $F_{n1} > 0$ and $F_{n1} < 0$. The information conveyed in each of the two cases is as follows :

I $F_{n1} > 0$: $Q_1 < 0$, $Q_n > 0$, so that $\theta < 0$.
II $F_{n1} < 0$: $Q_1 \gtrless 0$, $Q_n \gtrless 0$, so that $\theta \gtrless 0$.

The combination of cases I and II with the independently determined cases (*a*), (*b*), (*c*), (*d*) gives a total of eight cases. These are tabulated below, showing the information which can be derived about the signs of Q_i, Q_n and θ, and the consequent information about P using the conditions (9.2).

TABLE I

Case		Sign of Q_i	Sign of Q_n	Sign of θ	*Relationship of Price to marginal cost for* x_i
I $F_{n1}>0$	(*a*) F_{ij}, $F_{ni}>0$	?	+	—	?
	(*b*) F_{ij}, $F_{ni}<0$	?	+	—	?
	(*c*) $F_{1i}>0$, $F_{ni}<0$	+	+	—	?
	(*d*) $F_{1i}<0$, $F_{ni}>0$	—	+	—	Price exceeds marginal cost
II $F_{ni}<0$	(*a*) F_{1i}, $F_{ni}>0$	?	?	?	?
	(*b*) F_{1i}, $F_{ni}<0$	?	?	?	?
	(*c*) $F_{1i}>0$, $F_{ni}<0$	+	?	?	?
	(*d*) $F_{1i}<0$, $F_{ni}>0$	—	?	?	?

Of the eight cases tabulated, the signs of Q_i, Q_n and θ are simultaneously determinate in only two, I(*c*) and I(*d*), and in only one of these two, I(*d*), does this lead to a determinate relationship between price and marginal cost. This sole case leads to the only *a priori* statement that can be made about the nature of second best solutions on the basis of the signs of the mixed second order partial derivatives of the utility function :

If the monopolised commodity is complementary (in the Edgeworth-Pareto sense) to the numéraire, and the *i*'th commodity is also complementary to the numéraire, but a substitute for the monopolised good, then, in order to attain a second best solution, the price of the *i*'th commodity must be set higher than its marginal cost.

Since knowledge of the sign alone of the derivatives F_{ij} reveals only one determinate case, it would seem worth while to examine the situation if more heroic assumptions can be made about the knowledge of the utility function. The additional information which is assumed is that two commodities may be known to be " weakly related ", that is, that the derivative F_{ij} is either zero or of the second order relative to other quantities.

In the expression $Q_i = \dfrac{F_nF_{1i} - F_1F_{ni}}{F_n^2}$, for example, if the *i*'th commodity and the numéraire are weakly related in this sense, then the term $F_1\ F_{ni}$ can be neglected relative to the term $F_n\ F_{1i}$, and the sign of Q_i is wholly determined by the sign of F_{1i}.

If the monopolised good and the numéraire are weakly related, then $Q_1 < 0$ and $Q_n > 0$. This is similar to the case I, in which the two goods were complements, leading to the same conclusions. There are now, however, four additional cases to add to (*a*), (*b*), (*c*), (*d*), for various combinations of weak relatedness with substitution and complementarity as between the *i*'th commodity and the monopolised good and the numéraire. All the cases which can be given in terms of the three relationships (weakly related, complements, substitutes) are tabulated in Table II. There are now three determinate cases, which can be summarised as follows :

If the monopolised good and the numéraire are either complements or only weakly related, then the second best solution will certainly require the price of the *i*'th good to be set above its marginal cost either if the good is a substitute for the monopolised good and either complementary or only weakly related to the numéraire, or if the good is weakly related to the monopolised good but complementary to the numéraire.

With any other combinations of relatedness among the goods, it cannot be determined, *a priori*, whether the second best solution will require the price of any particular good to be above or below its marginal cost. In particular, if there is no complementarity between

TABLE II

Relationship between monopolised good and numeraire	*Relationship of i'th good to :* *Monopolised Good*	*Numeraire*	*Signs of* Q_i	Q_n	θ	*Price of i'th good relative to marginal cost*
Complements, or weak	Complements	Complements	?	+	—	?
	Substitutes	Substitutes	?	+	—	?
	Complements	Substitutes	+	+	—	?
	Substitutes	Complements	—	+	—	Higher
	Complements	Weak	+	+	—	?
	Substitutes	Weak	—	+	—	Higher
	Weak	Complements	—	+	—	Higher
	Weak	Substitutes	+	+	—	?
Substitutes	Any	Any	$\left.\begin{array}{c}+\\-\\?\end{array}\right\}$	?	?	?

any pairs of goods, and the relationship between the monopolised commodity and the numéraire is not weak, then there are no determinate cases.

As a matter of interest it is possible to work out conditions that may be likely to bring about any particular result. For example, a possible case in which the price of a good might be set below its marginal cost would be that in which the monopolised good, the numéraire, and the other good were all substitutes, but the rate at which marginal utility diminished was small in the case of the monopolised good (so that Q_1, Q_n would both be positive, with Q_1 large compared with Q_n, giving a positive value for θ), and the relationship of the good under discussion was much stronger with the monopolised good than with the numéraire (so that Q_i might be negative). There can be few real cases, however, where such guesses about the magnitudes of the quantities involved could be made.

X THE PROBLEM OF MULTIPLE-LAYER OPTIMA

In all the preceding analysis, the problems have been conceived in terms of a single-layer optimum. It has been assumed that the constraint which defined the Paretian optimum (the transformation function, for example) was a technically fixed datum, and was not, itself, the result of an optimisation process at a lower level.

The characteristic of general economic systems is, however, that they usually involve several successive processes of optimisation, of increasing generality. The transformation function, for example, may have been derived as the result of competitive firms maximising their profits. Firms are assumed to have minimised their costs before proceding to maximise their profits, and these costs are themselves derived from processes involving optimisation by the owners of the various factors of production.

It is of the nature of the economic process, therefore, that optimisation takes place at successive levels, and that the maximisation of a welfare function subject to a transformation function is only the topmost of these. It is also of the nature of Paretian optima (due to the simple proportionality of the conditions) that the optimisation at the different levels can be considered as independent problems.

In the case of a second best solution, however, the neat proportionality of the Paretian conditions disappears : this immediately poses the question whether a second best solution in the circumstances of a multiple-layer economic system will require a breaking of the Paretian conditions at lower levels of the system, as well as at the level at which the problem was initiated.

The present paper does not propose to examine the problem, for it is a subject that would seem to merit full-scale treatment of its own. There seems reason to suppose, however, that there may well be cases in which a breaking of the Paretian rules at lower levels of the process (moving off the transformation function, for example) may enable a higher level of welfare to be obtained than if the scope of policy is confined to one level only.

A two-dimensional geometric illustration that is suggestive, although not conclusive, is set out in Figure 1. Ox, Oy represent the quantities of two goods x, y. The line AB represents a transformation function (to be considered as a boundary condition) and CD a constraint condition. In the absence of the constraint CD the optimum position will be some point, such as P, lying on the transformation line at the point of its tangency with one of the contours of the welfare function.

If the constraint condition must be satisfied, only points along CD can be chosen, and the optimum point P is no longer attainable. A point on the transformation line (Q) is still attainable. Will the second best solution be at the point Q, or should the economy move off the transformation line ? If the welfare contours and the constraint line are as shown in the diagram, then the second best point will be at the point R, inside the transformation line.

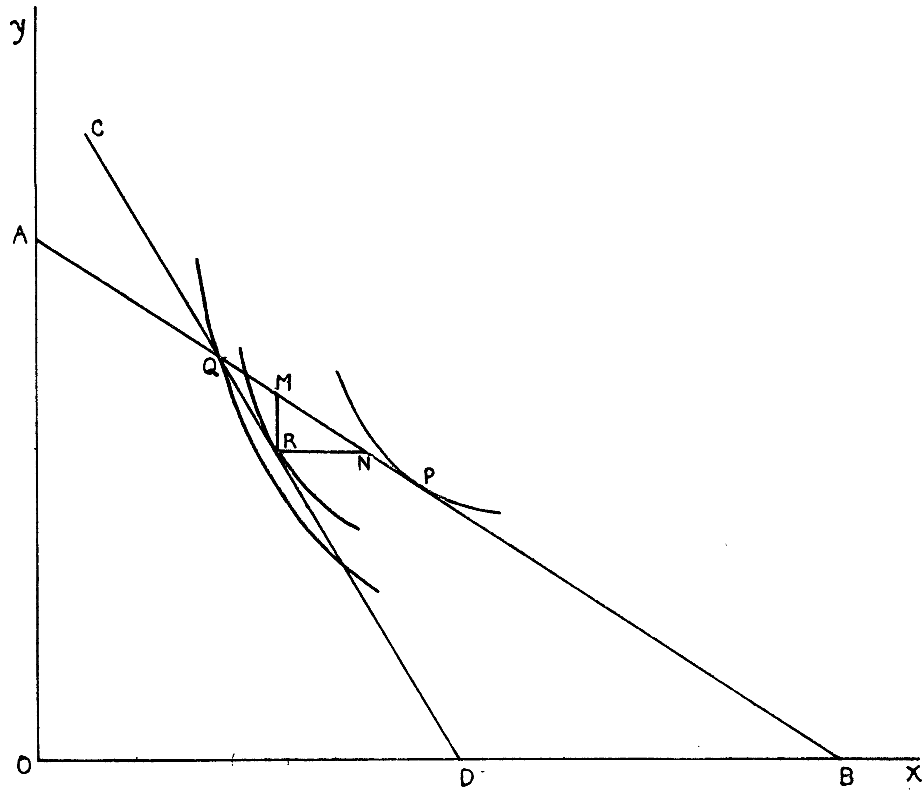

FIG. 1

It is obvious, of course, that the second best will never be at a point which is technically inefficient (has less of one commodity and no more of the other) relative to any *attainable* point. Although there are points (the segment *MN*) on the transformation line which are technically more efficient than *R*, these are not attainable. *R* is *not* technically inefficient relative to *Q*, even though *R* lies inside the transformation line.

If the line *CD* had a positive slope (as have the types of constraint which have been exemplified in the preceding analyses), the second best would always lie at its point of intersection with the transformation line, since all other points on *CD* would be technically inefficient relative to it.

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