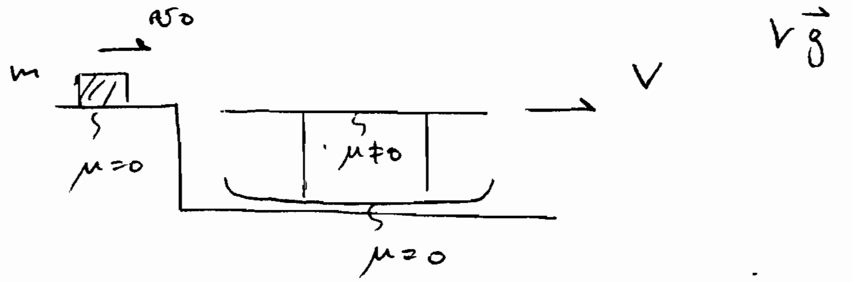
Anexo P1 Aux 09 11/4/2006



En clases vimos que:

sistema: masa + trineo

(a) $F_x^{ext} = 0 \Rightarrow P_x^{inicial} = P_x^{final}$

$$\Rightarrow m v_0 = (m+M)V$$

$$\Rightarrow \boxed{V = \frac{m}{m+M} v_0}$$

(b) $W(\text{froce}) = \Delta E_{m+M} = \Delta K_{m+M}$ (no hay cambios de energía potencial)

$$K_i = \frac{1}{2} m v_0^2$$

$$K_f = \frac{1}{2}(m+M)V^2 = \frac{1}{2}(m+M)\frac{m^2}{(m+M)^2}v_0^2 = \frac{1}{2}\frac{m^2}{m+M}v_0^2$$

$$\Rightarrow W\binom{\text{froce}}{\text{sobre } m+M} = \frac{1}{2}\frac{m^2}{m+M}v_0^2 - \frac{1}{2}m v_0^2 = \frac{1}{2}m\left(\frac{m}{m+M}-1\right)v_0^2$$

$$\Rightarrow \boxed{W\binom{\text{froce}}{\text{sobre } m+M} = -\frac{1}{2}\frac{mM}{m+M}v_0^2}$$

(c) $$W\binom{\text{froce}}{\text{sobre } m+M} = -\underbrace{\text{froce}}_{-\mu\cdot mg} \cdot \underbrace{\Delta X_{relativo}}_{D} = -\mu m g D$$

$$\Rightarrow -\mu m g D = -\frac{1}{2}\frac{mM}{m+M} \cdot v_0^2$$

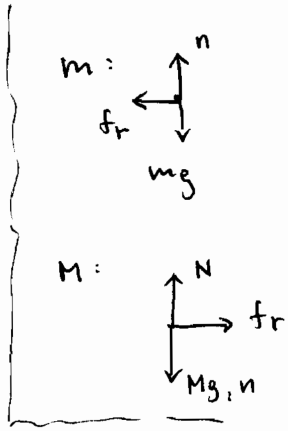
$$\Rightarrow \boxed{\mu = \frac{M}{m+M}\frac{v_0^2}{2gD}}$$

El término $\Delta X_{relativo}$ sale de:

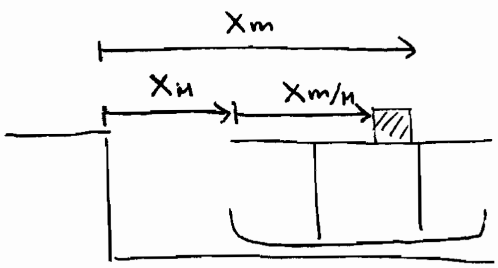
$$W\left(\begin{matrix}\text{froce}\\ \text{sobre } m+M\end{matrix}\right) = W\left(\begin{matrix}\text{froce}\\ \text{sobre } m\end{matrix}\right) + W\left(\begin{matrix}\text{froce}\\ \text{sobre } M\end{matrix}\right)$$

$$= -f_r \cdot \Delta X_m + f_r \cdot \Delta X_M$$

$$= -f_r\,\underbrace{(\Delta X_m - \Delta X_M)}$$



$\Delta X_{m/M}$ desplazamiento de m relativo a M



$$\Rightarrow \boxed{W\left(\begin{matrix}\text{froce}\\ \text{sobre } m+M\end{matrix}\right) = -f_r \cdot \Delta X_{relativo}}$$

Otra forma de haber calculado la parte (c) es mediante movimiento relativo

$$X_m = X_M + X_{m/M}$$

$$\Rightarrow \ddot{X}_m = \ddot{X}_M + \ddot{X}_{m/M}$$

pero; Newton sobre m $\longrightarrow$ $-\mu m g = m\ddot{X}_m$

$$\Rightarrow \ddot{X}_m = -\mu g$$

Newton sobre M $\longrightarrow$ $\mu m g = M\ddot{X}_M$

$$\Rightarrow \ddot{X}_M = \mu \frac{m}{M} g$$

Además, por cinemática:

$$\cancel{v_f}^{\,2\ 0} - \underbrace{v_i}_{v_0}{}^2 = 2\ddot{X}_{m/M} \cdot D \Rightarrow \ddot{X}_{m/M} = -\frac{v_0^2}{2D}$$

Entonces, $\ddot{X}_m = \ddot{X}_M + \ddot{X}_{m/M} \Rightarrow -\mu g = \mu \frac{m}{M} g - \frac{v_0^2}{2D}$

$$\Rightarrow \boxed{\mu = \frac{M}{m+M} \cdot \frac{v_0^2}{2gD}}$$