Pauta P1-C2

$$L = \frac{I}{2}\left(\dot{\theta}^2 + \dot{\varphi}^2 \operatorname{sen}^2\theta\right) + \frac{I_3}{2}\left(\dot{\psi} + \dot{\varphi}\cos\theta\right)^2 - mgl\cos\theta$$

$$\Rightarrow \frac{\partial L}{\partial \dot{\varphi}} = I_3(\dot{\psi} + \dot{\varphi}\cos\theta)\cos\theta + I\dot{\varphi}\operatorname{sen}^2\theta = P_\varphi$$

$$\frac{\partial L}{\partial \dot{\psi}} = I_3(\dot{\psi} + \dot{\varphi}\cos\theta) = P_\psi$$

$$\Rightarrow \boxed{\dot{\psi} = \frac{P_\psi}{I_3} - \frac{(P_\varphi - P_\psi\cos\theta)\cos\theta}{I\operatorname{sen}^2\theta}}$$

$$\Rightarrow P_\psi\cos\theta + I\dot{\varphi}\operatorname{sen}^2\theta = P_\varphi$$

$$\Rightarrow \boxed{\dot{\varphi} = \frac{P_\varphi - P_\psi\cos\theta}{I\operatorname{sen}^2\theta}}$$

Ec. de mov:

$$I\ddot{\theta} = I\dot{\varphi}^2\operatorname{sen}\theta\cos\theta - I_3(\dot{\psi} + \dot{\varphi}\cos\theta)\operatorname{sen}\theta\,\dot{\varphi} + mgl\operatorname{sen}\theta$$

$$= I\operatorname{sen}\theta\cos\theta\,\frac{(P_\varphi - P_\psi\cos\theta)^2}{I^2\operatorname{sen}^4\theta} - \frac{P_\psi\operatorname{sen}\theta\,(P_\varphi - P_\psi\cos\theta)}{I\operatorname{sen}^2\theta} + mgl\operatorname{sen}\theta$$

$$= \frac{(P_\varphi - P_\psi\cos\theta)^2\cos\theta}{I\operatorname{sen}^3\theta} - \frac{P_\psi(P_\varphi - P_\psi\cos\theta)\operatorname{sen}^2\theta}{I\operatorname{sen}^3\theta} + mgl\operatorname{sen}\theta$$

$$= -\frac{(P_\varphi - P_\psi\cos\theta)(P_\psi - P_\varphi\cos\theta)}{I\operatorname{sen}^3\theta} + mgl\operatorname{sen}\theta$$

$$\Rightarrow \frac{\partial V_{eff}}{\partial \theta} = \frac{(P_\varphi - P_\psi\cos\theta)(P_\psi - P_\varphi\cos\theta)}{I\operatorname{sen}^3\theta} - mgl\operatorname{sen}\theta$$

Condición que $\theta = 0$ sea pto eq:

$$\Rightarrow \left.\frac{\partial V_{eff}}{\partial \theta}\right|_{\theta^* = 0} = 0 \Rightarrow \frac{(P_\varphi - P_\psi)(P_\psi - P_\varphi)}{I \operatorname{sen}^3\theta_{\theta\to 0}} = 0$$

Como $\operatorname{sen}\theta^* \to 0$ es necesario que $P_\psi = P_\varphi$ para que exista límite.

$$\Rightarrow \frac{\partial V_{eff}}{\partial \theta} = \frac{P^2(1-\cos\theta)^2}{I\operatorname{sen}^3\theta} - mgl\operatorname{sen}\theta$$

$$\Rightarrow \left.\frac{\partial V_{eff}}{\partial \theta}\right|_{\theta^* = 0} = \frac{P^2(1-\cos\theta_{\theta\to 0})^2}{I\operatorname{sen}^3\theta_{\theta\to 0}} \to \frac{0}{0}$$

Por l'Hôpital:

$$\frac{P^2(1-\cos\theta)^2}{I\operatorname{sen}^3\theta} \to \frac{P^2 \cancel{\operatorname{sen}\theta}(1-\cos\theta)}{3I\operatorname{sen}^{\cancel{2}}\theta\cos\theta} = \frac{P^2(1-\cos\theta)}{3I\operatorname{sen}\theta\cos\theta} \to \frac{0}{0}$$

Nuevamente L'Hôpital:

$$\frac{(1-\cos\theta)}{3I\operatorname{sen}\theta\cos\theta} \to \frac{\operatorname{sen}\theta}{3I\cos^2\theta - 3I\operatorname{sen}^2\theta} \to \frac{0}{3I} = 0$$

En efecto, $\left.\frac{\partial V_{eff}}{\partial \theta}\right|_{\theta^*=0} = 0 \Rightarrow \theta^* = 0$ es pto. de eq. cuando $\boxed{P_\varphi = P_\psi}$

Para que sea estable:

$$\left.\frac{\partial^2 V_{eff}}{\partial \theta^2}\right|_{\theta^*=0} > 0 \Rightarrow \frac{\partial^2 V_{eff}}{\partial \theta^2} = \frac{2P^2(1-\cos\theta)\operatorname{sen}\theta}{I\operatorname{sen}^3\theta} - \frac{3P^2(1-\cos\theta)^2\cos\theta}{I\operatorname{sen}^4\theta} - mgl\cos\theta$$

$$\Rightarrow \frac{\partial^2 V_{eff}}{\partial \theta^2} = \frac{P^2[2+\cos\theta(4\cos\theta-\cos^2\theta-5)]}{I\,sen^4\theta} - mgl\cos\theta$$

cuando $\theta \to 0$

$$\Rightarrow \left.\frac{\partial^2 V_{eff}}{\partial \theta^2}\right|_{\theta^*=0} \longrightarrow \left\{\frac{0}{0}\right\} - mgl$$

Por l'Hôpital:

$$\Rightarrow \frac{2+\cos\theta(4\cos\theta-\cos^2\theta-5)}{sen^4\theta} \to \frac{-8\cos\theta\,sen\theta+3\cos^2\theta\,sen\theta+5sen\theta}{4sen^3\theta\cos\theta}$$

$$= \frac{-8\cos\theta+3\cos^2\theta+5}{4sen^2\theta\cos\theta}$$

$$\to \frac{8sen\theta-6\cos\theta\,sen\theta}{8sen\theta\cos^2\theta-4sen^3\theta}$$

$$= \frac{8-6\cos\theta}{8\cos^2\theta-4sen^2\theta} \to \frac{2}{8} = \frac{1}{4}$$

$$\Rightarrow \left.\frac{\partial^2 V_{eff}}{\partial \theta^2}\right|_{\theta^*=0} = \frac{P^2}{4I} - mgl$$

condición:

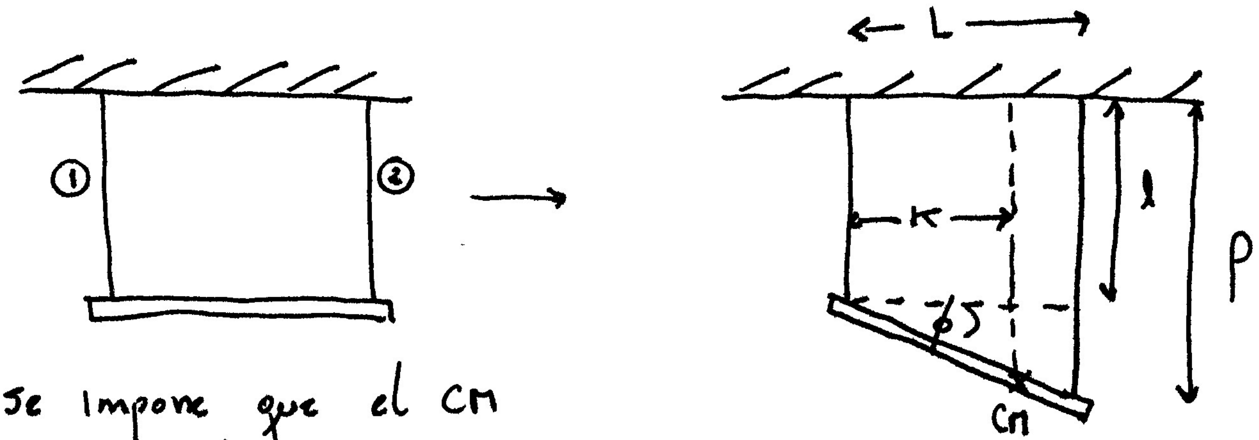
$$\frac{P^2}{4I} - mgl > 0$$

$$\Rightarrow \boxed{P_\varphi = P_\psi > \sqrt{4mglI}}$$

$$\omega = \sqrt{\frac{\left.\frac{\partial^2 V_{eff}}{\partial\theta^2}\right|_{\theta^*}}{\left.\frac{\partial^2 T}{\partial\dot\theta^2}\right|_{\dot\theta^*=0}}} = \sqrt{\frac{\frac{P_\varphi^2}{4I}-mgl}{I}} = \sqrt{\frac{P_\varphi^2}{4I^2}-\frac{mgl}{I}}$$

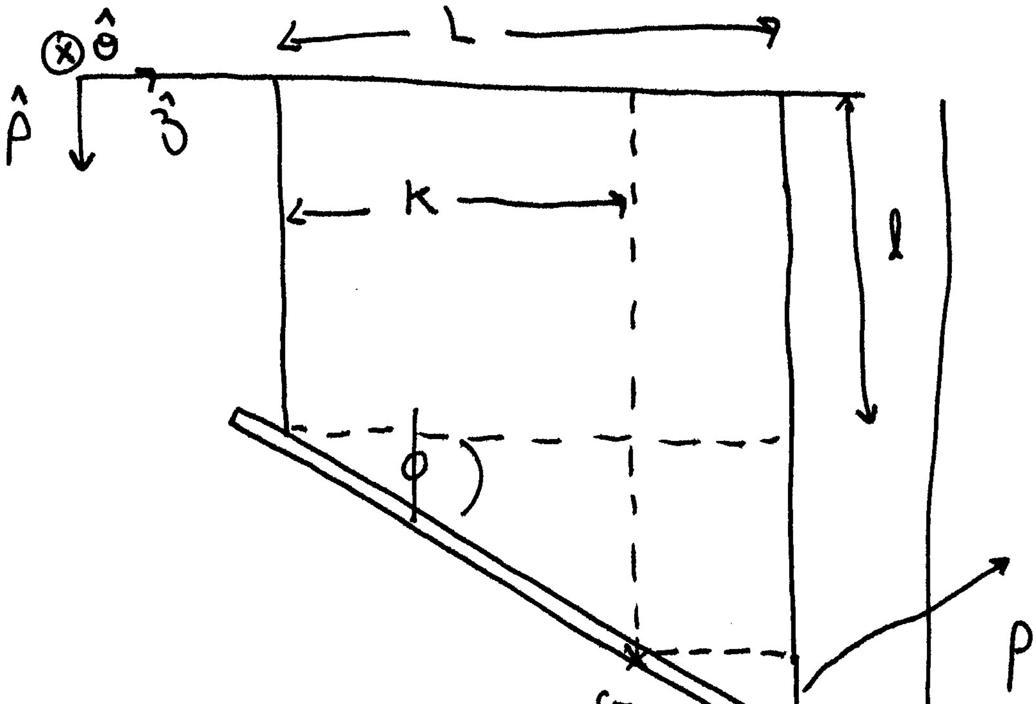
//

Pauta P2-C2



Se impone que el CM baja a distancia K y que la cuerda ② baja a una posición ρ medida del techo $\Rightarrow$ restricción : $\boxed{\rho - l = 0}$

Entonces:



$\tan\phi = \frac{\rho - l}{L}$

$x = (L-K)\tan\phi$

Asumimos $\phi \ll 1$

$$\Rightarrow \quad \phi = \frac{\rho - l}{L} \quad \Rightarrow \quad \dot{\phi} = \frac{\dot{\rho}}{L}$$

$$x = (L-K)\,\phi$$

$$\vec{r}_{cm} = (\rho - x)\hat{\rho} + k\hat{z}$$
$$= (\rho - (L-k)\phi)\,\hat{\rho} + k\hat{z}$$
$$= \left(\rho - (L-k)\frac{(\rho-l)}{L}\right)\hat{\rho} + k\hat{z}$$

$$\Rightarrow \vec{V}_{cm} = \left(\dot{\rho} - \frac{(L-k)}{L}\dot{\rho}\right)\hat{\rho} + \left(\rho - (L-k)\frac{(\rho-l)}{L}\right)\dot{\theta}\,\hat{\theta}$$
$$= \dot{\rho}\left(\frac{k}{L}\right)\hat{\rho} + \left[\rho\left(\frac{k}{L}\right) + \frac{(L-k)l}{L}\right]\dot{\theta}\,\hat{\theta}$$

$$\Rightarrow L = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}I\omega^2 + Mgh_{cm} + \lambda(\rho - l)$$
$$= \frac{1}{2}M\left(\dot{\rho}^2\frac{k^2}{L^2} + \left[\rho\frac{k}{L} + \frac{(L-k)l}{L}\right]^2\dot{\theta}^2\right) + \frac{1}{2}I\dot{\phi}^2$$
$$+ Mg\left[\rho\left(\frac{k}{L}\right) + \frac{(L-k)l}{L}\right]\cos\theta + \lambda(\rho - l)$$

$\frac{k}{L} = a.$

$$\Rightarrow L = \frac{1}{2}M\left(\dot{\rho}^2a^2 + (\rho a + (1-a)l)^2\dot{\theta}^2\right) + \frac{1}{2}I\frac{\dot{\rho}^2}{L^2}$$
$$+ Mg[\rho a + (1-a)l]\cos\theta + \lambda(\rho - l)$$

<u>ec. de mov θ)</u>

$$\frac{d}{dt}\left((\rho a + (1-a)l)^2\dot{\theta}M\right) = -Mg[\rho a + (1-a)l]\sin\theta$$

$$\Rightarrow 2(\rho a + (1-a)l)\dot{\rho}a\dot{\theta}M + (\rho a + (1-a)l)^2\ddot{\theta}M = -Mg[\rho a + (1-a)l]\sin\theta \quad (1)$$

Ec. de mov ρ

$$\frac{d}{dt}\left(M\dot{\rho}a^2 + I\frac{\dot{\rho}}{L^2}\right) = M(\rho a + (1-a)l)\dot{\theta}^2 \cdot a + Mga\cos\theta + \lambda$$

$$\Rightarrow M\ddot{\rho}a^2 + I\frac{\ddot{\rho}}{L^2} = M(\rho a + (1-a)l)\dot{\theta}^2 a + Mga\cos\theta + \lambda \quad (2)$$

Ec. de mov λ

$$\rho - l = 0 \Rightarrow \rho = l \Rightarrow \boxed{\dot{\rho} = 0}$$

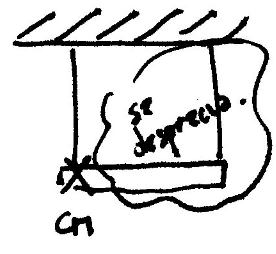
(2)

$$\Rightarrow 0 = M(la + (1-a)l)\dot{\theta}^2 a + Mga\cos\theta + \lambda$$

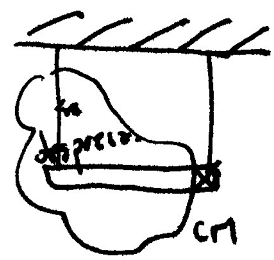
$$\Rightarrow \lambda = -(Mla\dot{\theta}^2 + Mga\cos\theta)$$

$$\boxed{\lambda = -\left(M\frac{lK}{L}\dot{\theta}^2 + Mg\frac{K}{L}\cos\theta\right)}$$

si $K = 0 \Rightarrow T = 0$ lo cual es lógico $\Rightarrow$



$K = L \Rightarrow T =$ péndulo simple $\Rightarrow$



y en (1)

$$\Rightarrow Ml^2\ddot{\theta} = -Mgl\sin\theta$$

$$\Rightarrow \boxed{\ddot{\theta} = \frac{-g}{l}\sin\theta}$$

esto ocurre ya que se supuso que ambas cuerdas oscilan en fase.