



Escuela de  
Ingeniería  
Universidad  
de Chile



# FI33A ELECTROMAGNETISMO

## Clase 15

### Magnetostática-II

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Departamento de Ingeniería Eléctrica  
Universidad de Chile



# INDICE

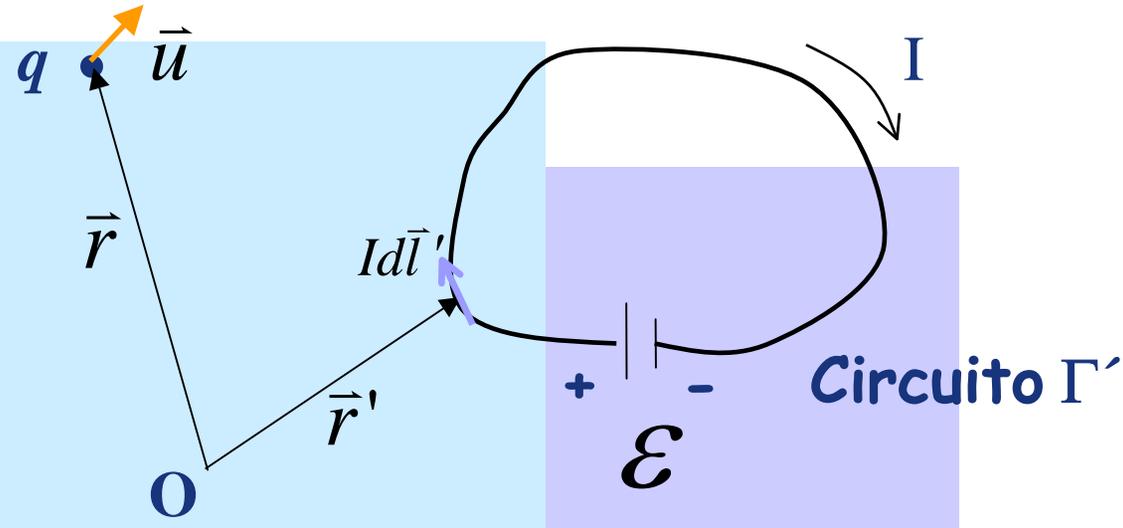
- Campo magnético
- Campo magnético de cargas y distribuciones
- Ley de Biot y Savarat
- Ley circuital de Ampere

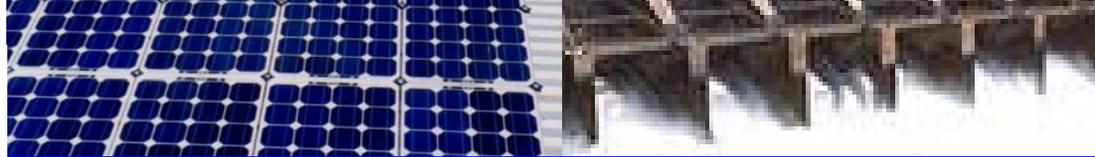


# Campo magnético

Campo producido  
por circuito  $\Gamma'$

$$\vec{B} = \oint_{\Gamma'} \frac{\mu_0 I d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$



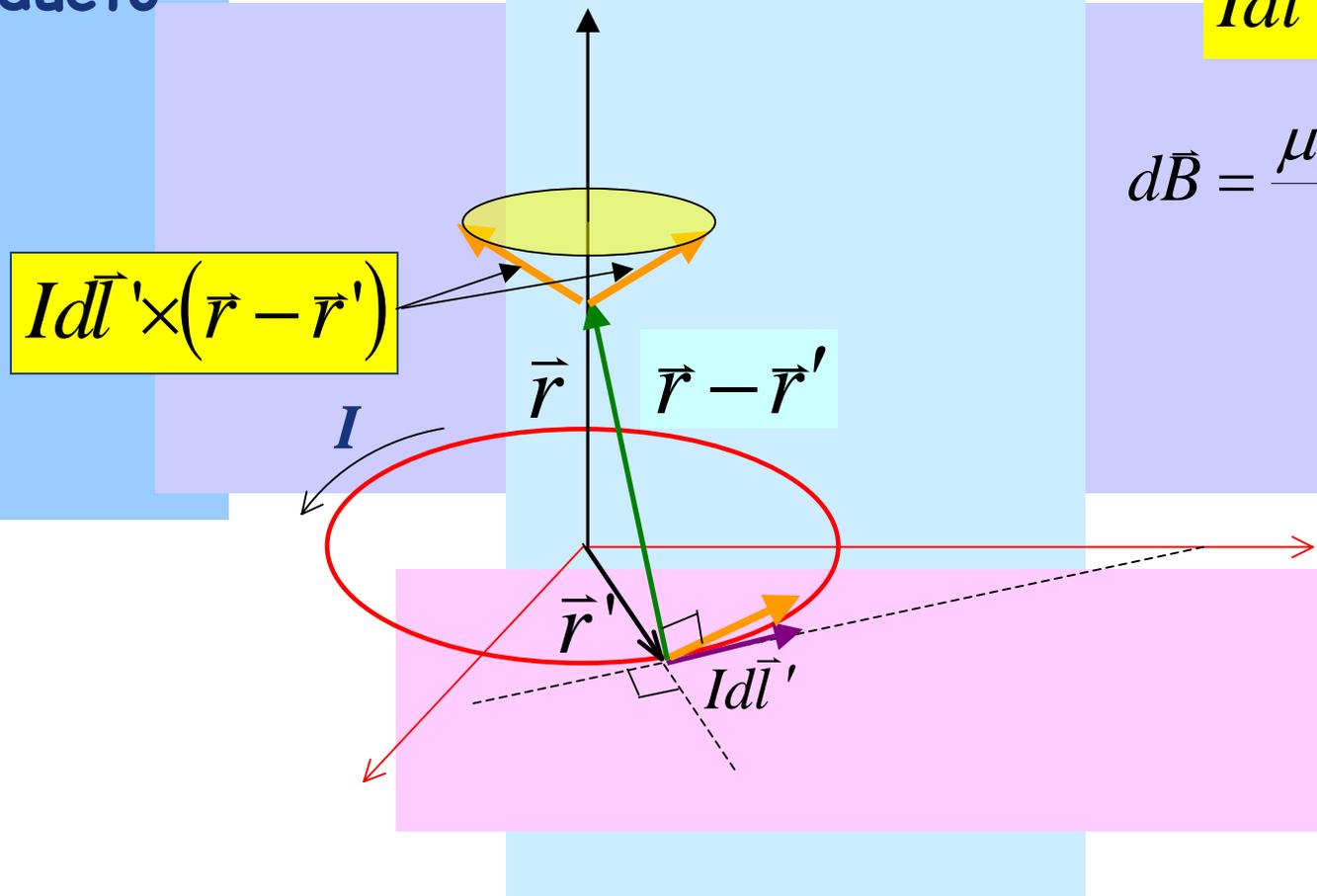


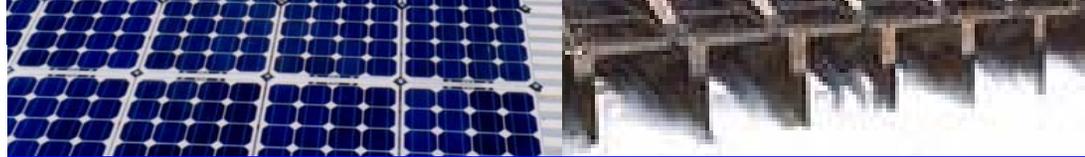
# Regla de la mano derecha

Dirección de campo está dado por el producto

$$Id\vec{l}' \times (\vec{r} - \vec{r}')$$

$$d\vec{B} = \frac{\mu_0 Id\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$

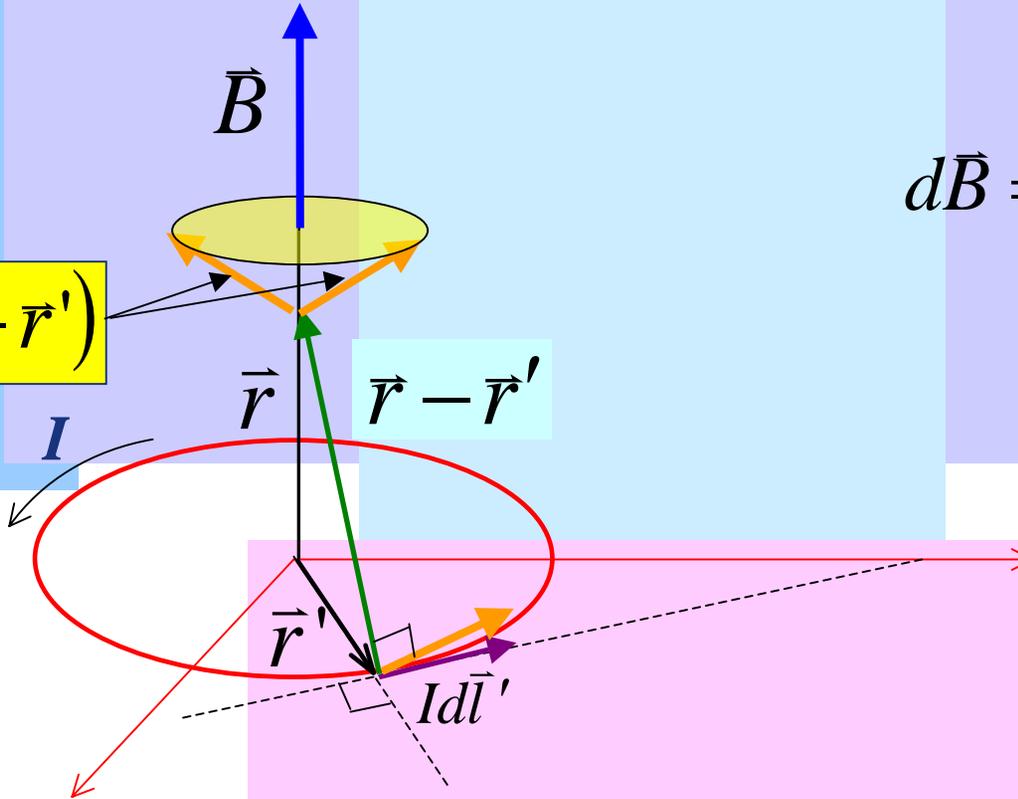




# Regla de la mano derecha

Campo magnético resultante sólo tiene dirección según eje z.

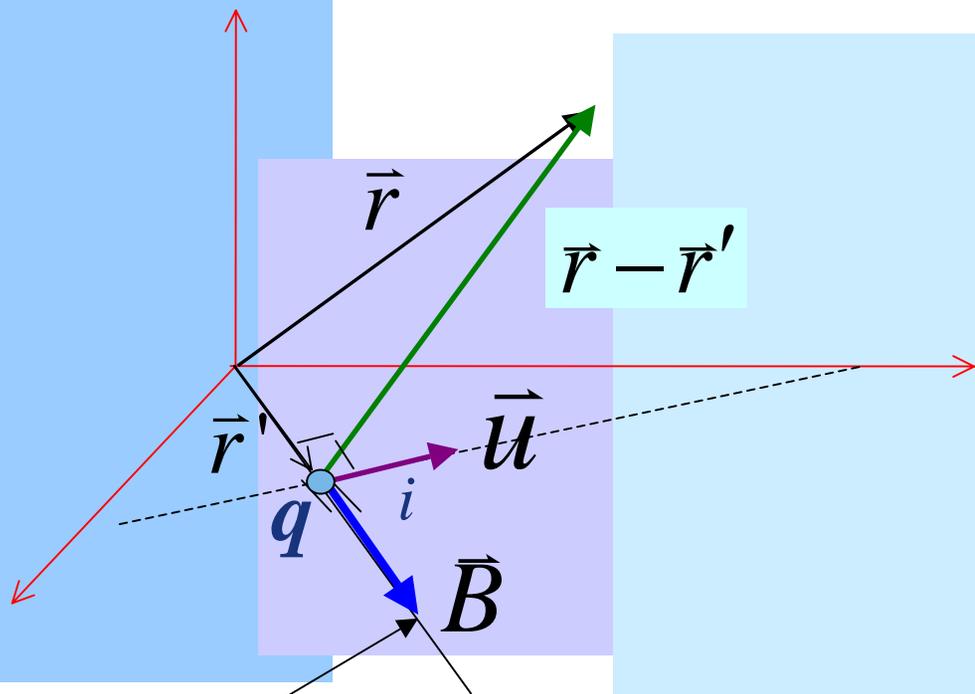
$$Id\vec{l}' \times (\vec{r} - \vec{r}')$$



$$d\vec{B} = \frac{\mu_0 Id\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$



# Campo Magnético de una Carga Puntual



$$d\vec{B} = \frac{\mu_0 I d\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$

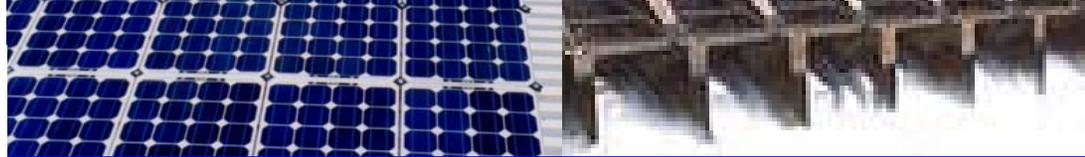
$$i d\vec{l}' = \frac{dq}{dt} dl \hat{u} = dq \frac{dl}{dt} \hat{u}$$

$$dq \rightarrow q, \quad \frac{dl}{dt} = \|\vec{u}\| \quad \gamma \quad d\vec{B} \rightarrow \vec{B}$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} q \vec{u} \times \frac{(\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3}$$

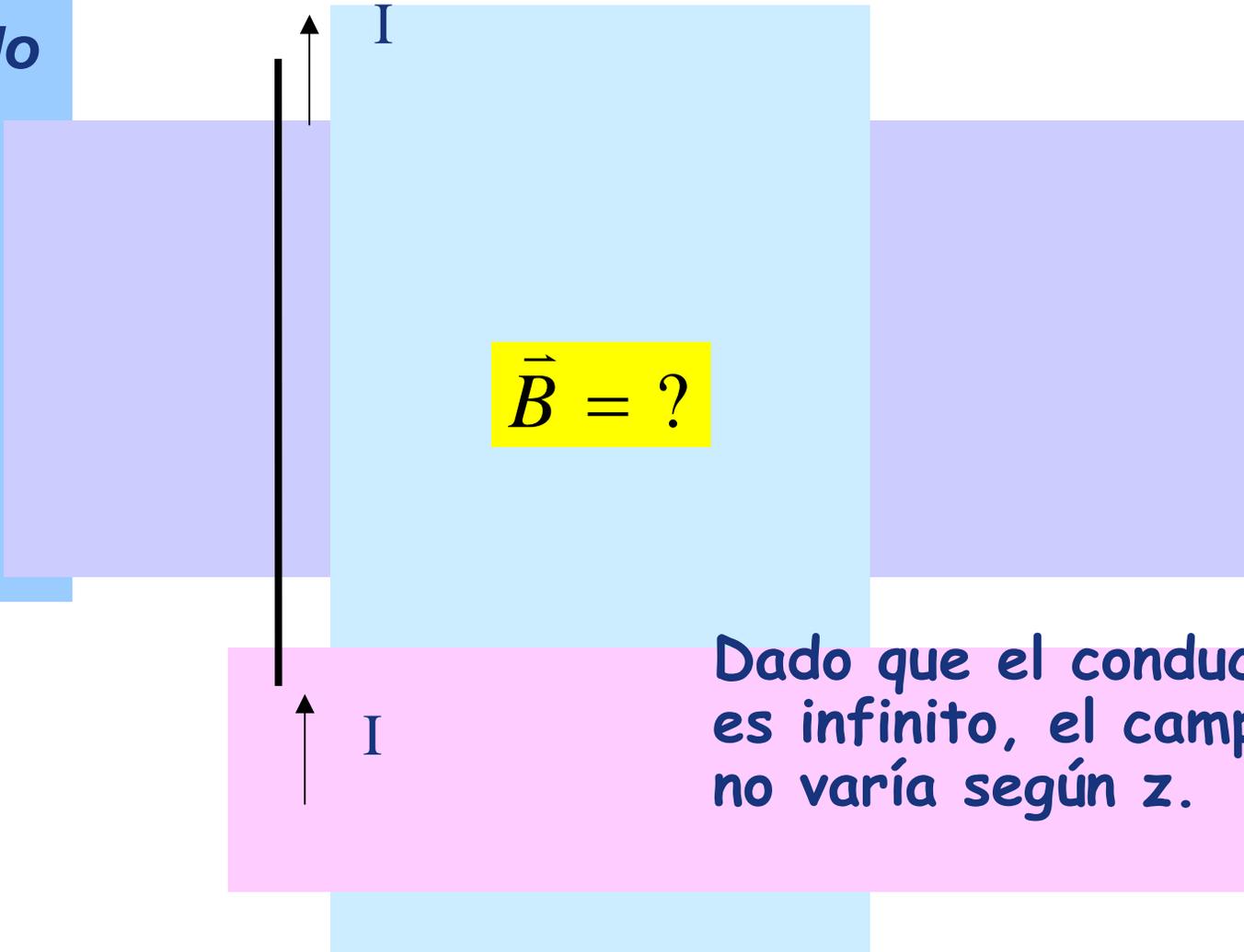
$$I d\vec{l}' \times (\vec{r} - \vec{r}')$$

Campo magnético resultante perpendicular a la velocidad



# Campo Magnético

*Ejemplo*

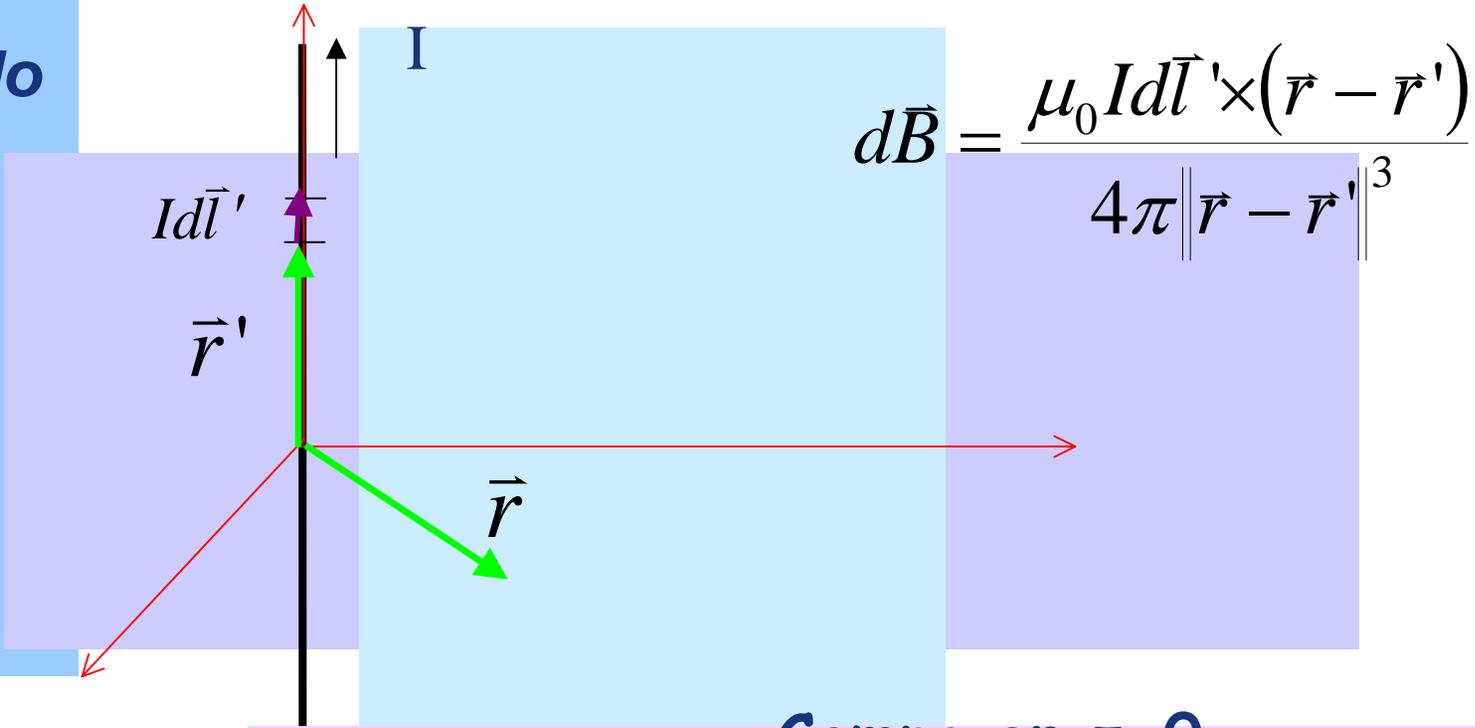


Dado que el conductor es infinito, el campo no varía según  $z$ .



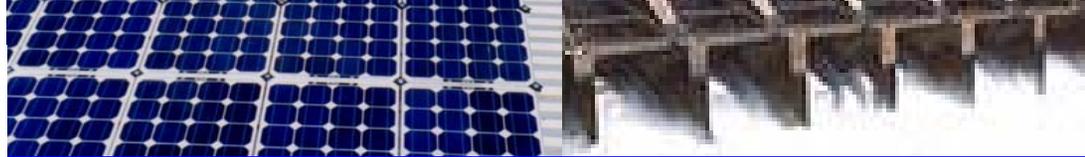
# Campo Magnético

**Ejemplo**



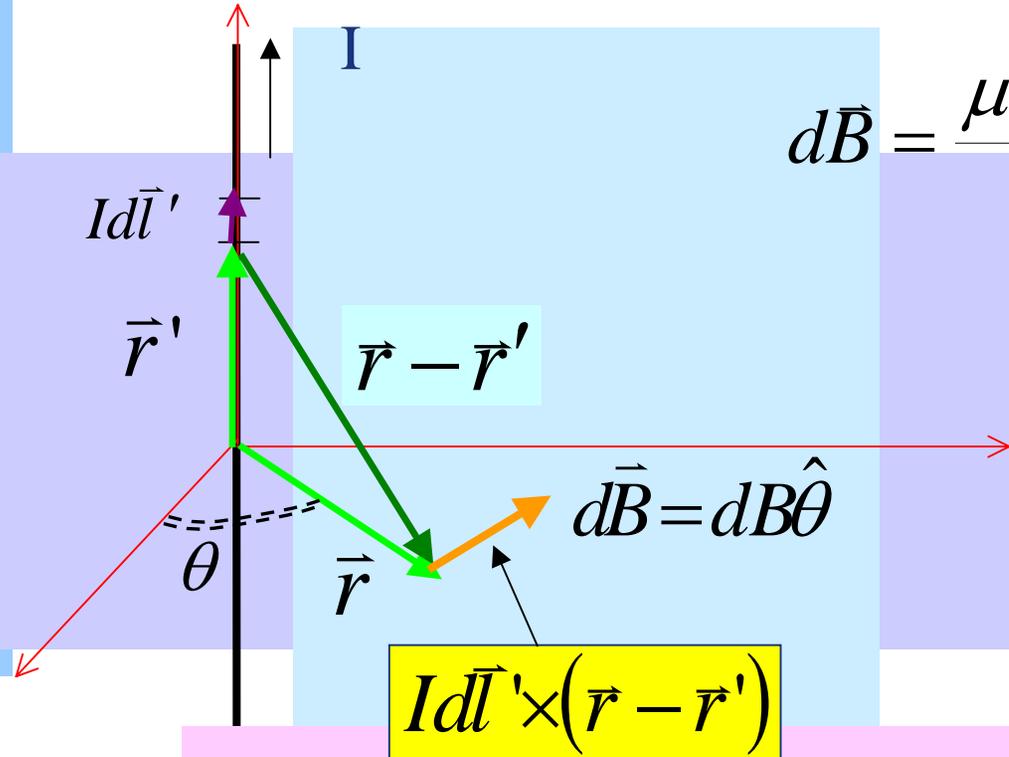
$$d\vec{B} = \frac{\mu_0 Id\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$

**Campo en z=0**

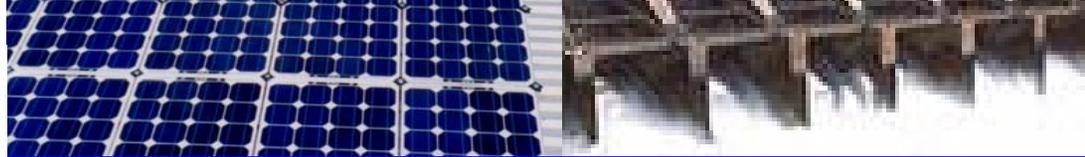


# Campo Magnético

Ejemplo

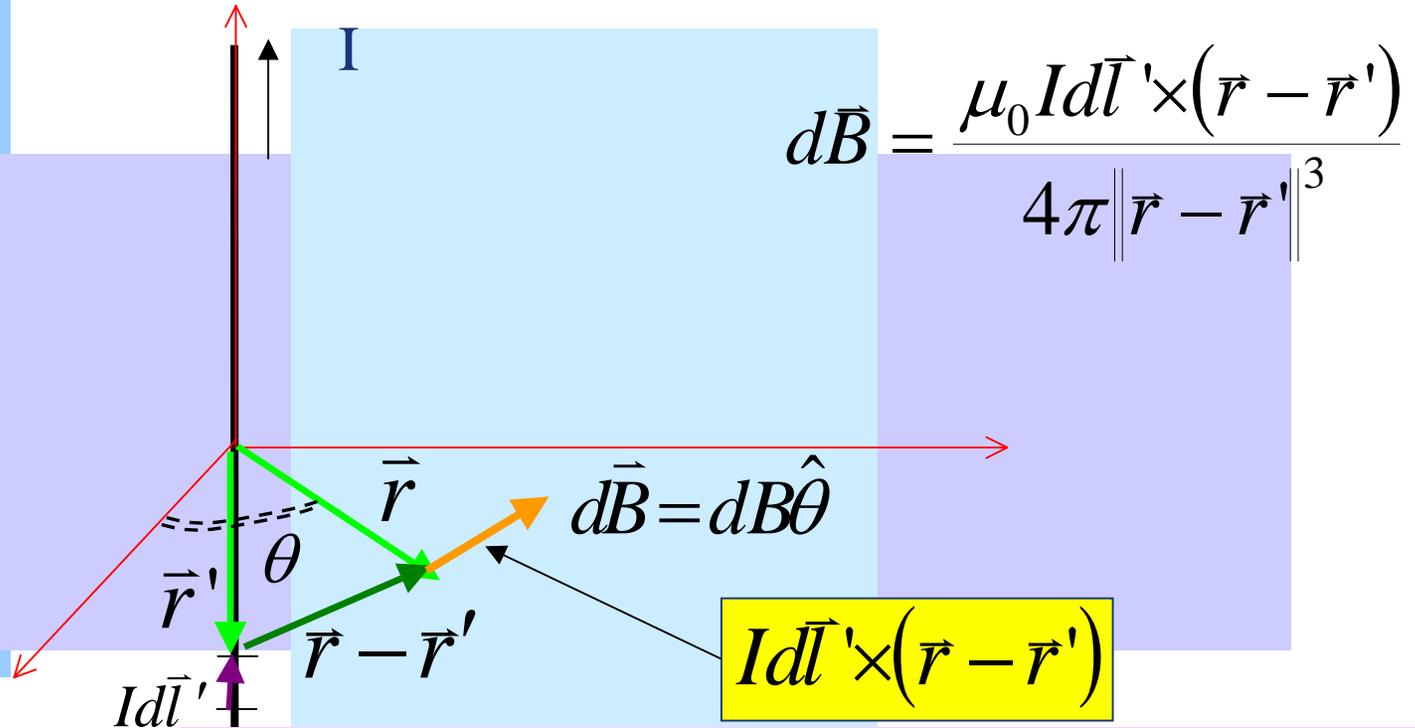


$$d\vec{B} = \frac{\mu_0 Id\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$



# Campo Magnético

Ejemplo

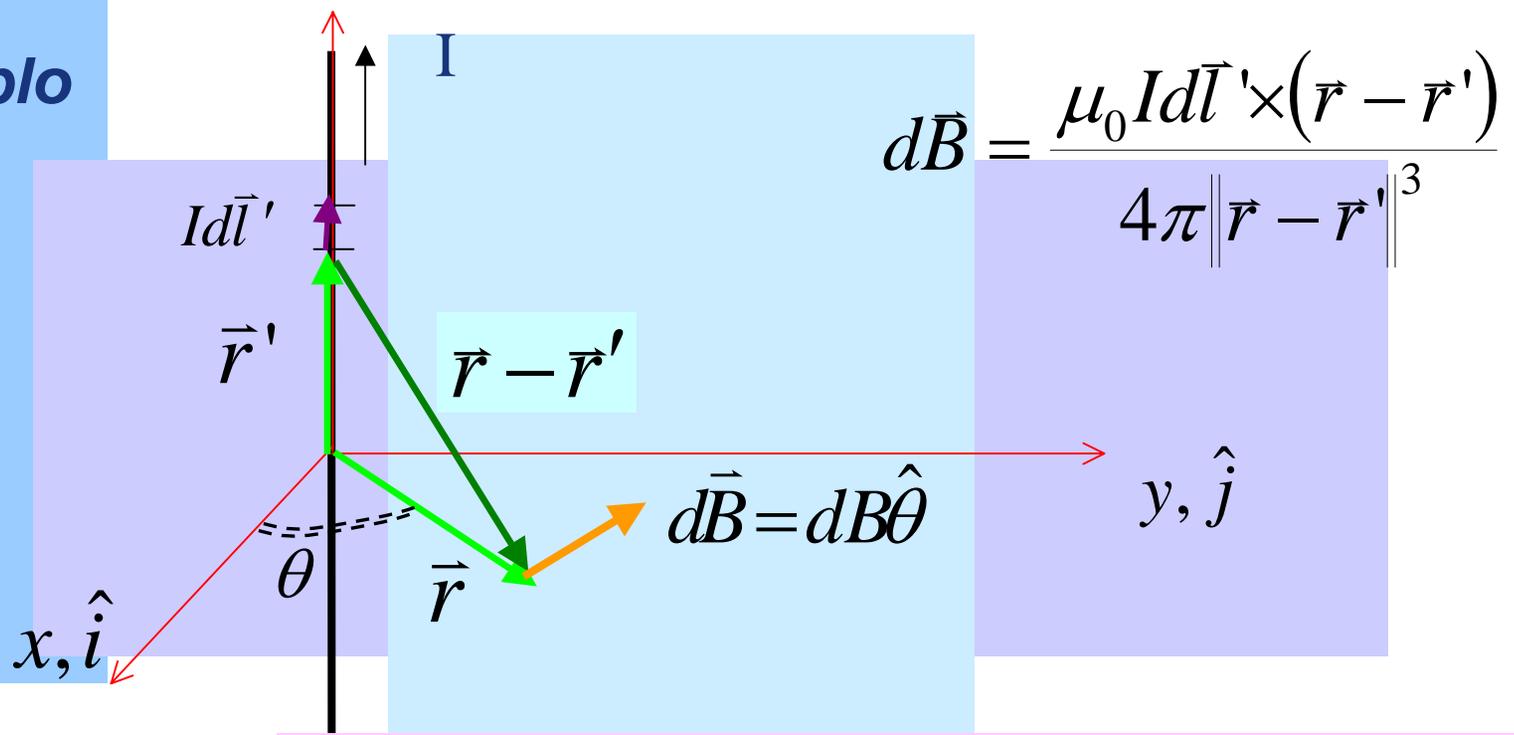


Notar que la contribución de todos los elementos de corriente tiene la misma dirección según  $\hat{\theta}$



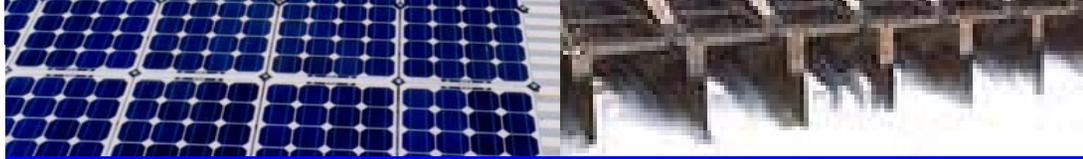
# Campo Magnético

**Ejemplo**



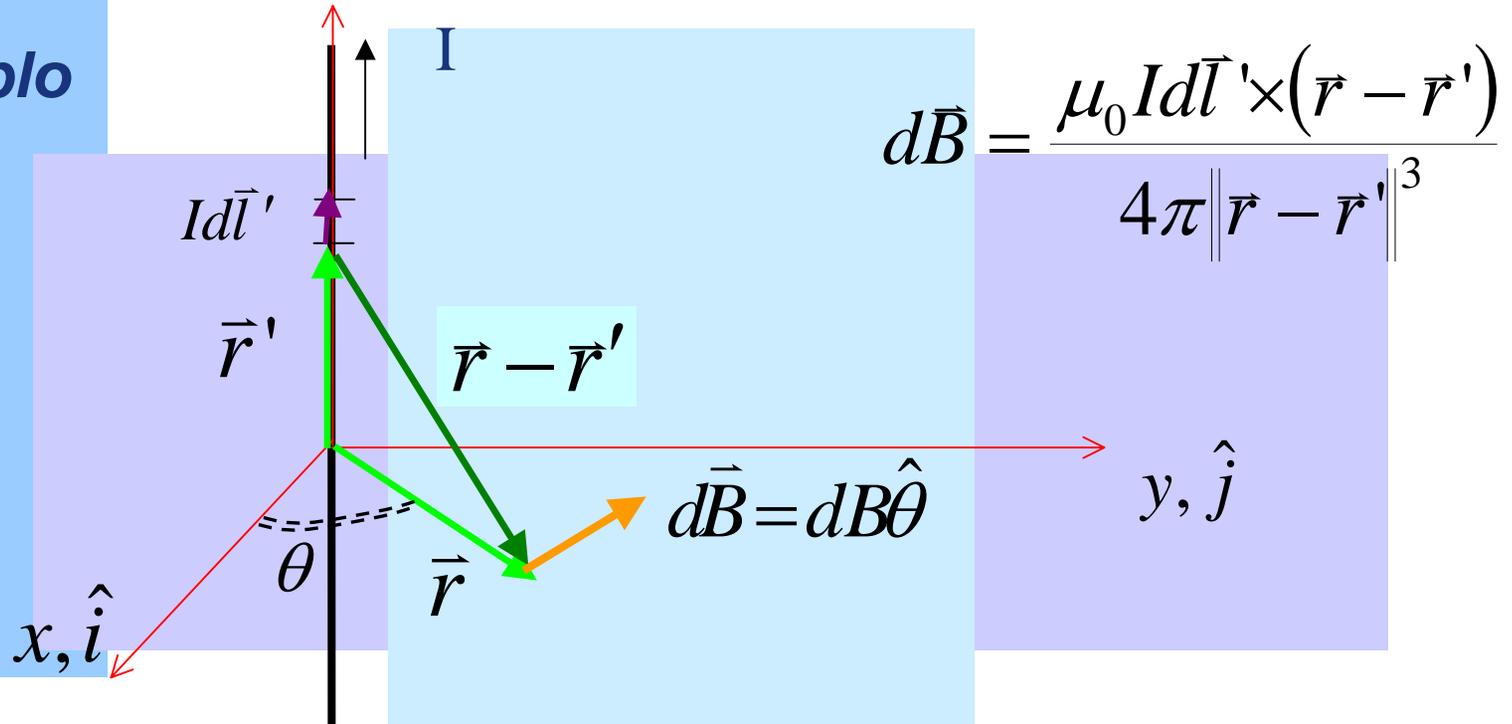
$$d\vec{B} = \frac{\mu_0 Id\vec{l}' \times (\vec{r} - \vec{r}')}{4\pi \|\vec{r} - \vec{r}'\|^3}$$

$$\left. \begin{aligned} \vec{r}' &= z'\hat{k} \\ \vec{r} &= r\cos\theta\hat{i} + r\sin\theta\hat{j} \\ Id\vec{l}' &= Idz'\hat{k} \end{aligned} \right\} d\vec{B} = \frac{\mu_0 Idz'\hat{k} \times (r\cos\theta\hat{i} + r\sin\theta\hat{j} - z'\hat{k})}{4\pi [r^2 + z'^2]^{3/2}}$$

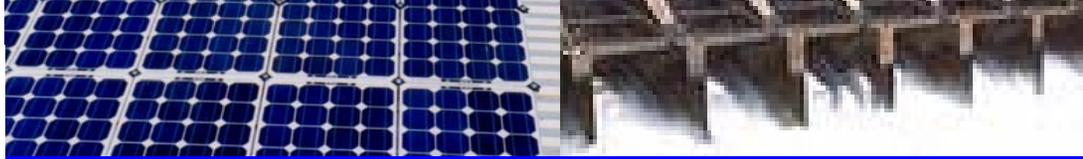


# Campo Magnético

Ejemplo

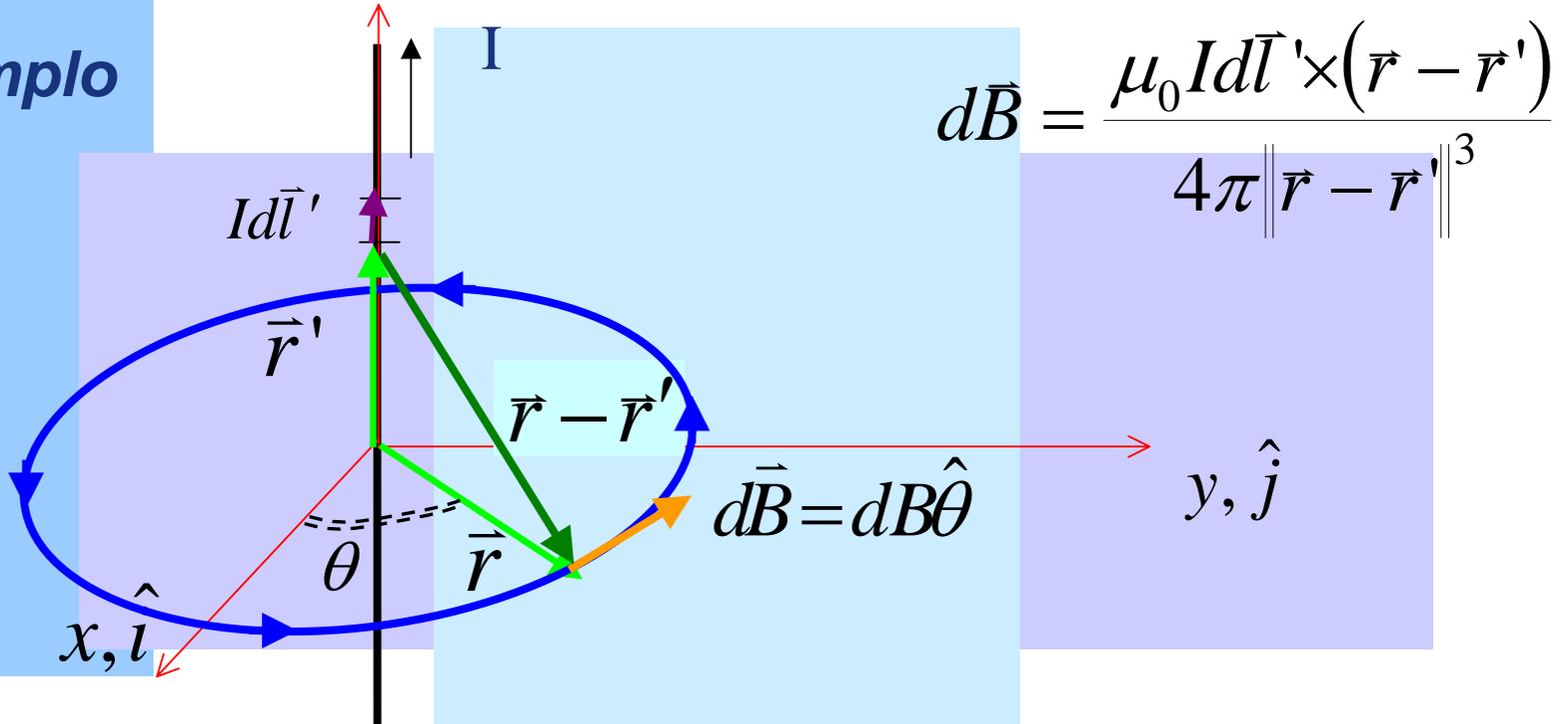


$$\vec{B} = \int_{z'=-\infty}^{z'=\infty} \frac{\mu_0 I r (\cos \theta \hat{j} - \sin \theta \hat{i}) dz'}{4\pi [r^2 + z'^2]^{3/2}}$$



# Campo Magnético

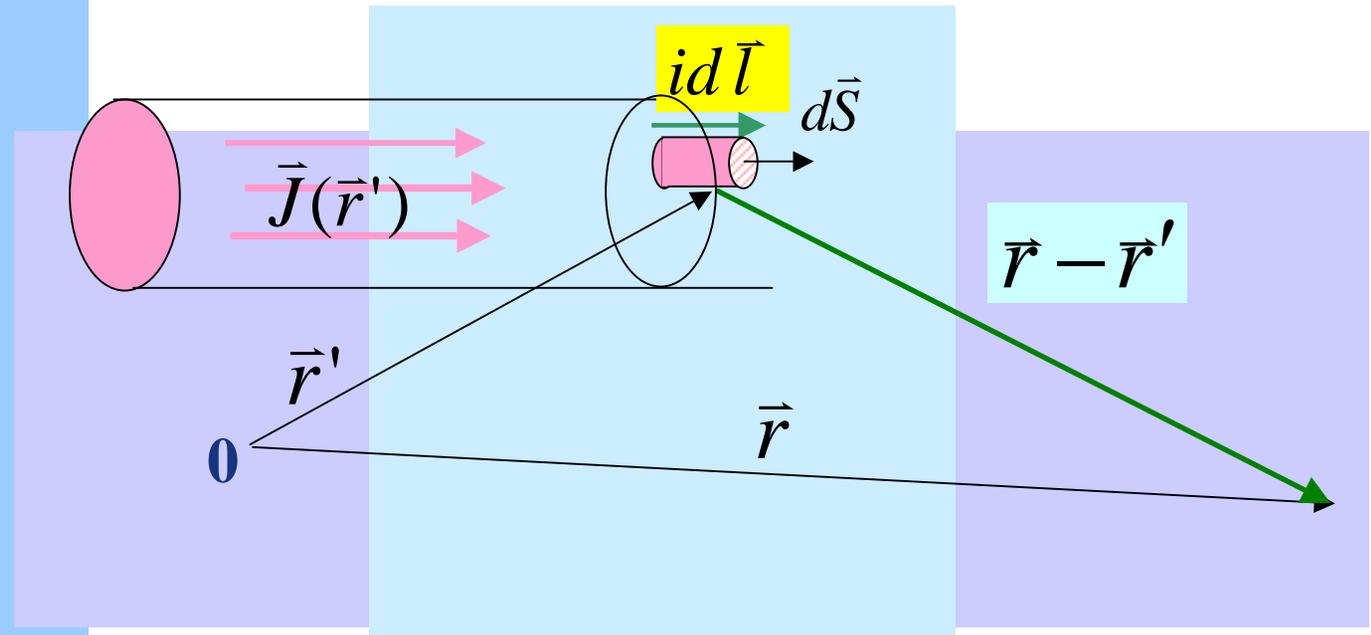
Ejemplo



$$\therefore \vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\theta}$$



# Campo magnético de distribuciones de corriente



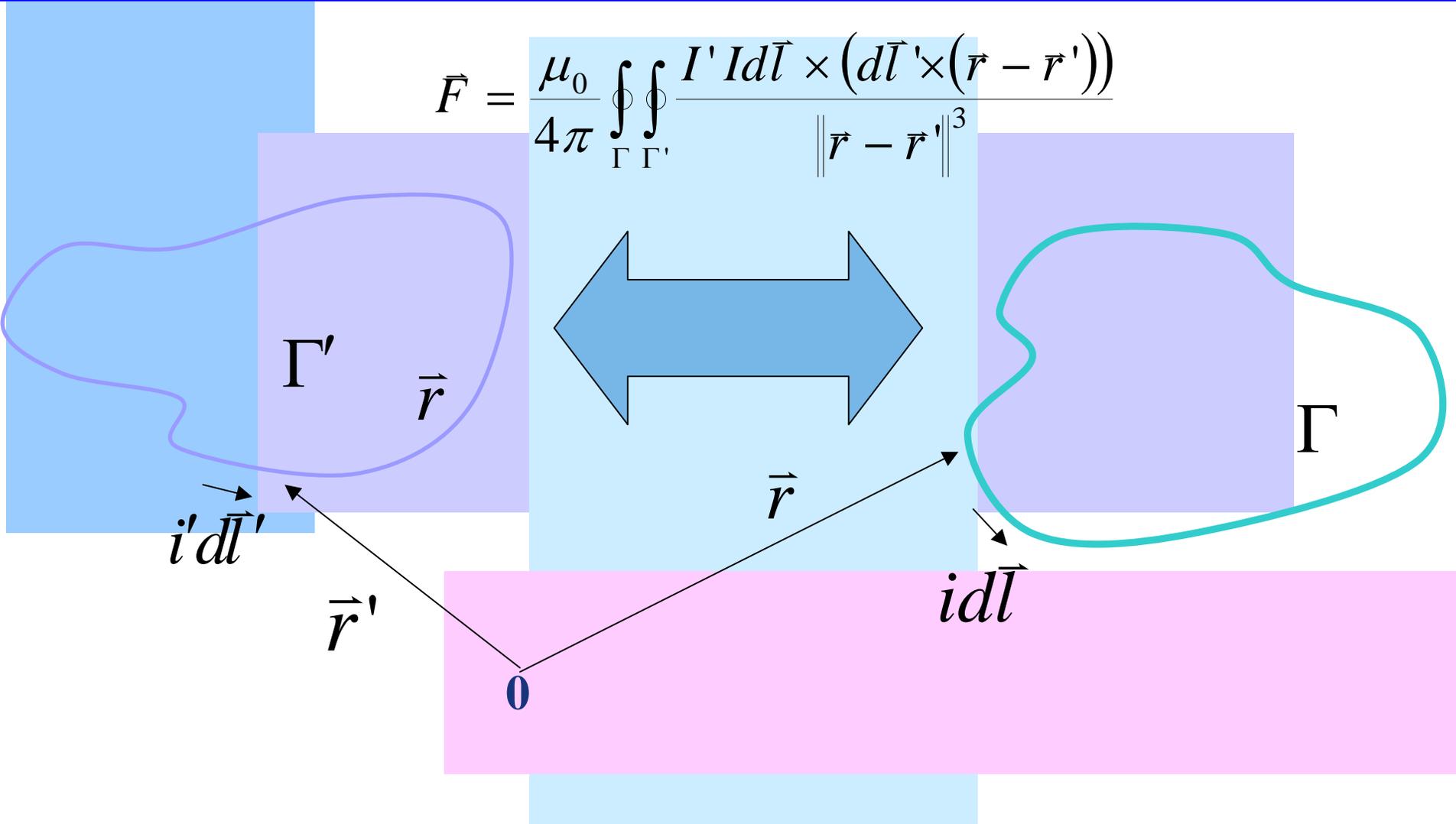
$$idl = \vec{J} \cdot d\vec{S} \cdot d\vec{l} = \vec{J} dV'$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_{V'} \frac{\vec{J}(\vec{r}') \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3} dV'$$



# Ley de Biot y Savarat

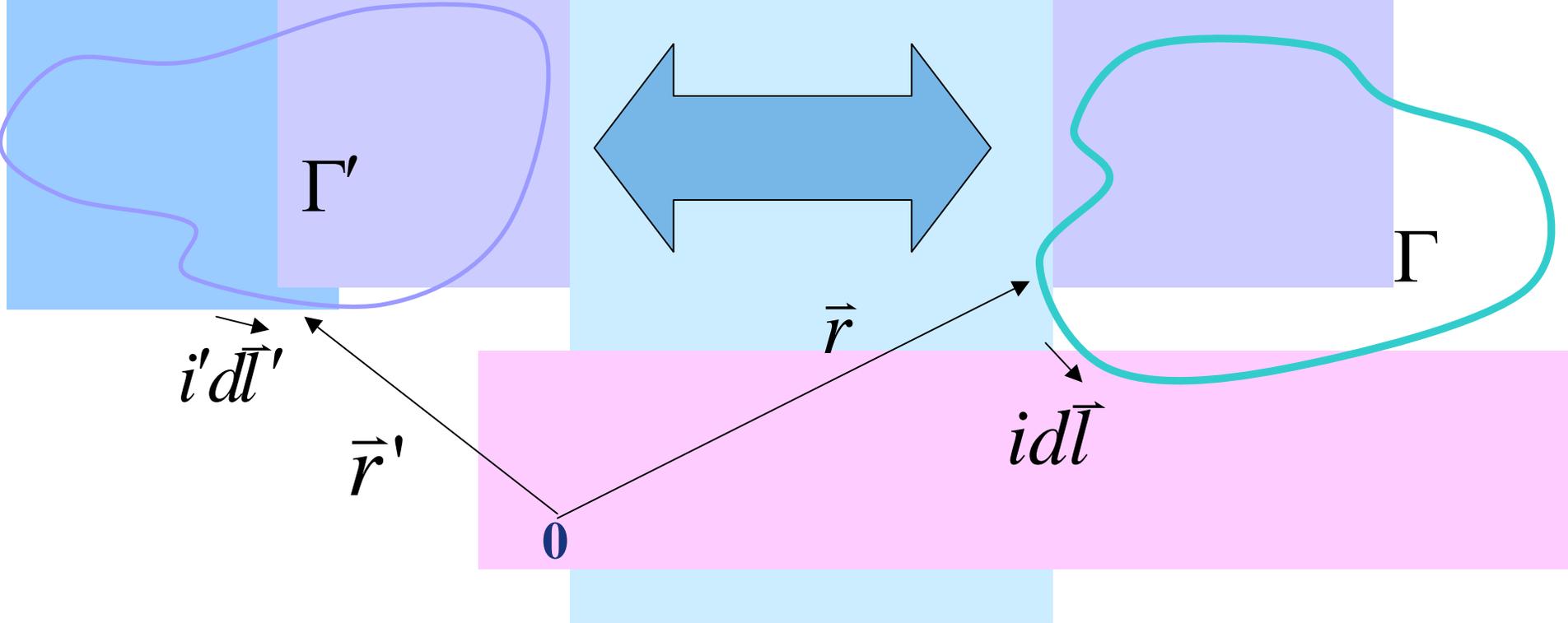
$$\vec{F} = \frac{\mu_0}{4\pi} \oint_{\Gamma} \oint_{\Gamma'} \frac{I' Id\vec{l} \times (d\vec{l}' \times (\vec{r} - \vec{r}'))}{\|\vec{r} - \vec{r}'\|^3}$$





# Ley de Biot y Savarat

$$\vec{F} = \frac{\mu_0}{4\pi} \oint_{\Gamma} \oint_{\Gamma'} \frac{I' Id\vec{l} \times (d\vec{l}' \times (\vec{r} - \vec{r}'))}{\|\vec{r} - \vec{r}'\|^3} \rightarrow d\vec{F} = \frac{Id\vec{l} \times \mu_0}{4\pi} \oint_{\Gamma'} \frac{I' d\vec{l}' \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3}$$

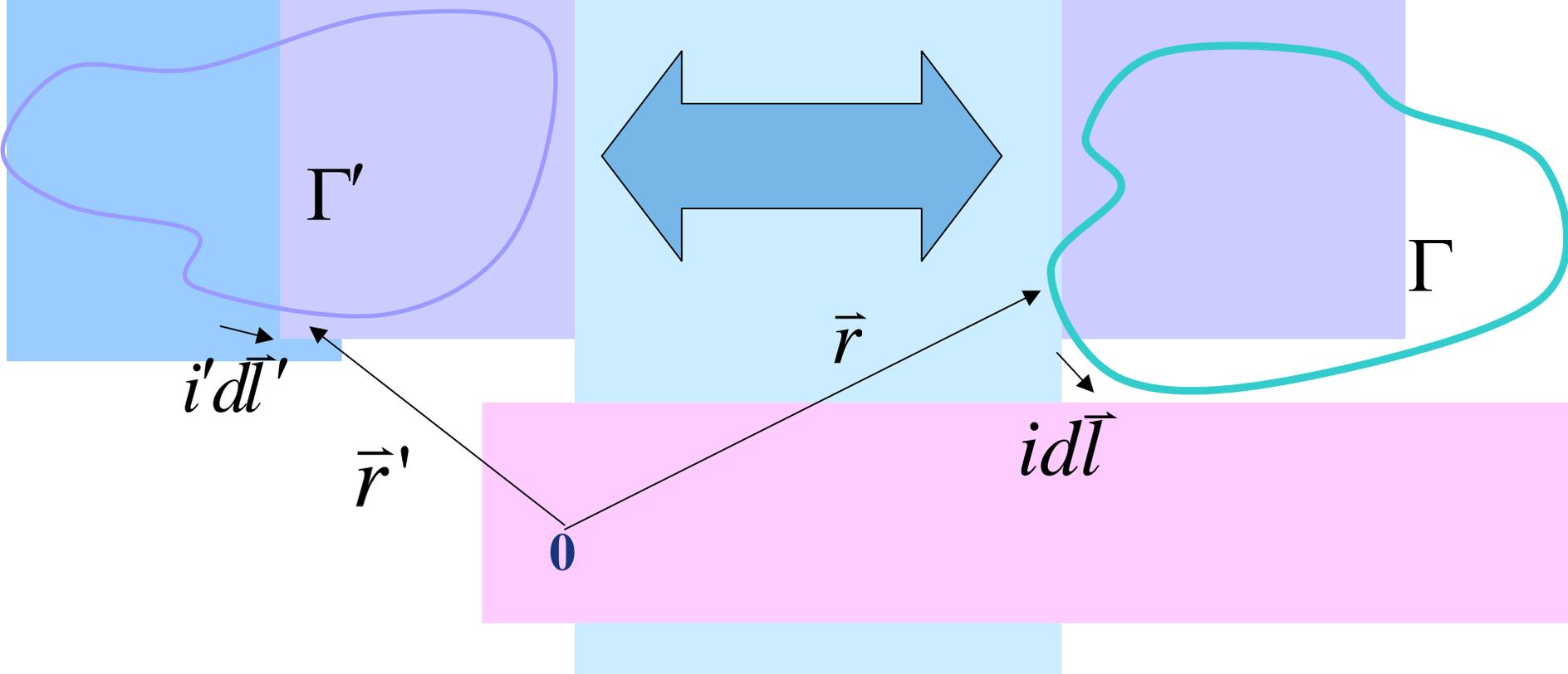


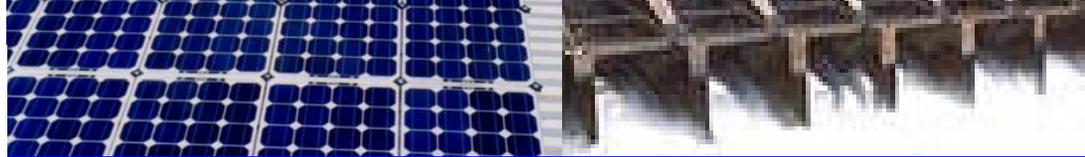


# Ley de Biot y Savarat

$$d\vec{F} = \frac{Id\vec{l} \times \mu_0}{4\pi} \oint_{\Gamma'} \frac{I' d\vec{l}' \times (\vec{r} - \vec{r}')}{\|\vec{r} - \vec{r}'\|^3}$$

$$\therefore d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$



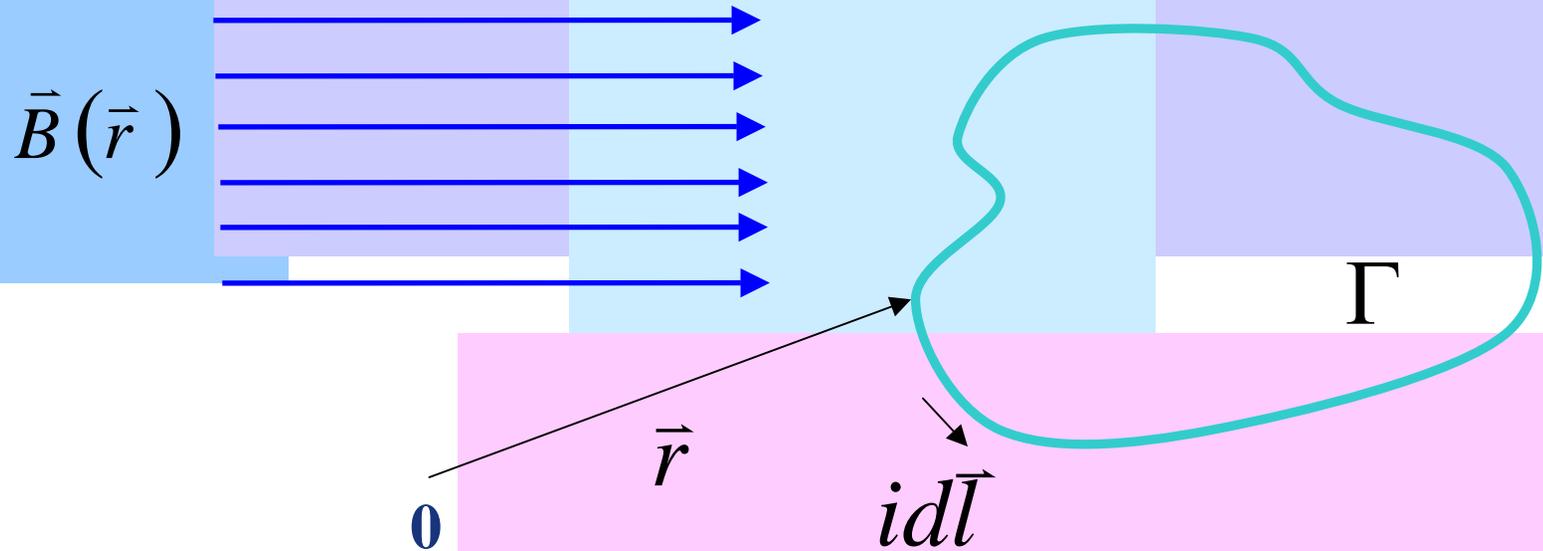


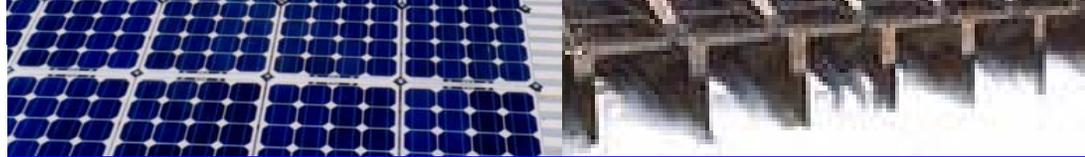
# Ley de Biot y Savarat

Así, un circuito en presencia de un campo magnético experimenta una fuerza dada por la ecuación

$$d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$

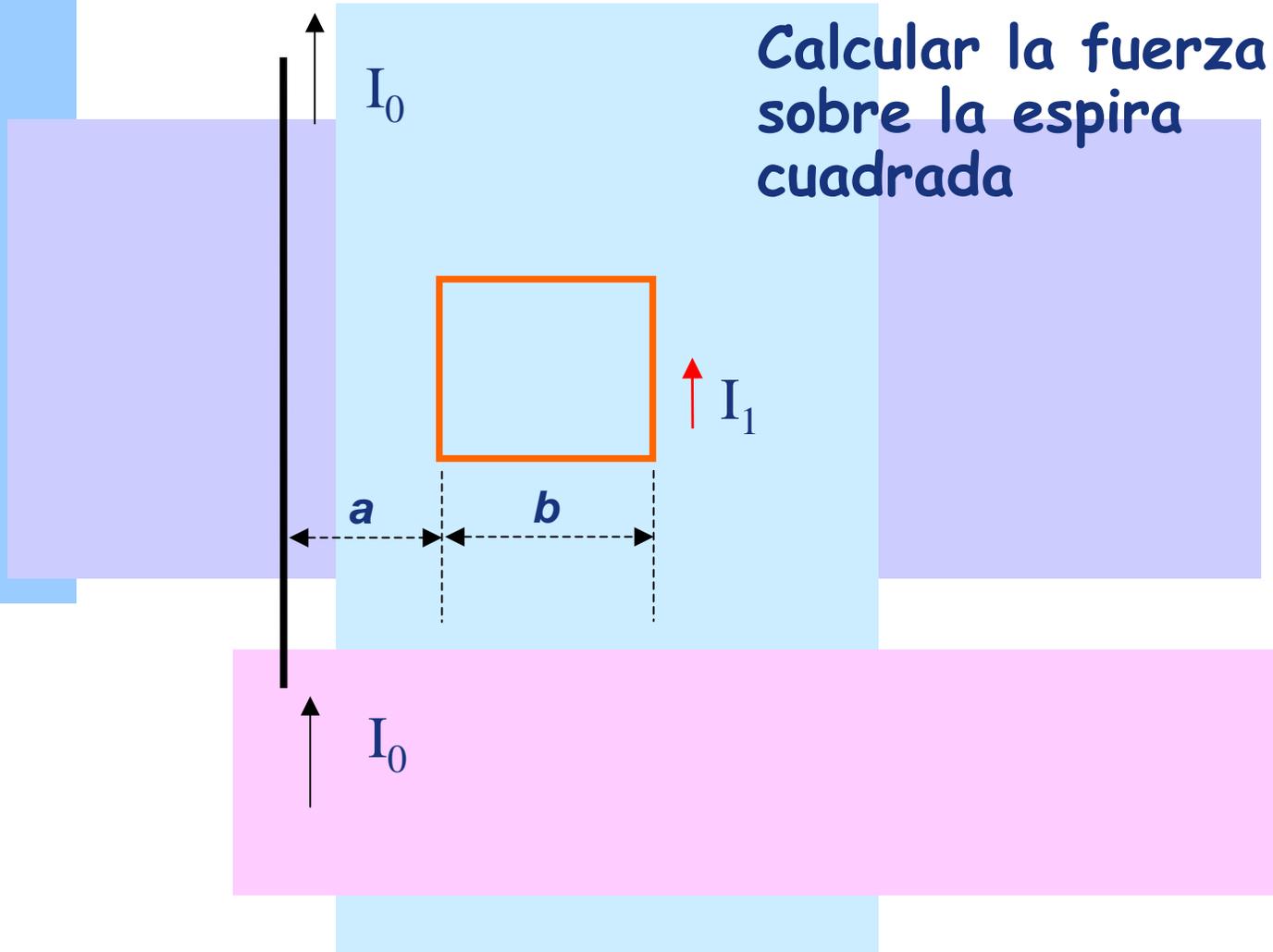
$$\therefore \vec{F} = \oint_{\Gamma} d\vec{F} = \oint_{\Gamma} Id\vec{l} \times \vec{B}(\vec{r})$$

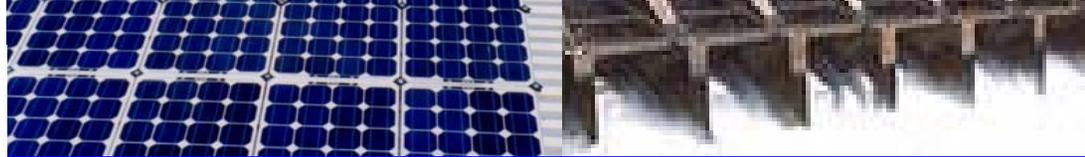




# Ley de Biot y Savarat

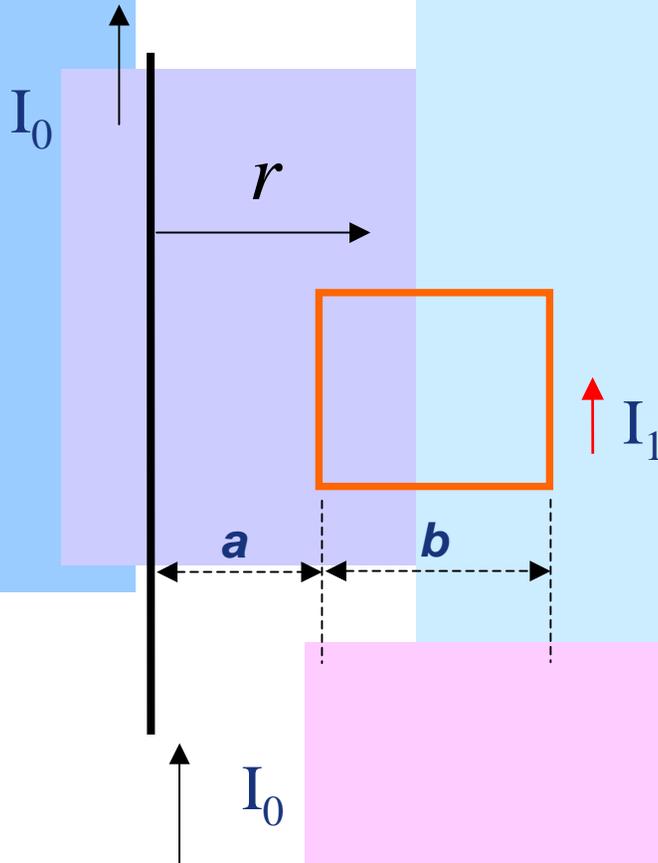
## Ejemplo





# Ley de Biot y Savarat

## Ejemplo



Campo producido por el conductor infinito es

$$\vec{B} = \frac{\mu_0 I_0}{2\pi r} \hat{\theta}$$

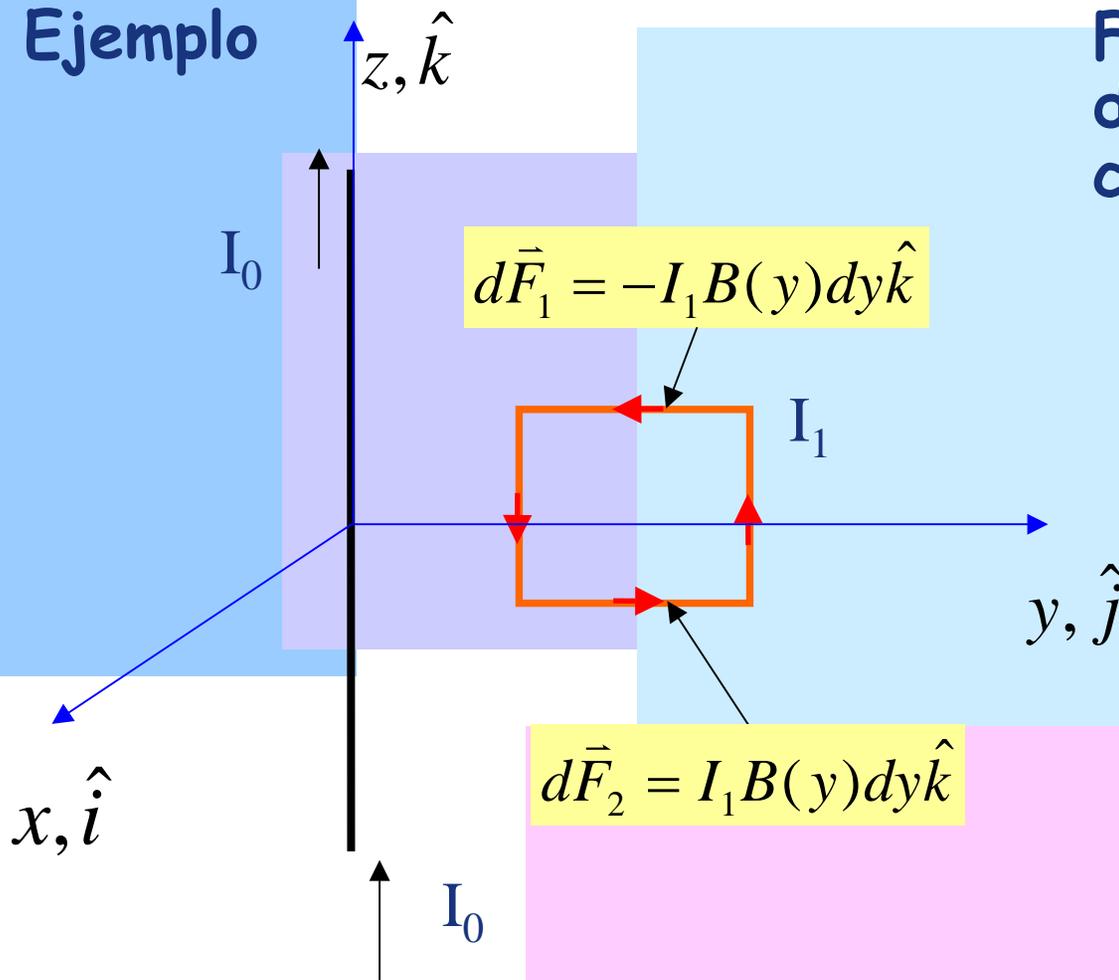
Fuerza sobre elemento de corriente de espira cuadrada

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$



# Ley de Biot y Savarat

Ejemplo



Fuerza sobre elemento de corriente de espira cuadrada

$$d\vec{F} = I_1 d\vec{l} \times \vec{B}(\vec{r})$$

$$\vec{B}(\vec{r}) = -B(y)\hat{i}$$

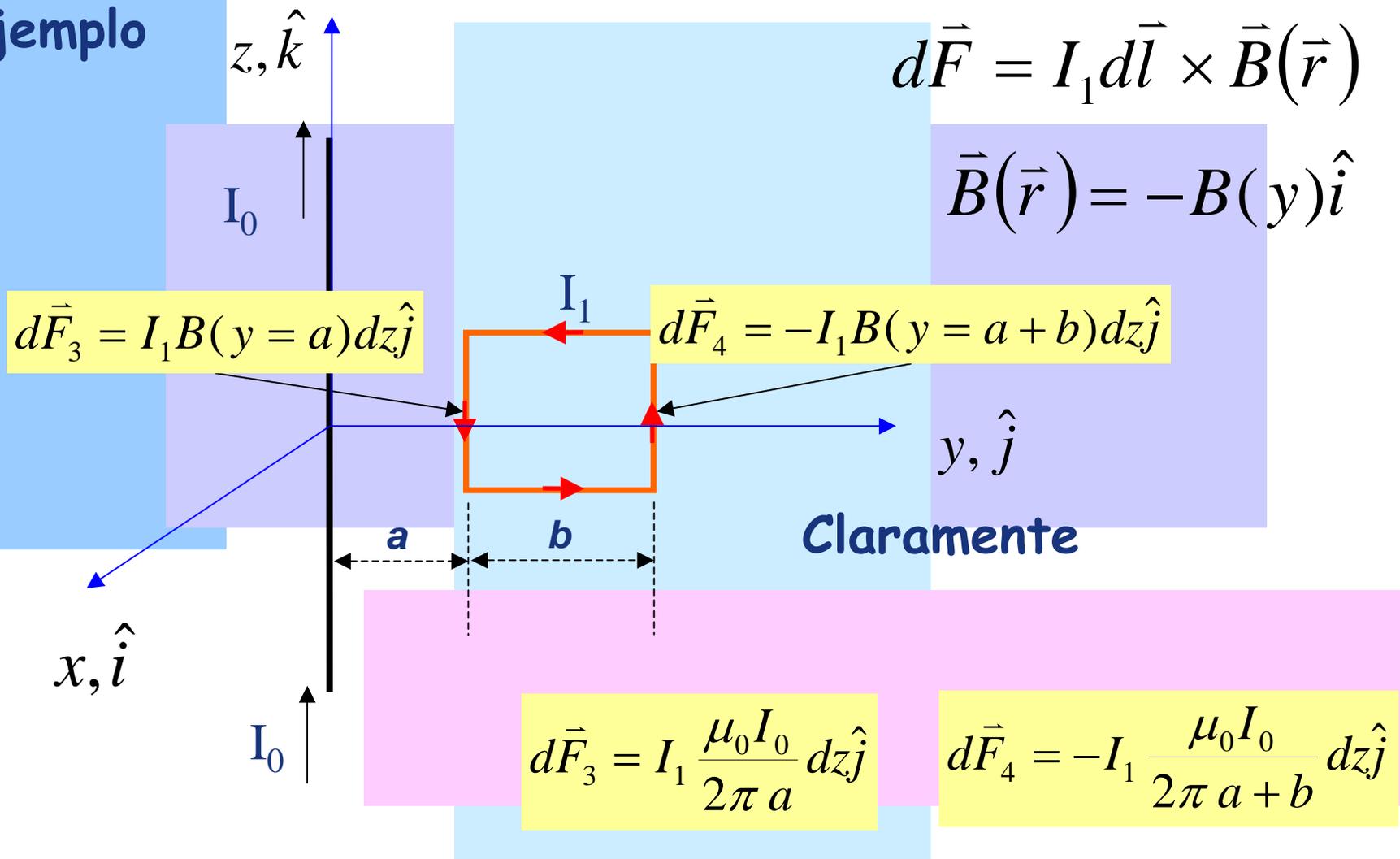
Claramente

$$d\vec{F}_1 = -d\vec{F}_2$$



# Ley de Biot y Savarat

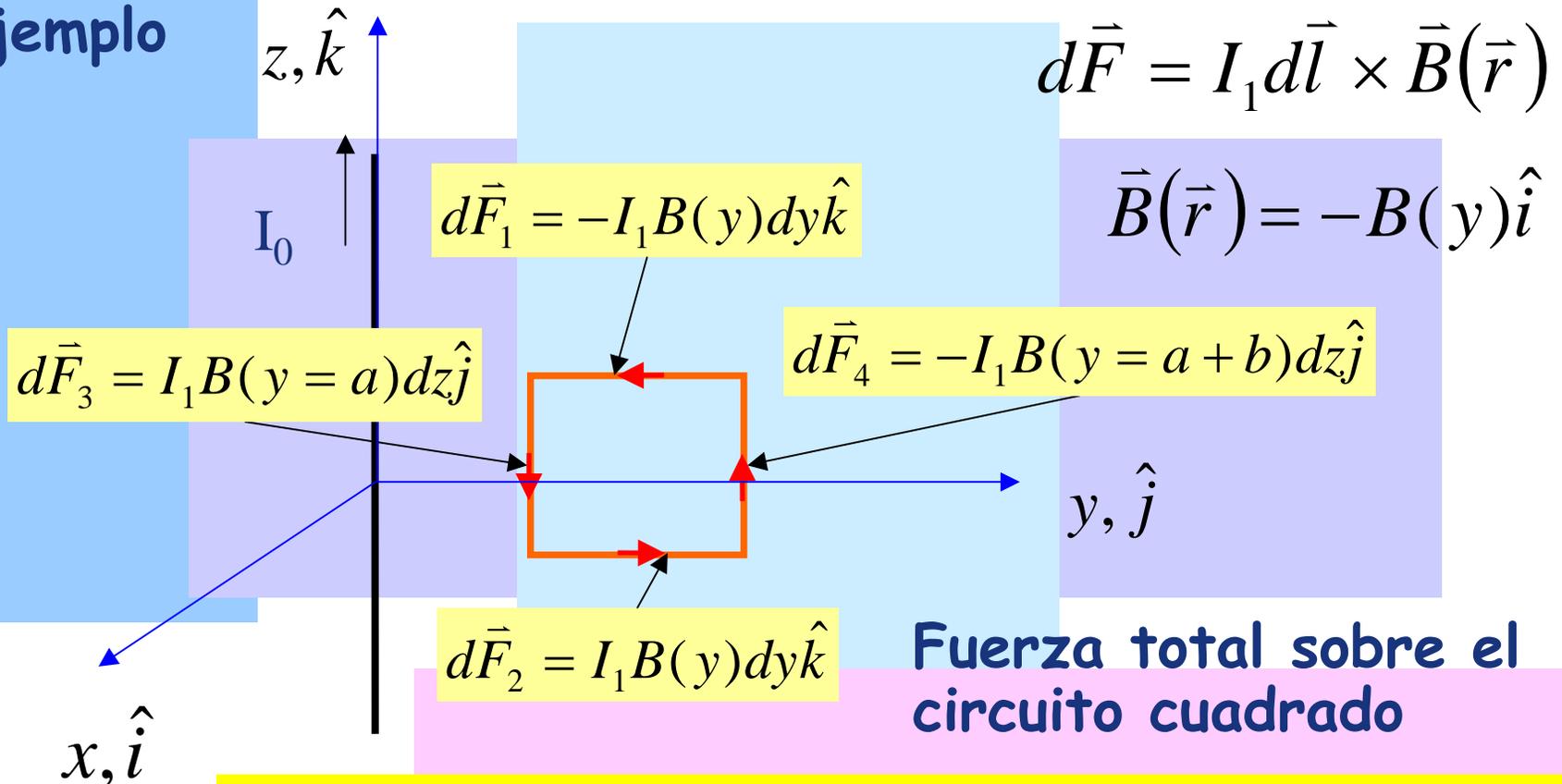
Ejemplo





# Ley de Biot y Savarat

## Ejemplo

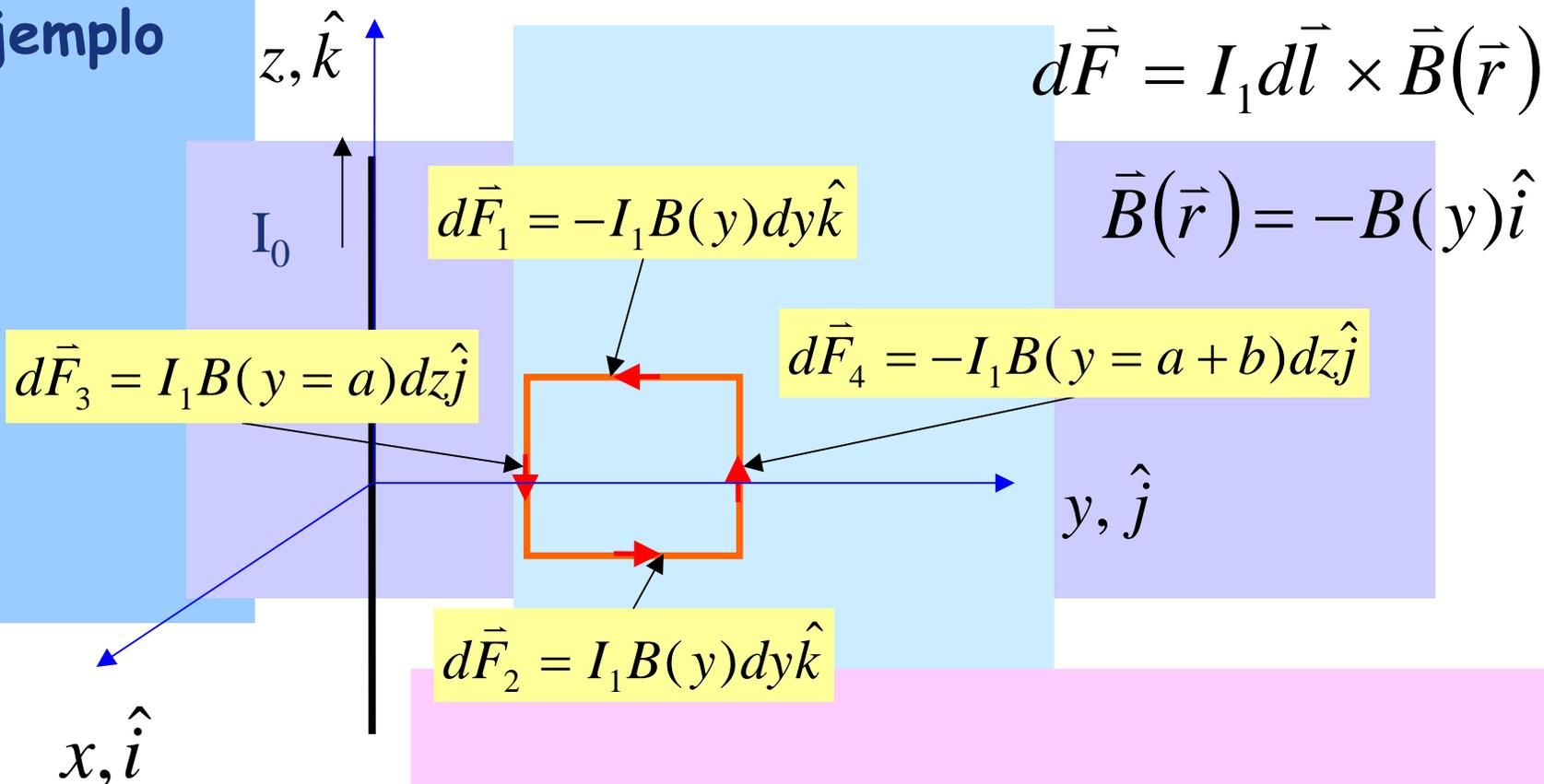


$$\therefore \vec{F} = \oint_{\Gamma} d\vec{F} = \int_{y=a+b}^{y=a} d\vec{F}_1 + \int_{z=b/2}^{z=-b/2} d\vec{F}_3 + \int_{y=a}^{y=a+b} d\vec{F}_2 + \int_{z=-b/2}^{z=b/2} d\vec{F}_4$$

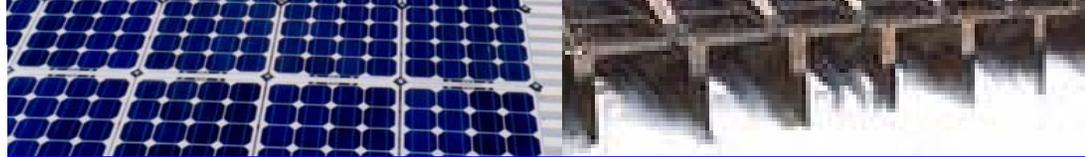


# Ley de Biot y Savarat

Ejemplo



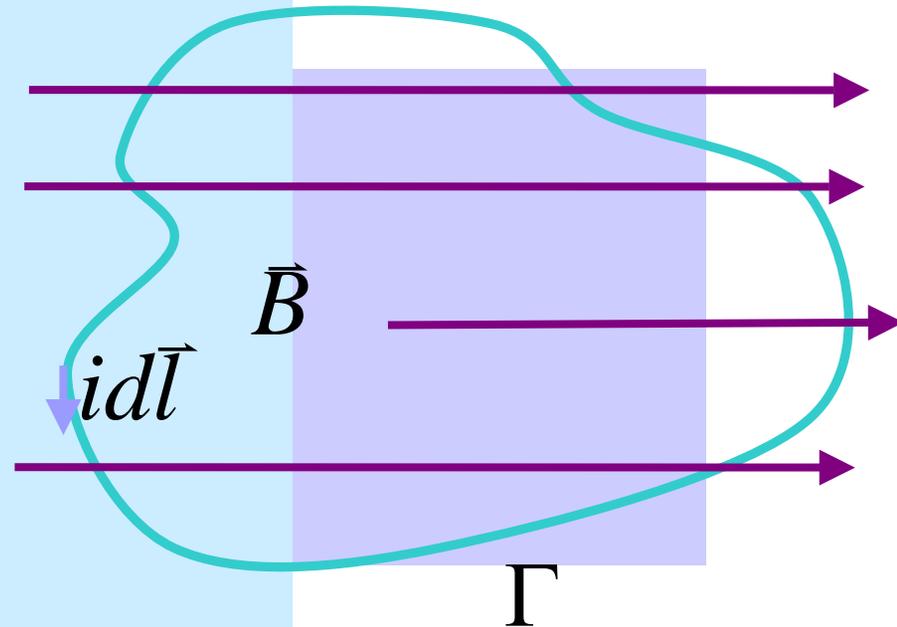
$$\vec{F} = \oint_{\Gamma} d\vec{F} = \int_{z=b/2}^{z=-b/2} \frac{\mu_0 I_1 I_0 \hat{j}}{2\pi a} dz - \int_{z=-b/2}^{z=b/2} \frac{\mu_0 I_1 I_0 \hat{j}}{2\pi(a+b)} dz = \frac{\mu_0 I_1 I_0 b^2}{2\pi(a+b)} \hat{j}$$

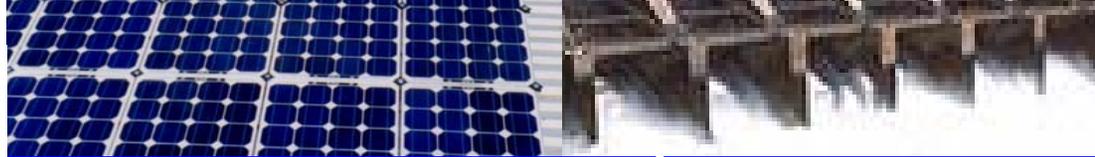


# Torque Magnético

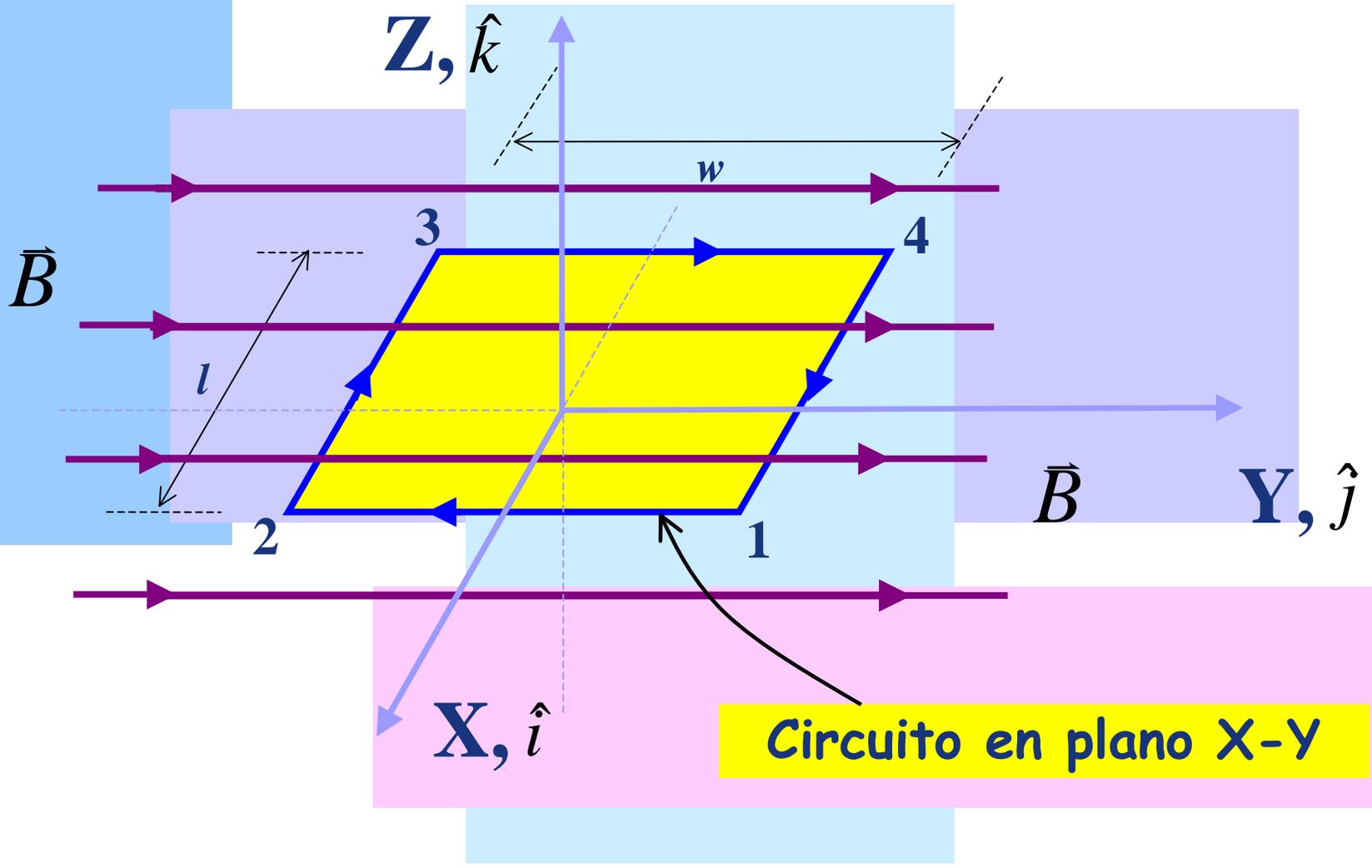
## Ley de Biot y Savarat

$$\therefore d\vec{F} = Id\vec{l} \times \vec{B}(\vec{r})$$



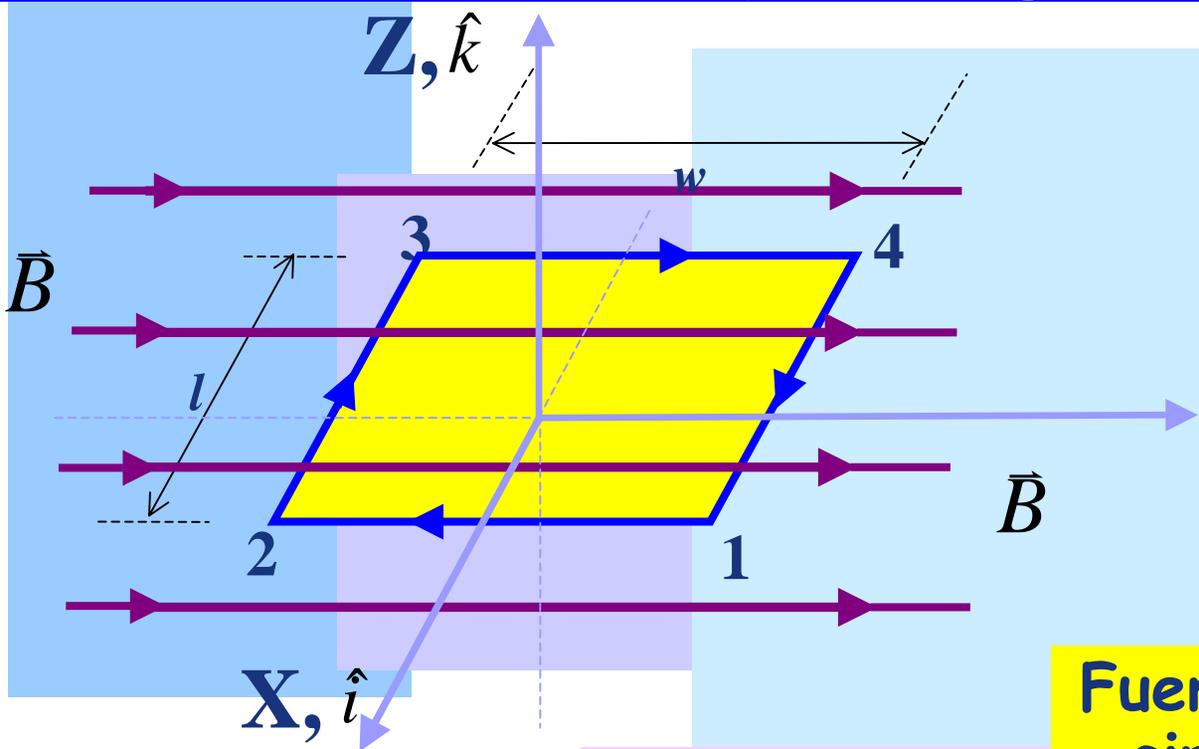


# Torque Magnético





# Torque Magnético



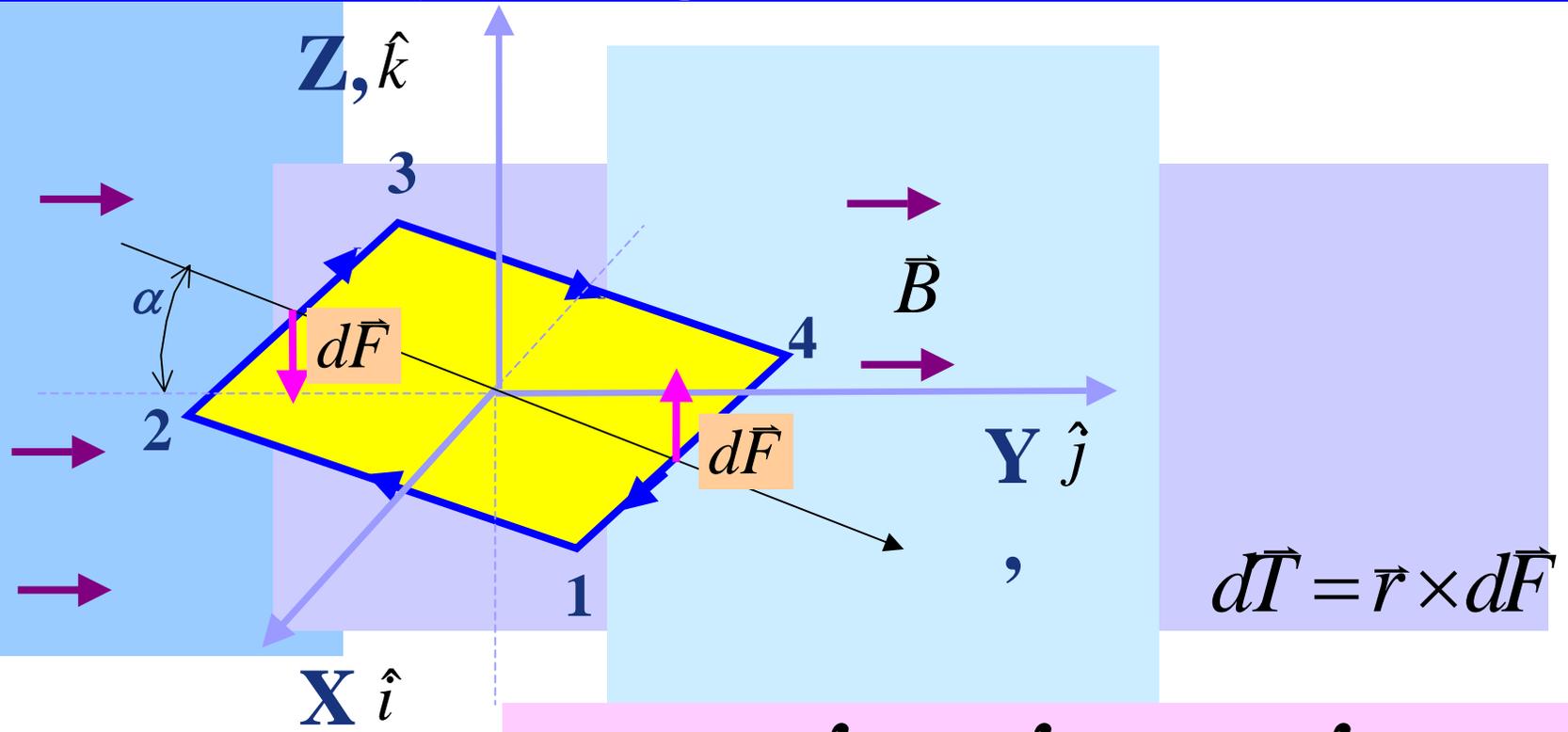
En lados 1-2 y 3-4  
 $I d\vec{l}$  es paralelo a  $\vec{B}$   
Luego  $F=0$

Fuerza neta nula sobre el  
circuito si B constante

$$\vec{F} = I \int_2^3 d\vec{l} \times \vec{B} + I \int_4^1 d\vec{l} \times \vec{B} \Rightarrow \vec{F} = I \int_2^3 dx (-\hat{i}) \times \vec{B} + I \int_4^1 dx (\hat{i}) \times \vec{B}$$

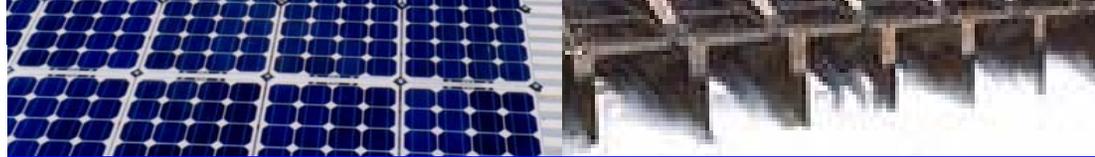


# Torque Magnético

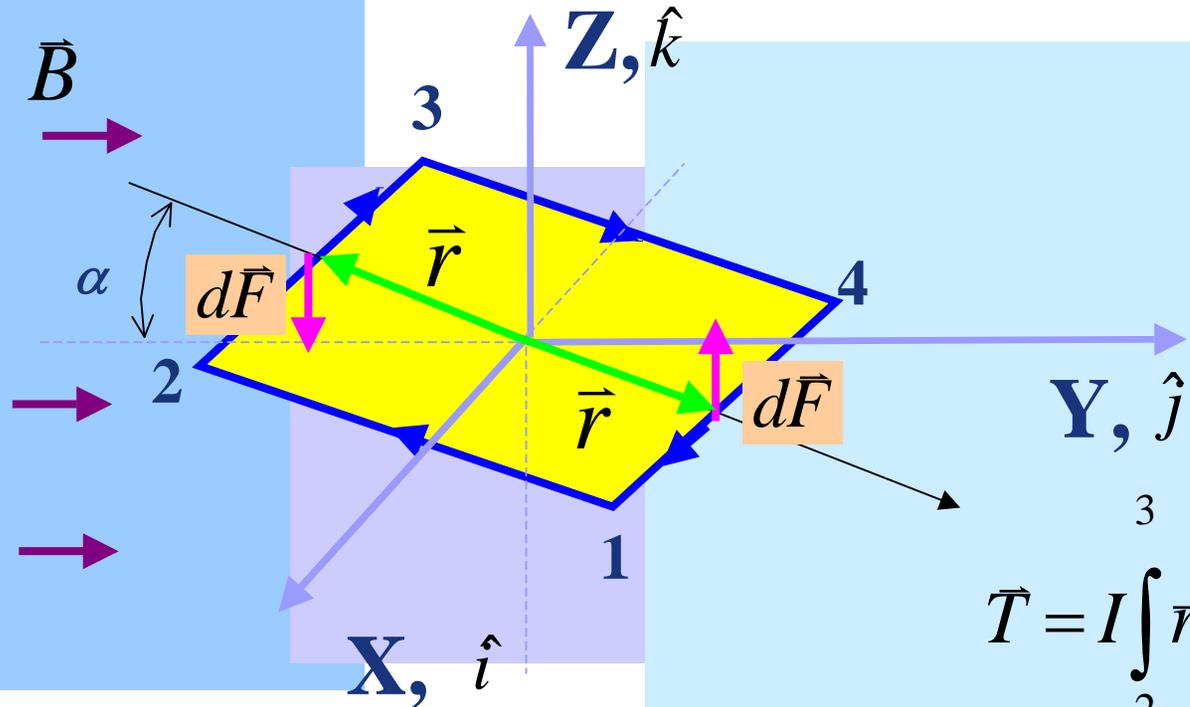


$$\vec{T} = \oint_c d\vec{T} = \oint_c \vec{r} \times d\vec{F} = \oint_c \vec{r} \times i d\vec{l} \times \vec{B}$$

**Torque neto no nulo sobre el circuito**



# Torque Magnético



$$\vec{T} = I \int_2^3 \vec{r} \times d\vec{x} \times \vec{B} + I \int_4^1 \vec{r} \times d\vec{x} \times \vec{B}$$

$$\vec{T} = \frac{Iwl}{2} \cos\alpha \hat{i} + \frac{Iwl}{2} \cos\alpha \hat{i}$$

Torque neto sobre el circuito  $\therefore \vec{T} = Iwl \cos\alpha \hat{i}$