

FI33A ELECTROMAGNETISMO

Clase 5

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INDICE

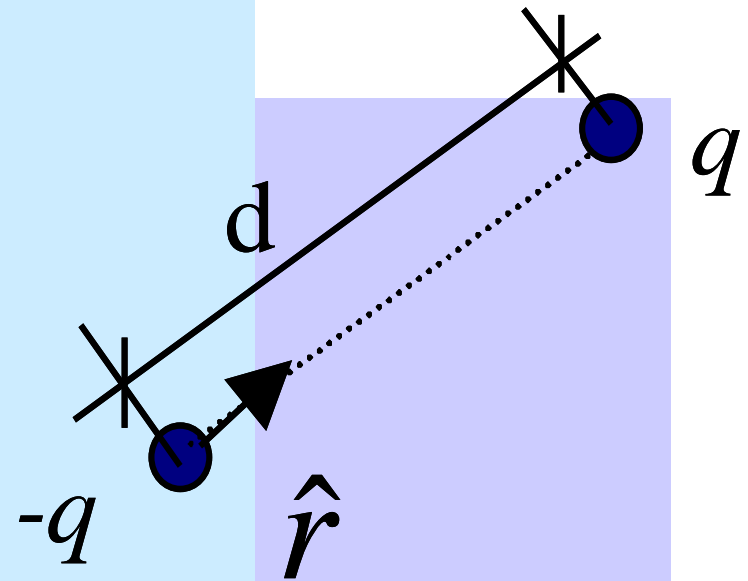
- Definición dipolo eléctrico,
- Dipolo de un conjunto de cargas,
- Dipolo de una distribución volumétrica de cargas
- Potencial a grandes distancias



Dipolo Eléctrico

Se define el dipolo eléctrico como

$$\vec{p} = qd\hat{r} \quad [\text{C}\cdot\text{m}]$$



Sistema de dos cargas de igual magnitud y signo contrario que se mantienen a una distancia constante entre ellas

Potencial de Dipolo Eléctrico

Potencial eléctrico de un dipolo

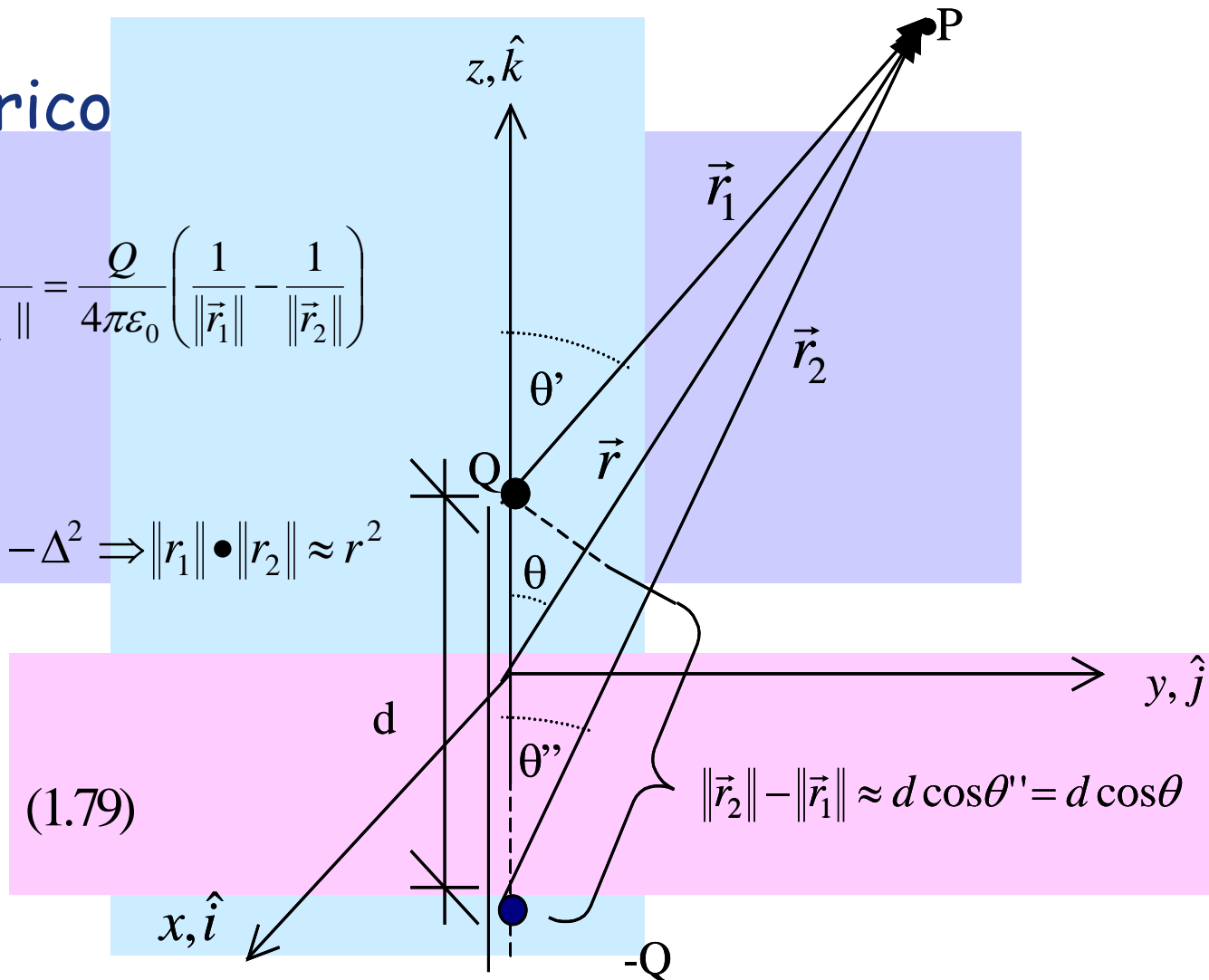
$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0 \|\vec{r}_1\|} + \frac{-Q}{4\pi\epsilon_0 \|\vec{r}_2\|} = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{\|\vec{r}_1\|} - \frac{1}{\|\vec{r}_2\|} \right)$$

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0} \frac{\|\vec{r}_2\| - \|\vec{r}_1\|}{\|\vec{r}_1\| \|\vec{r}_2\|}$$

$$\|\vec{r}_1\| \bullet \|\vec{r}_2\| \approx (r - \Delta)(r + \Delta) = r^2 - \Delta^2 \Rightarrow \|\vec{r}_1\| \bullet \|\vec{r}_2\| \approx r^2$$

$$r_2 - r_1 = d \cos \theta$$

$$\Rightarrow V(\vec{r}) = \frac{Q}{4\pi\epsilon_0} \left[\frac{d \cos \theta}{r^2} \right] \quad (1.79)$$



Potencial de Dipolo Eléctrico

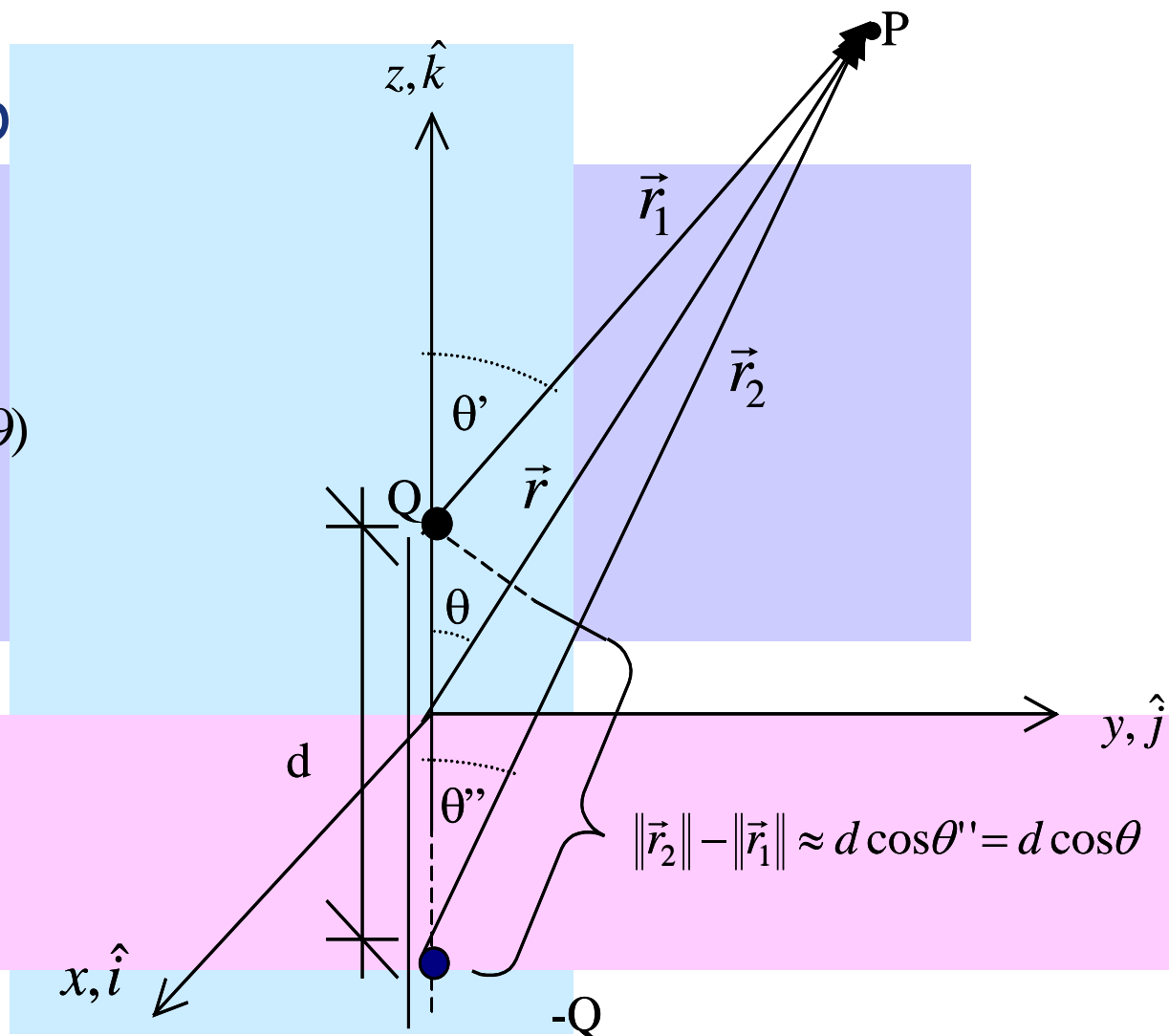
Potencial eléctrico
de un dipolo

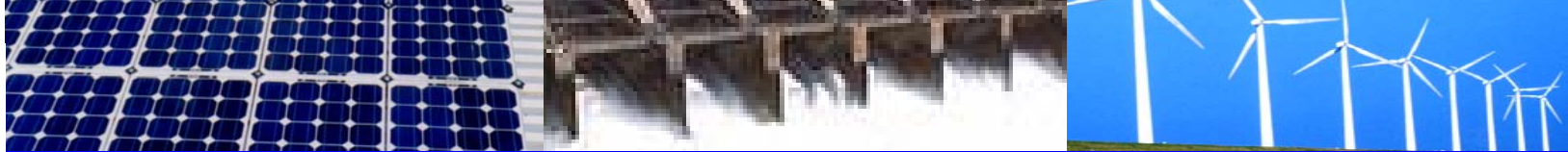
$$r_2 - r_1 = d \cos \theta$$

$$\Rightarrow V(\vec{r}) = \frac{Q}{4\pi\epsilon_0} \left[\frac{d \cos \theta}{r^2} \right] \quad (1.79)$$

$$d \cos \theta = d \hat{k} \bullet \hat{r}$$

$$V(\vec{r}) = \frac{\vec{p} \bullet \hat{r}}{4\pi\epsilon_0 \|\vec{r}\|^2}$$



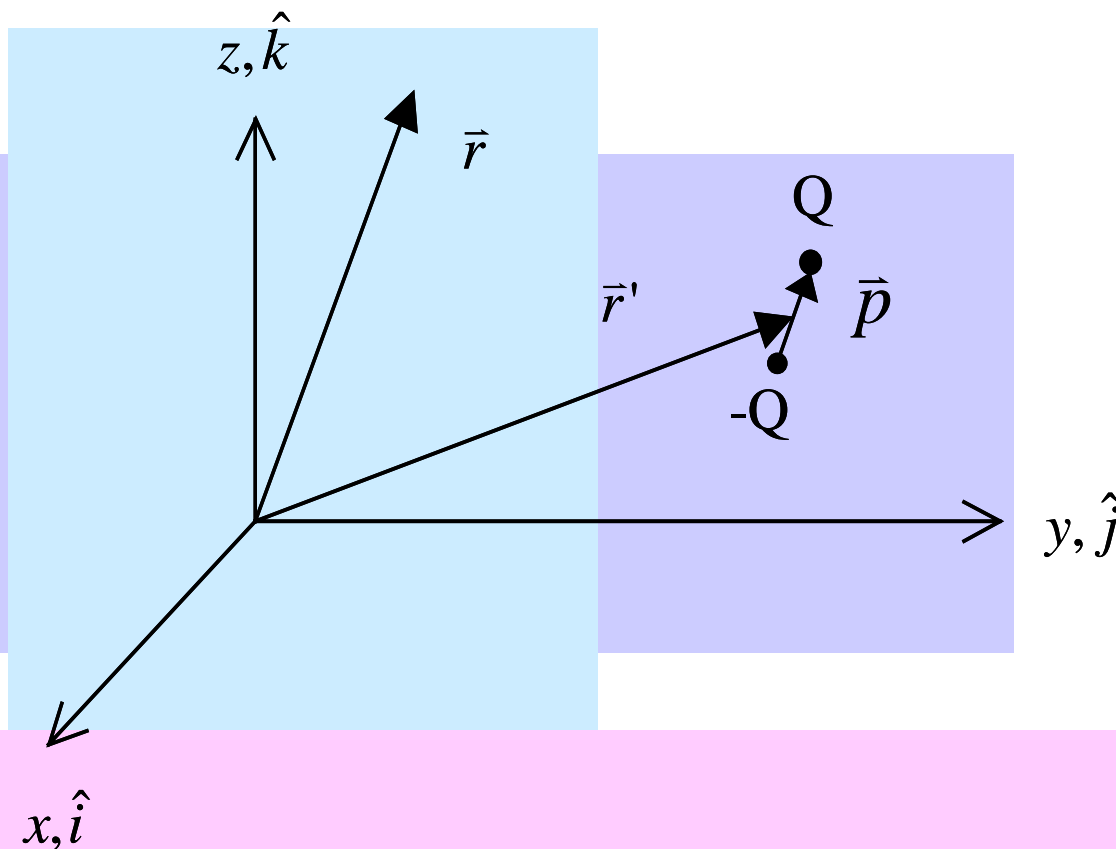


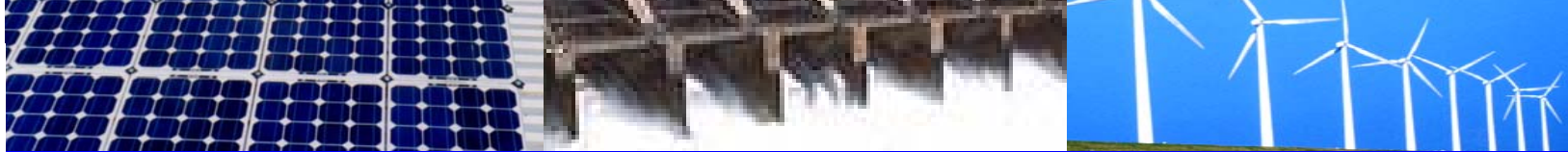
Potencial y Campo de Dipolo Eléctrico

$$V(\vec{r}) = \frac{\vec{p} \bullet (\vec{r} - \vec{r}')}{4\pi\epsilon_0 \|\vec{r} - \vec{r}'\|^3}$$

Campo eléctrico

$$\vec{E} = -\nabla V$$





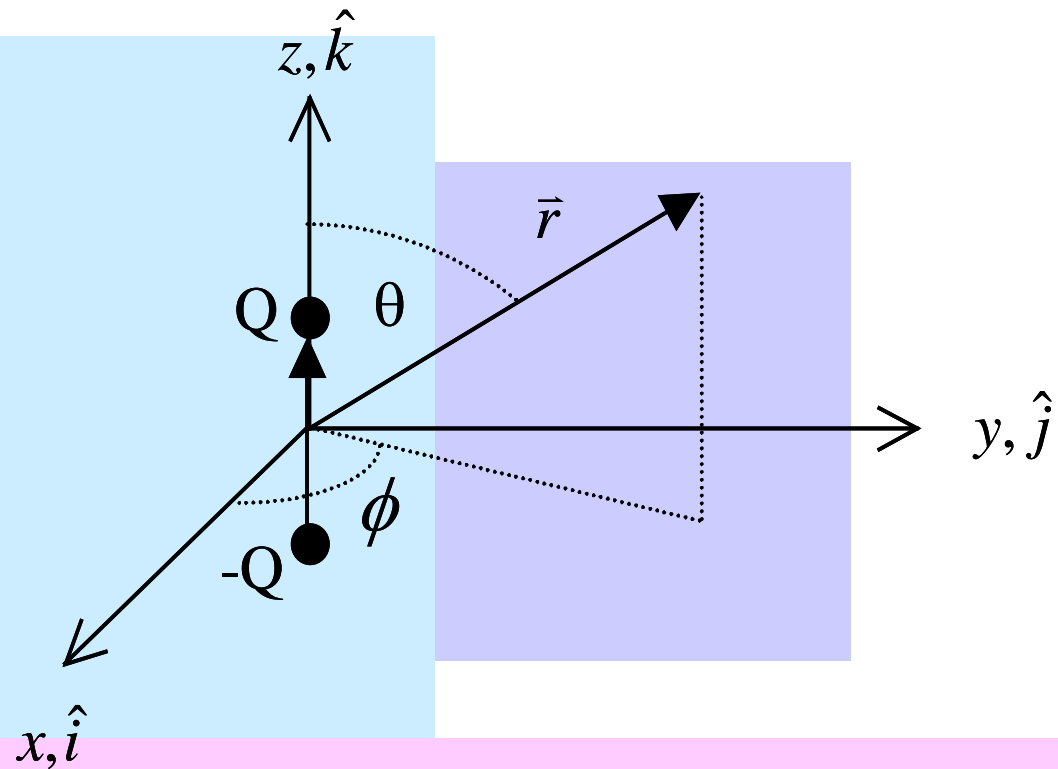
Ejemplo

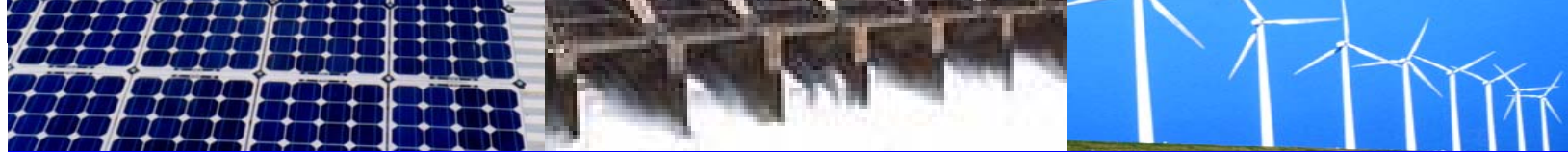
Ejemplo 13

$$\vec{r}' = 0$$

$$V(\vec{r}) = \frac{\vec{p} \bullet \vec{r}}{4\pi\epsilon_0 \|\vec{r} - \vec{r}'\|^3}$$

$$V(\vec{r}) = \frac{p \cos\theta}{4\pi\epsilon_0 r^2}$$





Ejemplo

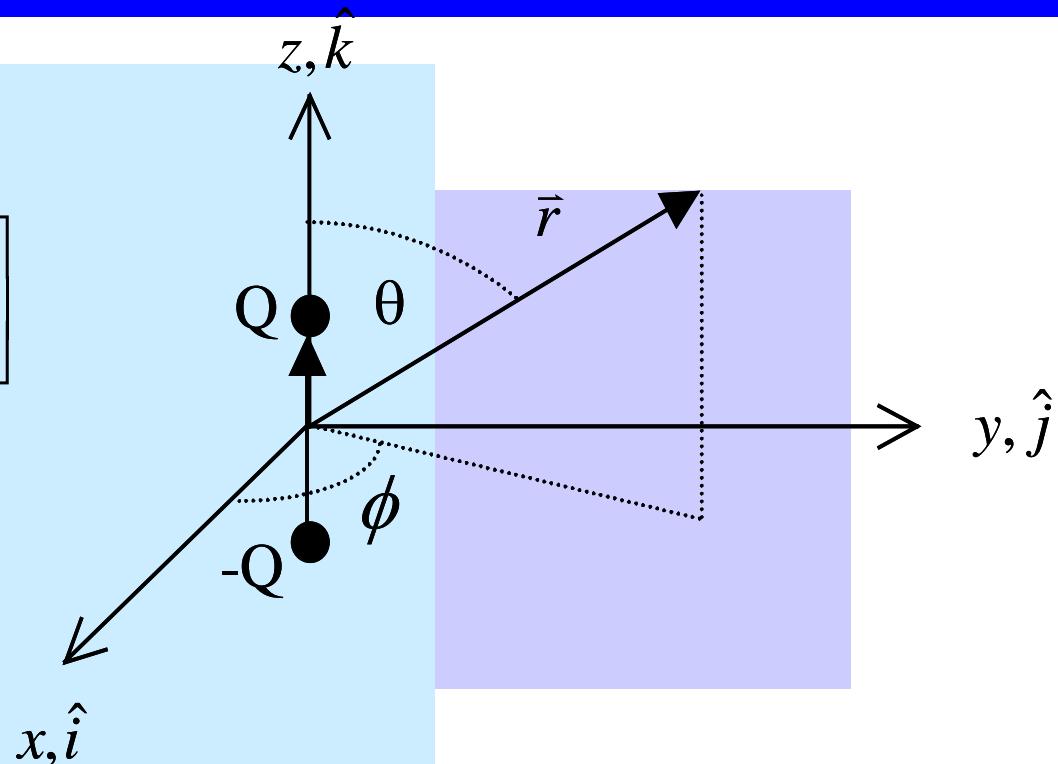
Ejemplo 13

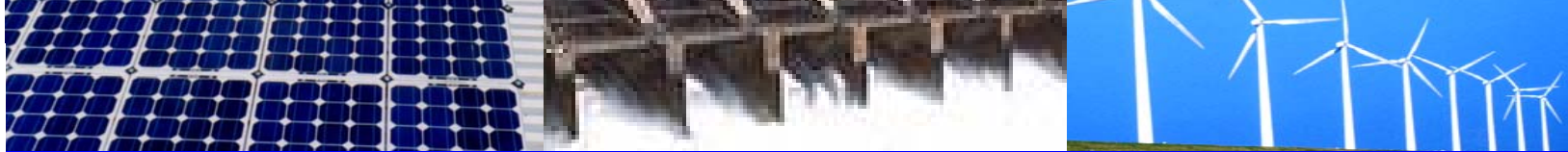
$$\nabla V = \left[\frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \hat{\phi} \right]$$

V solo depende de r y θ , luego

$$\nabla V = \frac{p \cos \theta}{4\pi\epsilon_0} (-2r^{-3}) \hat{r} + \frac{1}{r} \frac{p}{4\pi\epsilon_0} (-\sin \theta) \hat{\theta}$$

$$\therefore \vec{E} = \frac{p}{4\pi\epsilon_0} (2 \cos \theta \hat{r} + \sin \theta \hat{\theta})$$





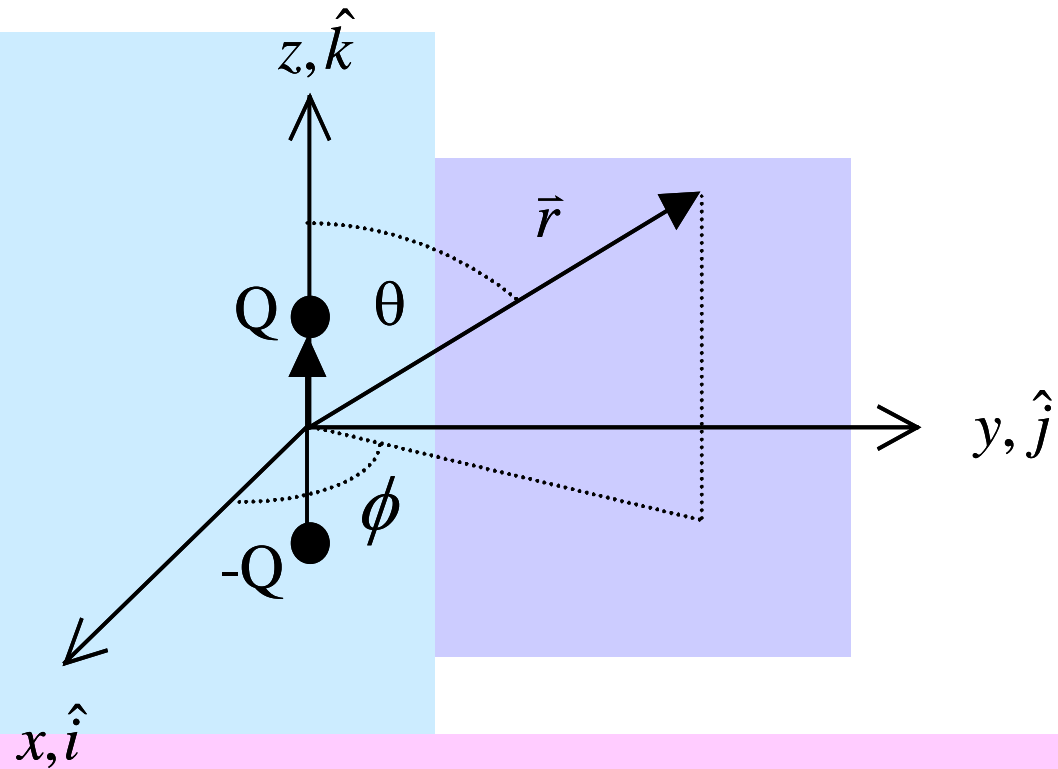
Ejemplo

Ejemplo 13. Dipolo centrado en el origen

$$\vec{r}' = 0$$

$$V(\vec{r}) = \frac{\vec{p} \bullet \vec{r}}{4\pi\epsilon_0 \|\vec{r} - \vec{r}'\|^3}$$

$$V(\vec{r}) = \frac{p \cos\theta}{4\pi\epsilon_0 r^2}$$





Ejemplo

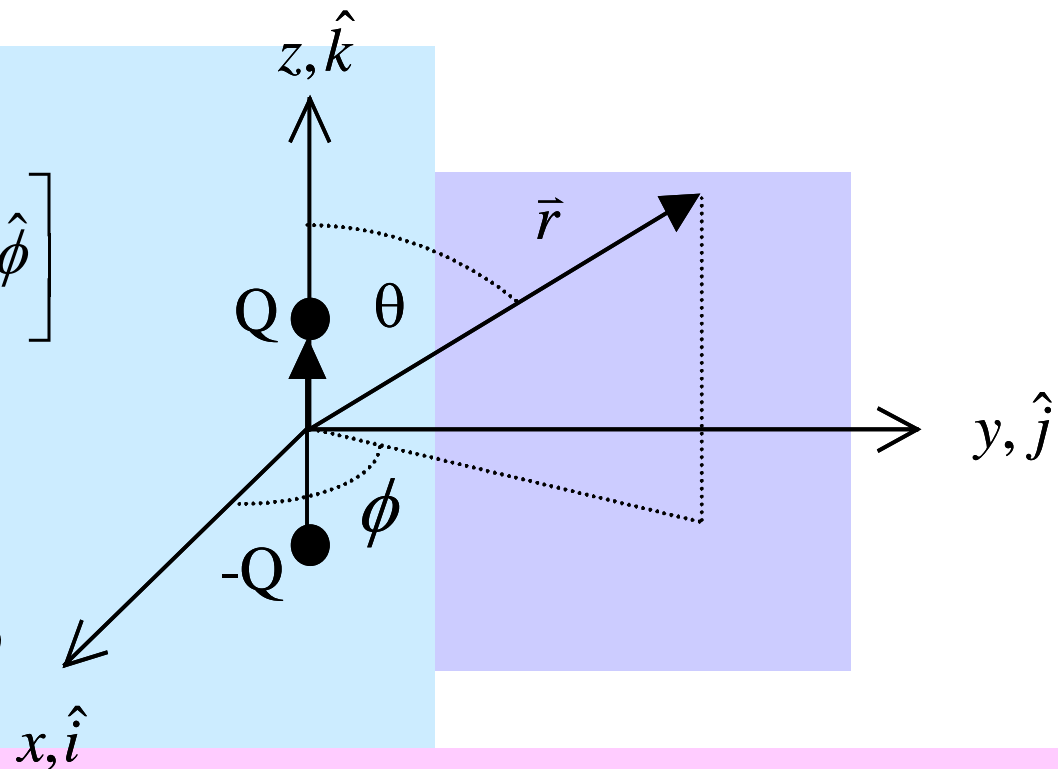
Ejemplo 13

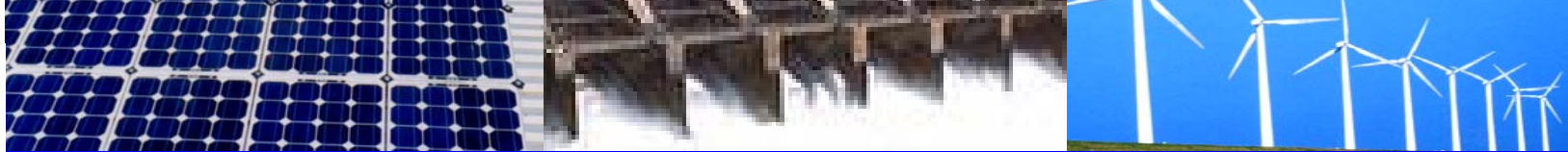
$$\nabla V = \left[\frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \hat{\phi} \right]$$

V solo depende de r y θ , luego

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$$\therefore \vec{E} = \frac{p}{4\pi\epsilon_0} (2\cos\theta \hat{r} + \sin\theta \hat{\theta})$$

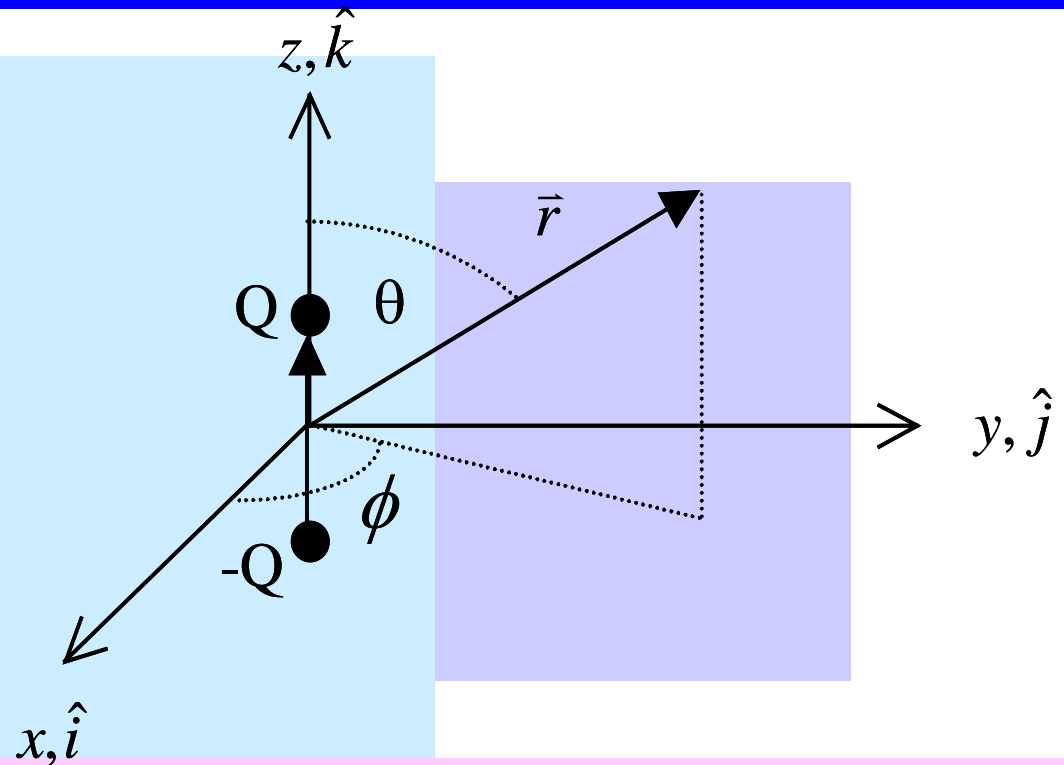




Dipolo Eléctrico

Para puntos muy
alejados

$$\vec{E}(\vec{p}) \propto \frac{1}{r^3}, \quad V(\vec{p}) \propto \frac{1}{r^2}$$



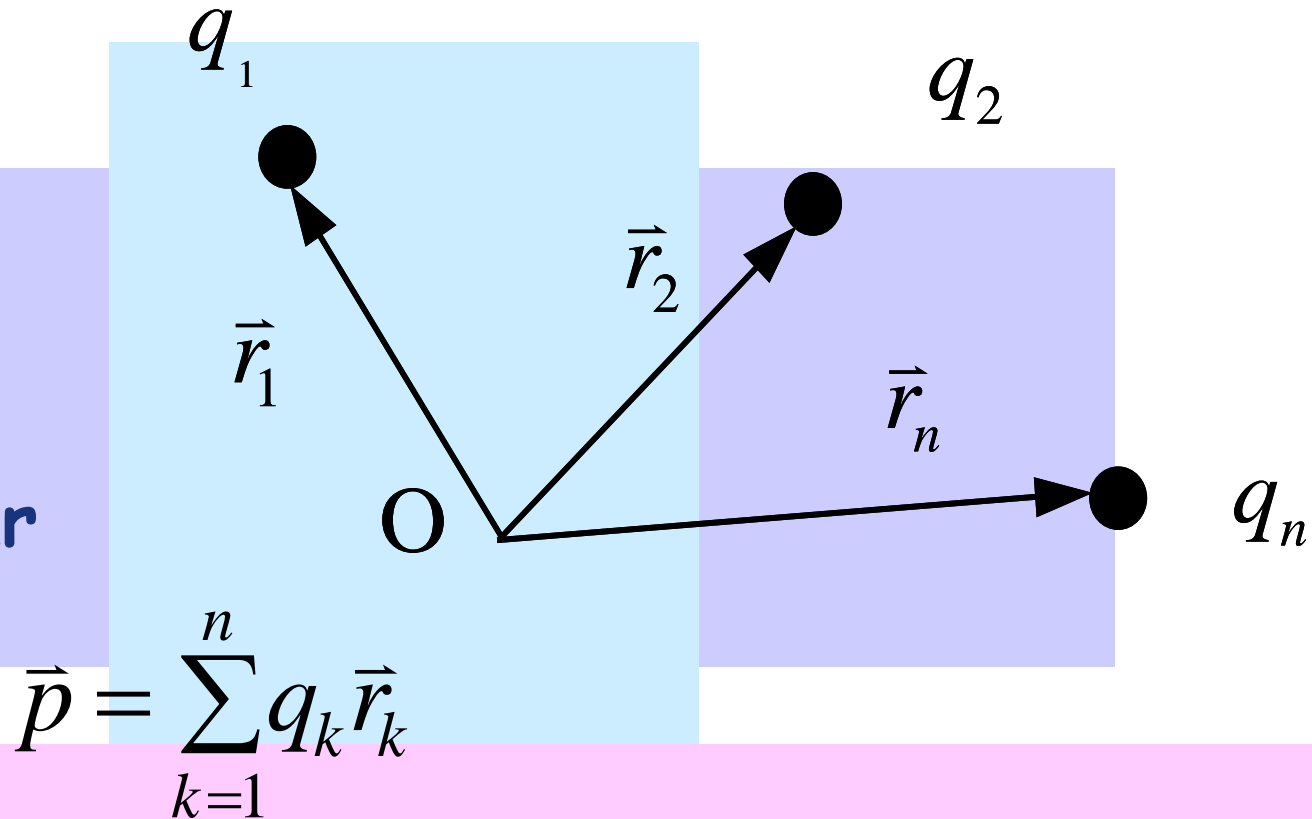


Dipolo de un Conjunto de Cargas

Condición

$$\sum_{k=1}^n q_k = 0$$

Momento Dipolar
Eléctrico



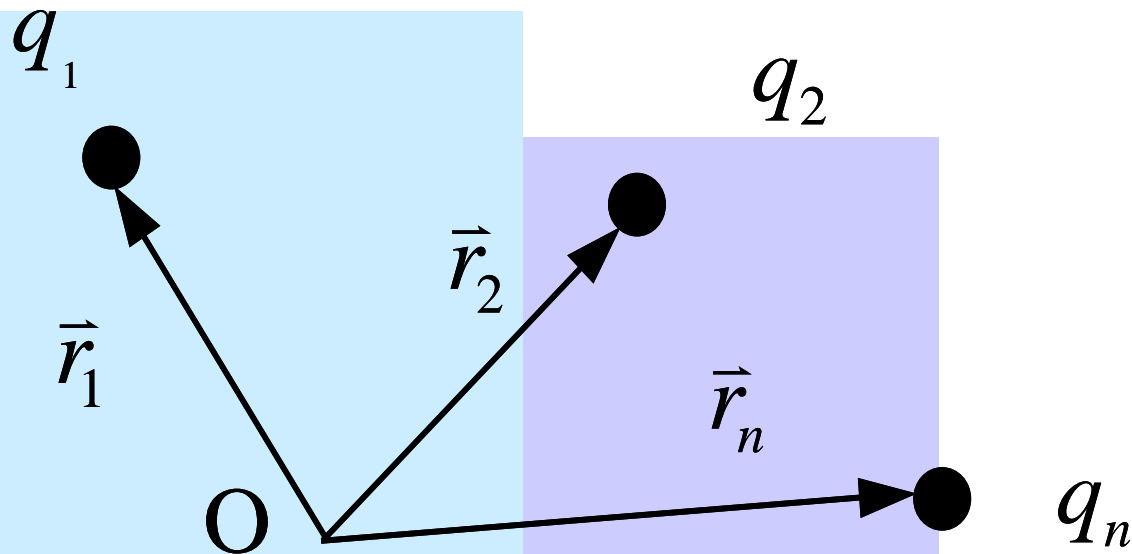


Dipolo de un Conjunto de Cargas

Momento Dipolar Eléctrico

$$\vec{p} = \sum_{k=1}^n q_k \vec{r}_k$$

Para $n=2$ se tiene



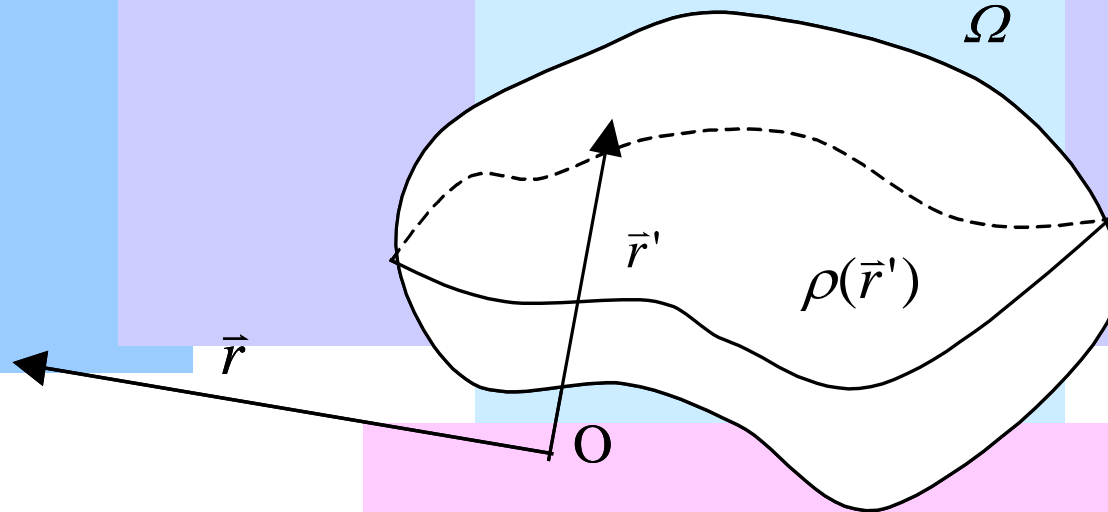
$$\vec{p} = q_1 \vec{r}_1 + q_2 \vec{r}_2 \Rightarrow q_1 = -q_2 = Q \Rightarrow \vec{p} = Q(\vec{r}_1 - \vec{r}_2) = Q\vec{d}$$

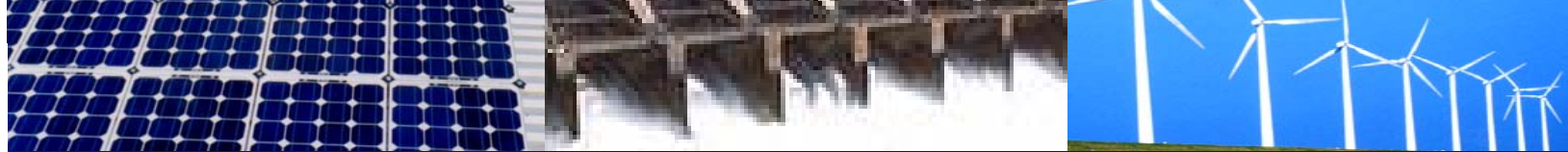
no depende del origen !



Momento dipolar de distribución volumétrica

$$\vec{p} = \sum_{k=1}^n q_k \vec{r}_k \Rightarrow \vec{p} = \iiint_{\Omega} \vec{r}' dq' = \iiint_{\Omega} \vec{r}' \rho(\vec{r}') dv'$$





Ejemplo

EJEMPLO 14

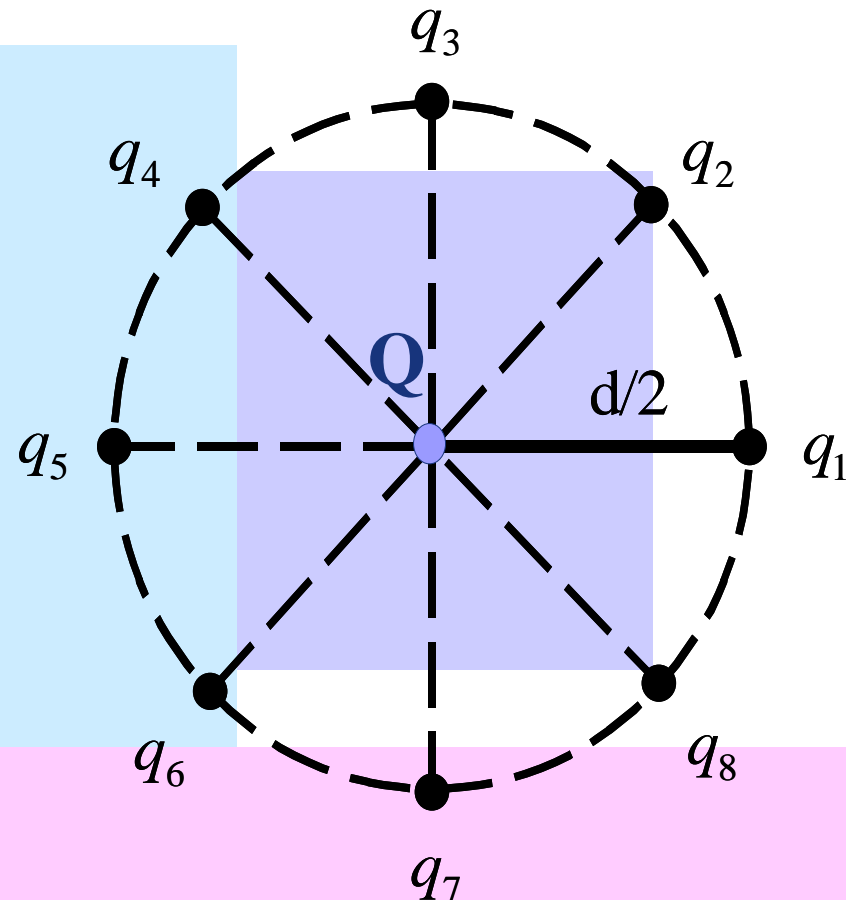
Calcular momento dipolar

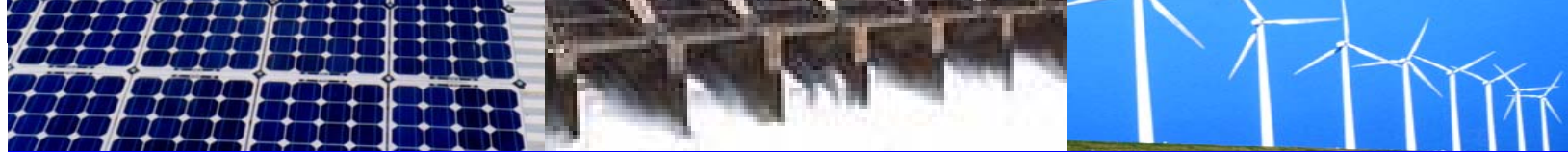
si:

$$Q = -\sum_{i=1}^{\infty} q_i$$

Solⁿ

$$\vec{p} = \sum_{i=1}^8 q_i \times \vec{r}_i + Q \times 0 = 0$$





Ejemplo

EJEMPLO 14

Calcular momento dipolar

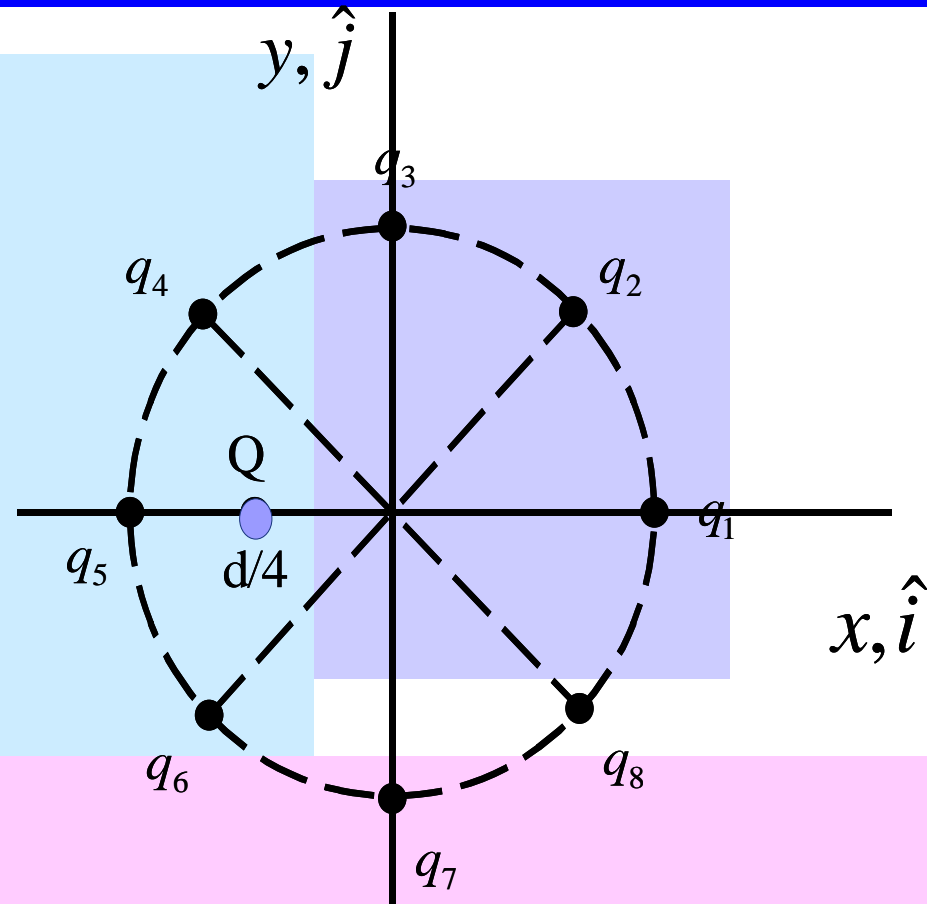
si:

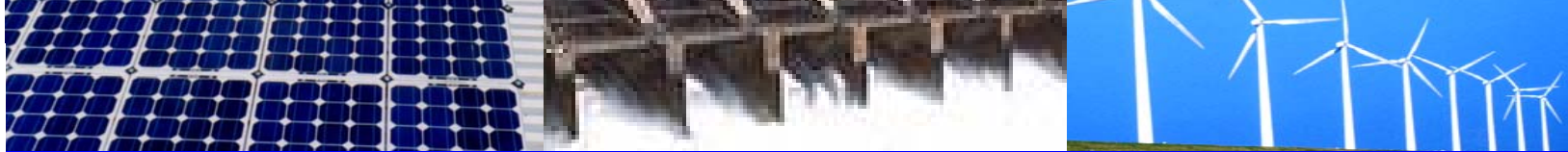
$$Q = -\sum_{i=1}^{\infty} q_i$$

Solⁿ

$$\vec{p} = \sum q_i \vec{r}_i - Q \left(\frac{d}{4} \right) \hat{i}$$

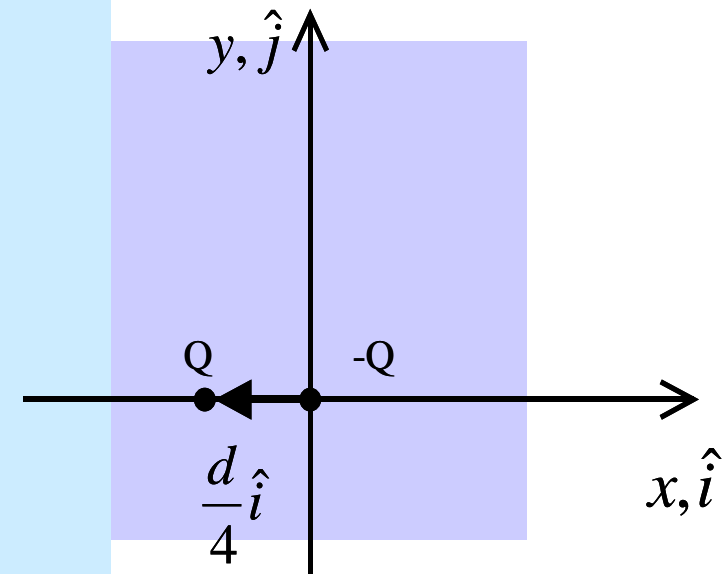
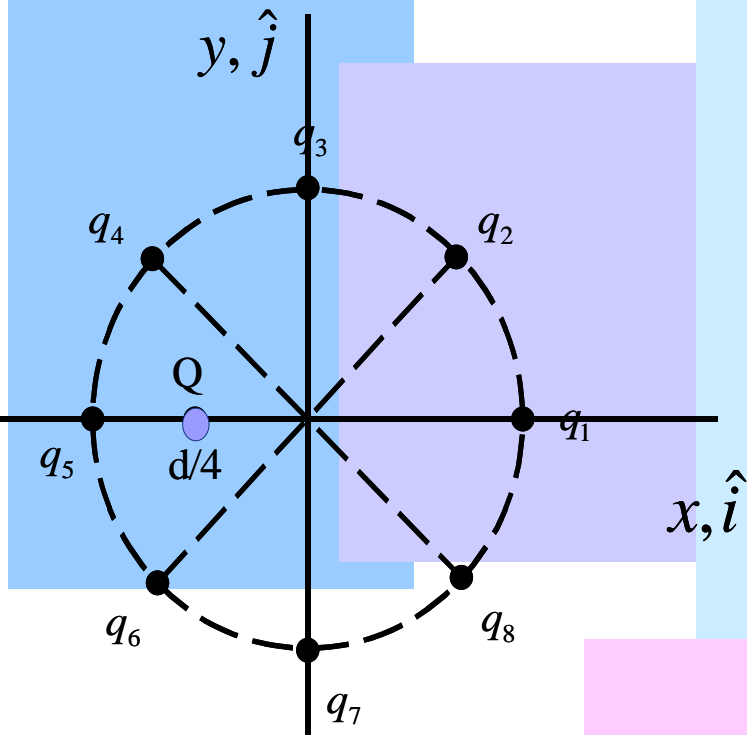
$$\vec{p} = -Q \frac{d}{4} \hat{i}$$





Ejemplo

EJEMPLO 14





Potencial a grandes distancias

Si

$$\|\vec{r}\| \gg \|\vec{r}'\| \Rightarrow$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\vec{r}') dv}{\|\vec{r} - \vec{r}'\|}$$

$$\frac{1}{\|\vec{r} - \vec{r}'\|} = \frac{1}{\|\vec{r}\|} + \frac{\vec{r} \bullet \vec{r}'}{\|\vec{r}\|^3} +$$

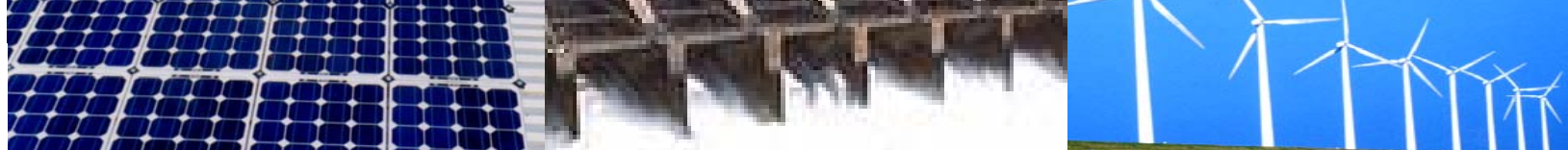


Potencial a grandes distancias

$$\Rightarrow V(\vec{r}) \approx \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\vec{r}')}{\|\vec{r}\|} dv' + \frac{1}{4\pi\epsilon_0} \iiint \frac{\vec{r} \bullet \vec{r}'}{\|\vec{r}\|^3} \rho(\vec{r}') dv' + TOS$$

$$V(\vec{r}) \approx \frac{1}{4\pi\epsilon_0} \frac{1}{\|\vec{r}\|} \iiint \rho(\vec{r}') dv' + \frac{1}{4\pi\epsilon_0} \frac{\vec{r} \bullet}{\|\vec{r}\|^3} \iiint \vec{r}' \rho(\vec{r}') dv' + TO$$

$$\therefore V(\vec{r}) = \frac{Q_{Total}}{4\pi\epsilon_0 \|\vec{r}\|} + \frac{\vec{r} \bullet \vec{p}}{4\pi\epsilon_0 \|\vec{r}\|^3} + TOS$$



A Grandes Distancias

Configuración	Potencial Eléctrico	Campo Eléctrico
Una carga $q \bullet$	$\propto 1 / r$	$\propto 1 / r^2$
Dos cargas Dipolo $q \bullet -q \bullet$	$\propto 1 / r^2$	$\propto 1 / r^3$
Cuatro cargas Dos dipolos $q \bullet -q \bullet$ $-q \bullet q \bullet$	$\propto 1 / r^3$	$\propto 1 / r^4$