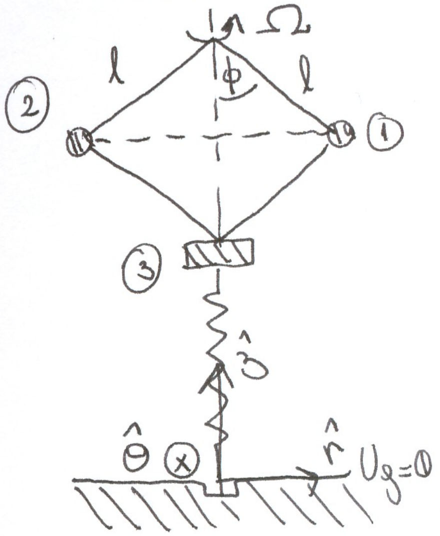
Pauta Ej. 2



En coord. cilíndricos:

$$\vec{v}_{①} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$$

pero $r = l\,\mathrm{sen}\phi \Rightarrow \dot{r} = l\cos\phi\,\dot{\phi}$

$\dot{\theta} = \Omega$

$z_1 = 2l - l\cos\phi \Rightarrow \dot{z} = l\,\mathrm{sen}\phi\,\dot{\phi}$

$$\Rightarrow \vec{v}_{①} = l\cos\phi\,\dot{\phi}\,\hat{r} + l\,\mathrm{sen}\phi\,\Omega\,\hat{\theta} + l\,\mathrm{sen}\phi\,\dot{\phi}\,\hat{z}$$

Notar que: $\vec{v}_{①} = \vec{v}_{②}$

$\vec{v}_3 = \dot{z}^3\,\hat{z}$ donde $z^3 = 2l - 2l\cos\phi$

$\Rightarrow \dot{z}^3 = 2l\,\mathrm{sen}\phi\,\dot{\phi}$

$$\Rightarrow T = \frac{1}{2}m'4l^2\mathrm{sen}^2\phi\,\dot{\phi}^2 + m\left(l^2\dot{\phi}^2 + l^2\mathrm{sen}^2\phi\,\Omega^2\right)$$

$$U = 2mg(2l - l\cos\phi) + m'gl(2 - 2\cos\phi) + \frac{1}{2}K(2l - 2l\cos\phi)^2$$

$$\Rightarrow L = m(l^2\dot{\phi}^2 + l^2\mathrm{sen}^2\phi\,\Omega^2) + 2m'l^2\mathrm{sen}^2\phi\,\dot{\phi}^2 - 2mgl(2 - \cos\phi)$$
$$- 2m'gl(1 - \cos\phi) + 2Kl^2(1 - \cos\phi)^2$$

Ecs.

$$\frac{\partial L}{\partial \phi} = 2ml^2\mathrm{sen}\phi\cos\phi\,\Omega^2 + 4m'l^2\mathrm{sen}\phi\cos\phi\,\dot{\phi}^2 - 2mgl\,\mathrm{sen}\phi - 2m'gl\,\mathrm{sen}\phi$$
$$+ 4Kl^2(1 - \cos\phi)\,\mathrm{sen}\phi$$

$$\frac{\partial L}{\partial \dot{\phi}} = 2ml^2\dot{\phi} + 4m'l^2\mathrm{sen}^2\phi\,\dot{\phi} \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = 2ml^2\ddot{\phi} + 8m'l^2\mathrm{sen}\phi\cos\phi\,\dot{\phi}^2$$
$$+ 4m'l^2\mathrm{sen}^2\phi\,\ddot{\phi}$$

$$\Rightarrow \ddot{\phi}\left(m + 2m'\mathrm{sen}^2\phi\right) = -m'\mathrm{sen}(2\phi)\,\dot{\phi}^2 + m\,\mathrm{sen}\phi\cos\phi\,\Omega^2 - (m+m')gl\,\mathrm{sen}\phi$$
$$+ 2K(1 - \cos\phi)\,\mathrm{sen}\phi$$

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