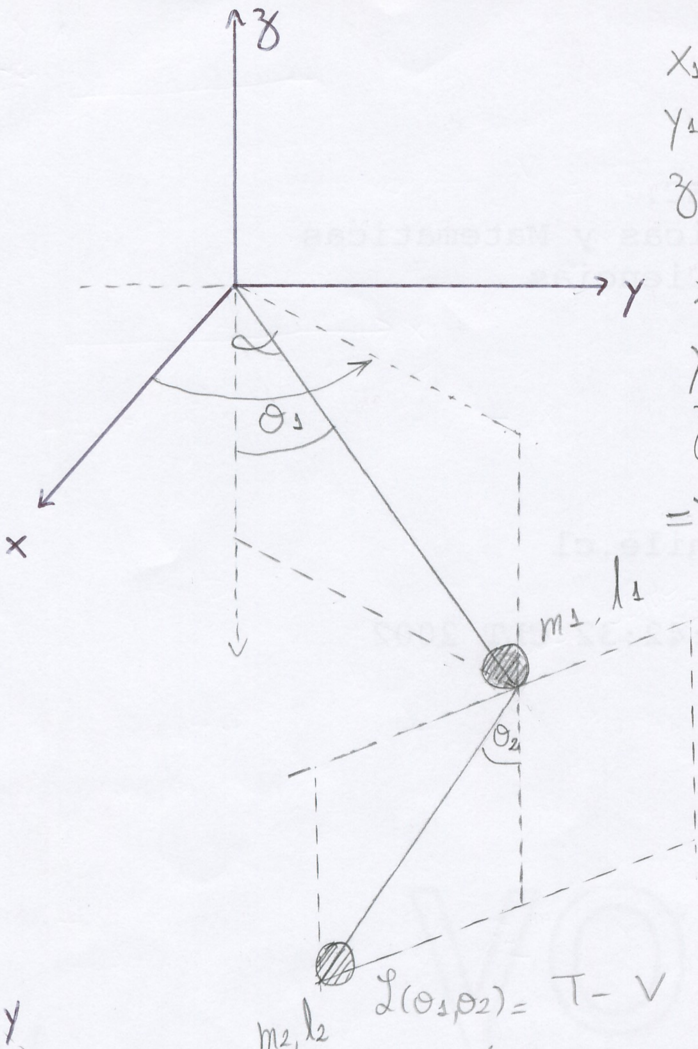



$$x_1(\theta_1) = l_1 \cos\alpha \, \mathrm{sen}\,\theta_1$$
$$y_1(\theta_1) = l_1 \mathrm{sen}\,\alpha \, \mathrm{sen}\,\theta_1$$
$$z_1(\theta_1) = -l_1 \cos\theta_1$$

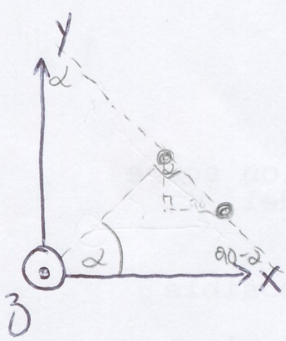
$$x_2(\theta_1,\theta_2) = l_1 \cos\alpha \, \mathrm{sen}\,\theta_1 + l_2 \mathrm{sen}\,\alpha \, \mathrm{sen}\,\theta_2$$
$$y_2(\theta_1,\theta_2) = l_1 \mathrm{sen}\,\alpha \, \mathrm{sen}\,\theta_1 + l_2 \cos\alpha \, \mathrm{sen}\,\theta_2$$
$$z_2(\theta_1,\theta_2) = -l_1 \cos\theta_1 - l_2 \cos\theta_2$$

$$\Rightarrow \dot{x}_1(\theta_1) = l_1 \cos\alpha \cos\theta_1 \, \dot{\theta}_1$$
$$\dot{y}_1(\theta_1) = l_1 \mathrm{sen}\,\alpha \cos\theta_1 \, \dot{\theta}_1$$
$$\dot{z}_1(\theta_1) = l_1 \mathrm{sen}\,\theta_1 \, \dot{\theta}_1$$

$$\dot{x}_2(\theta_1,\theta_2) = l_1 \cos\alpha \cos\theta_1 \, \dot{\theta}_1 + l_2 \mathrm{sen}\,\alpha \cos\theta_2 \, \dot{\theta}_2$$
$$\dot{y}_2(\theta_1,\theta_2) = l_1 \mathrm{sen}\,\alpha \cos\theta_1 \, \dot{\theta}_1 + l_2 \cos\alpha \cos\theta_2 \, \dot{\theta}_2$$
$$\dot{z}_2(\theta_1,\theta_2) = l_1 \mathrm{sen}\,\theta_1 \, \dot{\theta}_1 + l_2 \mathrm{sen}\,\theta_2 \, \dot{\theta}_2$$

$$\mathcal{L}(\theta_1,\theta_2) = T - V$$

$$T = \frac{m_1}{2}\left(\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2\right) + \frac{m_2}{2}\left(\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2\right)$$




$$= \frac{m_1}{2}\left(l_1^2 \cos^2\alpha \cos^2\theta_1 \, \dot{\theta}_1^2 + l_1^2 \mathrm{sen}^2\alpha \cos^2\theta_1 \, \dot{\theta}_1^2 + l_1^2 \mathrm{sen}^2\theta_1 \, \dot{\theta}_1^2\right) + \%$$

$$= \frac{m_1}{2}\left(l_1^2 \cos^2\theta_1 \, \dot{\theta}_1^2 + l_1^2 \mathrm{sen}^2\theta_1 \, \dot{\theta}_1^2\right) + \%$$

$$= \frac{m_1}{2}\left(l_1^2 \dot{\theta}_1^2\right) + \frac{m_2}{2}\left(l_1^2\dot{\theta}_1^2 + l_2^2\dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \left[\mathrm{sen}\,2\alpha \left(\cos\theta_1 \cos\theta_2\right) + \mathrm{sen}\,\theta_1 \mathrm{sen}\,\theta_2\right]\right)$$

$$V = -m_1 g \, l_1 \cos\theta_1 - m_2 g \, l_1 \cos\theta_1 - m_2 g \, l_2 \cos\theta_2$$
$$= -(m_1+m_2) g \, l_1 \cos\theta_1 - m_2 g \, l_2 \cos\theta_2$$

$\theta_1]$

$$\frac{d\mathcal{L}}{d\dot\theta_1} = m_1 l_1^2 \dot\theta_1 + m_2 l_1^2 \dot\theta_1 + m_2 l_1 l_2 \dot\theta_2 \left[ sen\, 2\alpha\, (\cos\theta_1 \cos\theta_2) + sen\,\theta_1\, sen\,\theta_2) \right]$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot\theta_1}\right) = (m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 \left[ \underbrace{sen\, 2\alpha}_{0} (\cos\theta_1 \cos\theta_2) + sen\,\theta_1\, sen\,\theta_2 \right]$$

$$+ m_2 l_1 l_2 \dot\theta_2 \frac{d}{dt} \left[ \alpha (\cos\theta_1 \cos\theta_2) + sen\,\theta_1\, sen\,\theta_2 \right]$$

$$\frac{d\mathcal{L}}{d\theta_1} = m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \left[ -\alpha\, sen\,\theta_1 \cos\theta_2 + \cos\theta_1\, sen\,\theta_2 \right] - (m_1+m_2)\, g\, l_1\, sen\,\theta_1$$

Ec. 1

$$(m_1+m_2) l_1^2 \ddot\theta_1 + m_2 l_1 l_2 \ddot\theta_2 \left[ sen\, 2\alpha (\cos\theta_1 \cos\theta_2) + sen\,\theta_1\, sen\,\theta_2 \right]$$

$$+ m_2 l_1 l_2 \dot\theta_2^2 \left[ -sen\, 2\alpha \cos\theta_1\, sen\,\theta_2 + sen\,\theta_1 \cos\theta_2\, \dot\theta_2 \right] = -(m_1+m_2)\, g\, l_1\, sen\,\theta_1$$

$\theta_2]$

$$\frac{d\mathcal{L}}{d\dot\theta_2} = m_2 l_2^2 \dot\theta_2 + m_2 l_1 l_2 \dot\theta_1 \left[ sen\, 2\alpha\, (\cos\theta_1 \cos\theta_2) + sen\,\theta_1\, sen\,\theta_2 \right]$$

$$\frac{d}{dt}\left(\frac{d\mathcal{L}}{d\dot\theta_2}\right) = m_2 l_2^2 \ddot\theta_2 + m_2 l_1 l_2 \ddot\theta_1 \left[ sen\, 2\alpha (\cos\theta_1 \cos\theta_2) + sen\,\theta_1\, sen\,\theta_2 \right]$$

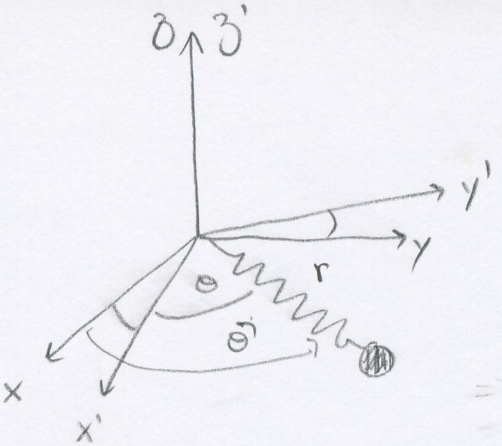
$$+ m_2 l_1 l_2 \dot\theta_1 \frac{d}{dt} \left[ sen\, 2\alpha\, (\cos\theta_1 \cos\theta_2) + sen\,\theta_1\, sen\,\theta_2 \right].$$

$$\frac{d\mathcal{L}}{d\theta_2} = m_2 l_1 l_2 \dot\theta_1 \dot\theta_2 \left[ -sen\, 2\alpha \cos\theta_1\, sen\,\theta_2 + sen\,\theta_1 \cos\theta_2 \right] - m_2\, g\, l_2\, sen\,\theta_2$$

Ec. 2

$$m_2 l_2^2 \ddot\theta_2 + m_2 l_1 l_2 \ddot\theta_1 \left[ sen\, 2\alpha (\cos\theta_1 \cos\theta_2) + sen\,\theta_1\, sen\,\theta_2 \right]$$

$$+ m_2 l_1 l_2 \dot\theta_1^2 \left[ sen\, 2\alpha\, sen\,\theta_1 \cos\theta_2\, \dot\theta_1 + \cos\theta_1\, sen\,\theta_2\, \dot\theta_1 \right] = -m_2\, g\, l_2\, sen\,\theta_2.$$




$$\vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\theta}'\hat{\theta}$$

$$= \dot{r}\hat{r} + r(\Omega(t) + \dot{\theta})\hat{\theta}$$

$$\Rightarrow L = \frac{1}{2}m\left(\dot{r}^2 + r^2(\Omega(t)+\dot{\theta})^2\right) - \frac{1}{2}kr^2$$

$$\frac{\partial L}{\partial r} = -kr + mr(\Omega(t)+\dot{\theta})^2$$

$$\frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

$$\Rightarrow m\ddot{r} = -kr + mr(\Omega(t)+\dot{\theta})^2$$

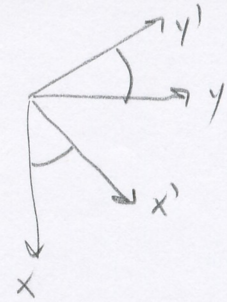
pero

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2(\Omega(t)+\dot{\theta}) = P_\theta$$

$$\Rightarrow (\Omega(t)+\dot{\theta}) = \frac{P_\theta}{mr^2}$$

$$\Rightarrow \boxed{m\ddot{r} = -kr + mr(\Omega(t)+\dot{\theta})^2}$$

$x' = r\cos\theta$
$y' = r\,\text{sen}\,\theta$




$\Rightarrow x = r\cos\theta\cos\alpha - r\,\text{sen}\,\theta\,\text{sen}\,\alpha$
$y = r\cos\theta\,\text{sen}\,\alpha + r\,\text{sen}\,\theta\cos\alpha$

$\alpha = \int \Omega(t)\,dt$

$x = x'\cos\alpha - y'\,\text{sen}\,\alpha$
$y = x'\,\text{sen}\,\alpha + y'\cos\alpha$

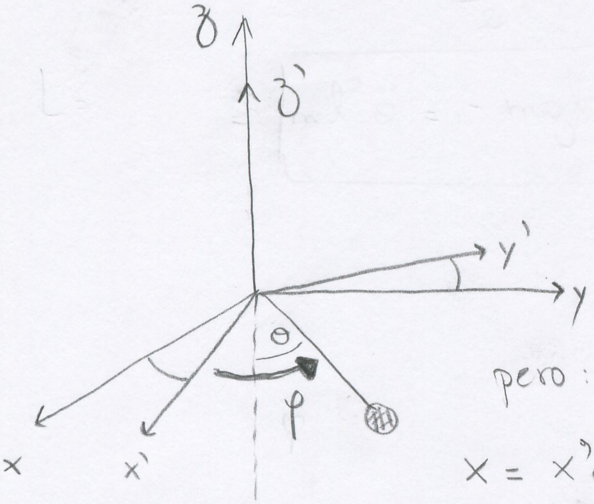
$\Rightarrow \dot{x} = \dot{r}(\cos\theta\cos\alpha - \text{sen}\,\theta\,\text{sen}\,\alpha) + r(-\text{sen}\,\theta\cos\alpha - \cos\theta\,\text{sen}\,\alpha)\dot{\theta}$
$+ r(-\cos\theta\,\text{sen}\,\alpha - \text{sen}\,\theta\cos\alpha)\dot{\alpha}$

$\dot{y} = \dot{r}(\cos\theta\,\text{sen}\,\alpha + \text{sen}\,\theta\cos\alpha) + r(-\text{sen}\,\theta\,\text{sen}\,\alpha + \cos\theta\cos\alpha)\dot{\theta}$
$+ r(\cos\theta\cos\alpha - \text{sen}\,\theta\,\text{sen}\,\alpha)\dot{\alpha}$

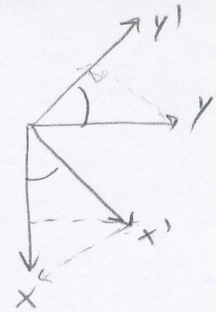
$$\Rightarrow \dot{x}^2 + \dot{y}^2 = \dot{r}^2(\cos^2\theta + \text{sen}^2\theta) + r^2\dot{\theta}^2(\text{sen}^2\theta + \cos^2\theta) + r^2\dot{\alpha}^2(\cos^2\theta + \text{sen}^2\theta)$$
$$+ 2r\dot{\theta}\,r\dot{\alpha}\,(\underbrace{\underbrace{\text{sen}^2\theta\cos^2\alpha + \cos^2\theta\,\text{sen}^2\alpha + \text{sen}^2\theta\,\text{sen}^2\alpha + \cos^2\theta\cos^2\alpha}_{\text{sen}^2\theta + \cos^2\theta}}_{1})$$

$$\Rightarrow \dot{x}^2 + \dot{y}^2 = \dot{r}^2 + r^2(\dot{\theta}^2 + \dot{\alpha}^2 + 2\dot{\theta}\dot{\alpha}) = \dot{r}^2 + r^2(\dot{\theta} + \Omega)^2$$

# Péndulo de Foucault




$$x' = l\,\mathrm{sen}\theta\cos\varphi$$
$$y' = l\,\mathrm{sen}\theta\,\mathrm{sen}\varphi$$
$$z' = -l\cos\theta$$




pero:

$$x = x'\cos\Omega t - y'\,\mathrm{sen}\,\Omega t$$
$$y = y'\cos\Omega t + x'\,\mathrm{sen}\,\Omega t$$
$$z = z'$$

$$\Rightarrow x = l\,\mathrm{sen}\theta\cos\varphi\cos\Omega t - l\,\mathrm{sen}\theta\,\mathrm{sen}\varphi\,\mathrm{sen}\,\Omega t$$
$$y = l\,\mathrm{sen}\theta\cos\varphi\,\mathrm{sen}\,\Omega t + l\,\mathrm{sen}\theta\,\mathrm{sen}\varphi\cos\Omega t$$
$$z = -l\cos\theta$$

$$\Rightarrow \dot{x} = l\cos\theta\,\dot{\theta}(\cos\varphi\cos\Omega t - \mathrm{sen}\varphi\,\mathrm{sen}\,\Omega t) + l\,\mathrm{sen}\theta(-\mathrm{sen}\varphi\cos\Omega t - \cos\varphi\,\mathrm{sen}\,\Omega t)\dot{\varphi}$$
$$+ l\,\mathrm{sen}\theta(-\cos\varphi\,\mathrm{sen}\,\Omega t - \mathrm{sen}\varphi\cos\Omega t)\Omega$$

$$\dot{y} = l\cos\theta\,\dot{\theta}(\cos\varphi\,\mathrm{sen}\,\Omega t + \mathrm{sen}\varphi\cos\Omega t) + l\,\mathrm{sen}\theta(-\mathrm{sen}\varphi\,\mathrm{sen}\,\Omega t + \cos\varphi\cos\Omega t)\dot{\varphi}$$
$$+ l\,\mathrm{sen}\theta(\cos\varphi\cos\Omega t - \mathrm{sen}\varphi\,\mathrm{sen}\,\Omega t)\Omega$$

$$\dot{z} = l\,\mathrm{sen}\theta\,\dot{\theta}$$

$$\dot{x}^2 + \dot{y}^2 = l^2\cos^2\theta\,\dot{\theta}^2(\cos^2\varphi + \mathrm{sen}^2\varphi) + l^2\,\mathrm{sen}^2\theta(\mathrm{sen}^2\varphi + \cos^2\varphi)\dot{\varphi}^2 + l^2\,\mathrm{sen}^2\theta(\cos^2\varphi + \mathrm{sen}^2\varphi)\Omega^2$$
$$+ 2l^2\,\mathrm{sen}^2\theta(\mathrm{sen}^2\varphi + \cos^2\varphi)\Omega\dot{\varphi}$$
$$= l^2\cos^2\theta\,\dot{\theta}^2 + l^2\,\mathrm{sen}^2\theta(\dot{\varphi}^2 + \Omega^2 + 2\Omega\dot{\varphi}) = l^2\cos^2\theta\,\dot{\theta}^2 + l^2\,\mathrm{sen}^2\theta(\dot{\varphi} + \Omega)^2$$

$$\Rightarrow T = \frac{1}{2}ml^2(\dot{\theta}^2 + \mathrm{sen}^2\theta(\dot{\varphi} + \Omega)^2)$$

$$\Rightarrow L = \frac{1}{2}ml^2(\dot{\theta}^2 + \mathrm{sen}^2\theta(\dot{\varphi} + \Omega)^2) + mgl\cos\theta$$

$$\frac{\partial L}{\partial \theta} = ml^2 \operatorname{sen}\theta \cos\theta \,(\dot\varphi + \Omega)^2 - mgl \operatorname{sen}\theta$$

$$\frac{\partial L}{\partial \dot\theta} = ml^2 \dot\theta$$

$$\Rightarrow ml^2 \ddot\theta = -mgl \operatorname{sen}\theta + ml^2 \operatorname{sen}\theta \cos\theta \,(\dot\varphi + \Omega)^2$$

pero

$$\frac{\partial L}{\partial \dot\varphi} = P_\varphi = ml^2 \operatorname{sen}^2\theta \,(\dot\varphi + \Omega)$$

$$\Rightarrow \frac{P_\varphi}{ml^2 \operatorname{sen}^2\theta} - \Omega = \dot\varphi$$

$$\Rightarrow \dot\varphi + \Omega = \frac{P_\varphi}{ml^2 \operatorname{sen}^2\theta}$$

$$\Rightarrow ml^2 \ddot\theta = -mgl \operatorname{sen}\theta + ml^2 \operatorname{sen}\theta \cos\theta \frac{P_\varphi^2}{m^2 l^4 \operatorname{sen}^4\theta}$$

$$\boxed{ml^2 \ddot\theta = -mgl \operatorname{sen}\theta + \frac{P_\varphi^2 \cos\theta}{ml^2 \operatorname{sen}^3\theta}}$$