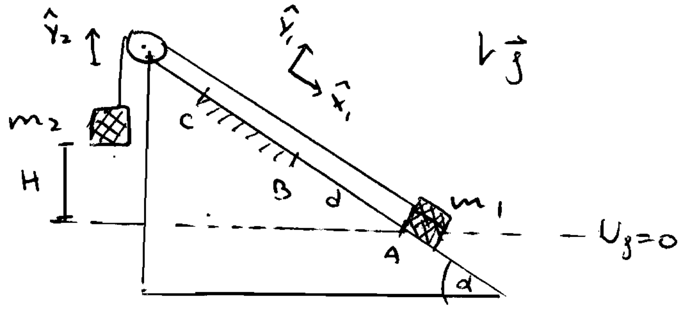
Solución Control 1 FI10B 2005

P1]



Datos : $m_1, m_2, d, \mu_c, \alpha$

(a) $E = \frac{1}{2} m_1 v_1^2 + m_1 g h_1 + \frac{1}{2} m_2 v_2^2 + m_2 g h_2$

$E_A = 0 + 0 + 0 + m_2 g H$

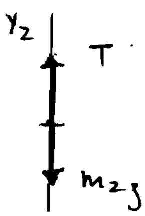
$E_B = \frac{1}{2} m_1 v_B^2 + m_1 g d \sin\alpha + \frac{1}{2} m_2 v_B^2 + m_2 g (H-d)$

$E_A = E_B \Rightarrow m_2 \cancel{g} H = \frac{1}{2}(m_1 + m_2) v_B^2 + m_1 g d \sin\alpha + m_2 \cancel{g} H - m_2 g d$

$$\Rightarrow \boxed{v_B^2 = 2 g d \, \frac{m_2 - m_1 \sin\alpha}{m_1 + m_2}}$$

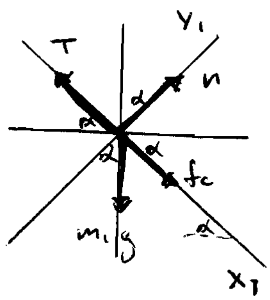
(b) Entre B y C

DCL m_2



$\hat{y}_2)$ $T - m_2 g = m_2 \ddot{y}_2$ (1)

DCL m_1



$\hat{y}_1)$ $n - m_1 g \cos\alpha = 0$ (2)

$\hat{x}_1)$ $\underbrace{f_c}_{\mu_c n} + m_1 g \sin\alpha - T = m_1 \ddot{x}_1$ (3)

(1) + (3) $\Rightarrow$ $\mu_c n + m_1 g \sin\alpha - m_2 g = m_1 \ddot{x}_1 + m_2 \ddot{y}_2$ pero $\ddot{x}_1 = \ddot{y}_2$

n de (2) $\Rightarrow$ $\mu_c m_1 g \cos\alpha + m_1 g \sin\alpha - m_2 g = (m_1 + m_2) \ddot{x}_1$

$$\Rightarrow \boxed{\ddot{x}_1 = \frac{m_1 (\sin\alpha + \mu_c \cos\alpha) - m_2}{m_1 + m_2} \, g}$$

(c) $v_f^2 = v_i^2 + 2a\,\Delta d$

$$v_C^2 = v_B^2 + 2a\,\Delta d_{BC}$$

$$0 = \cancel{2}g d.\frac{m_2 - m_1 \sin\alpha}{\cancel{m_1+m_2}} + \cancel{2}\frac{m_1(\sin\alpha + \mu_c \cos\alpha) - m_2}{\cancel{m_1+m_2}}\cancel{g}\,\Delta d_{BC}$$

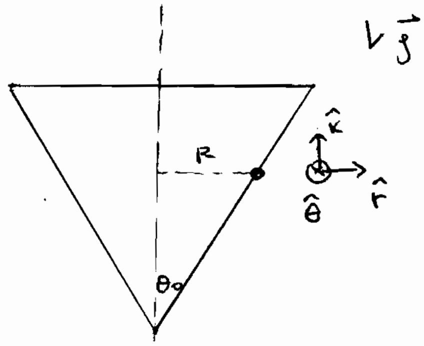
$$\Rightarrow \Delta d_{BC} = -\frac{m_2 - m_1 \sin\alpha}{m_1(\sin\alpha + \mu_c\cos\alpha) - m_2}\cdot d$$

$\Delta d_{BC} < 0$ pues m_1 se mueve en el sentido opuesto de $\hat{x}_1$

Entonces, la distancia total recorrida por m_1 es

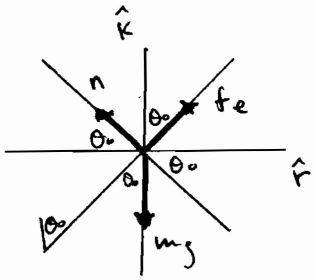
$$\boxed{d + \frac{m_1 \sin\alpha - m_2}{m_1(\sin\alpha + \mu_c\cos\alpha) - m_2}\cdot d}$$

P2]



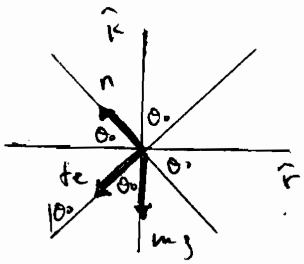
$\downarrow \vec{g}$ Datos: m, R, μ_e

(a) Si el cono gira "muy lento"



$$\hat{r})\ -n\cos\theta_0 + f_e \sin\theta_0 = -m\omega_{min}^2 R \quad (1)$$
$$\hat{k})\ n\sin\theta_0 + f_e\cos\theta_0 - mg = 0 \quad (2)$$
$$\hat{\theta})\ 0 = m\alpha R$$

Si el cono gira "muy rápido"



$$\hat{r})\ -n\cos\theta_0 - f_e\sin\theta_0 = -m\omega_{max}^2 R$$
$$\hat{k})\ n\sin\theta_0 - f_e\cos\theta_0 - mg = 0$$
$$\hat{\theta})\ 0 = m\alpha R$$

$f = \mu_e \cdot n$ en (1) y (2) $\Rightarrow$
$$\begin{cases} n(\cos\theta_0 - \mu_e\sin\theta_0) = m\omega_{min}^2 R & (3) \\ n(\sin\theta_0 + \mu_e\cos\theta_0) = mg & (4) \end{cases}$$

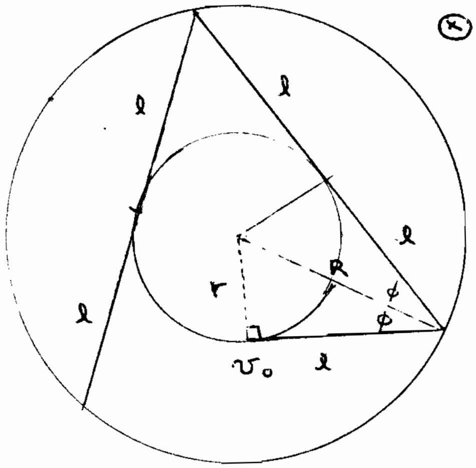
$$(3)/(4) \Rightarrow \omega_{min}^2 = \frac{\cos\theta_0 - \mu_e\sin\theta_0}{\sin\theta_0 + \mu_e\cos\theta_0}\cdot\frac{g}{R}$$

Para el otro caso es análogo

$$\Rightarrow \omega_{max}^2 = \frac{\cos\theta_0 + \mu_e\sin\theta_0}{\sin\theta_0 - \mu_e\cos\theta_0}\cdot\frac{g}{R}$$

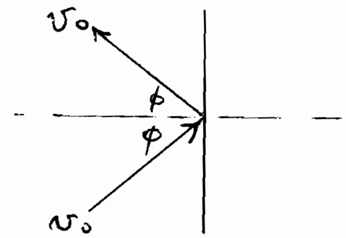
$$\therefore \boxed{\frac{\cos\theta_0 - \mu_e\sin\theta_0}{\sin\theta_0 + \mu_e\sin\theta_0}\cdot\frac{g}{R} \le \omega^2 \le \frac{\cos\theta_0 + \mu_e\sin\theta_0}{\sin\theta_0 - \mu_e\cos\theta_0}\cdot\frac{g}{R}}$$

P3



$\otimes \vec{g}$ Datos: r, R, w

Choque elástico

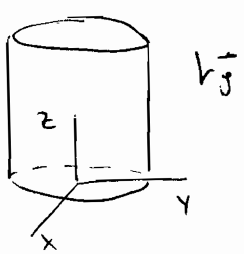


(a) $v_0 = w \cdot r$ $\qquad \ell = \sqrt{R^2 - r^2}$

N° rebote	distancia recorrida
1	ℓ
2	3ℓ
3	5ℓ
⋮	⋮
n	$(2n-1)\,\ell$

$$t = \frac{d}{v} \quad \Rightarrow \quad \boxed{t_{vuelo} = \frac{(2n-1)\sqrt{R^2 - r^2}}{w \cdot r}}$$

(b)



$$\cancel{z_f}^{\,0} = z_i + \cancel{v_{z_0}}^{\,0}\, t - \frac{1}{2} g t^2$$

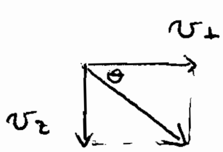
$$0 = h - \frac{1}{2} g\, t_{vuelo}^2$$

$$\Rightarrow \quad \boxed{h = \frac{1}{2} g \left[\frac{(2n-1)\sqrt{R^2 - r^2}}{wr} \right]^2}$$

(c) $v_\perp = w r$ velocidad en el plano XY, es cte

$$v_z = \cancel{v_{z_0}}^{\,0} - g\, t_{vuelo}$$

$$v_z = - g \frac{(2n-1)\,\ell}{wr}$$



$$\Rightarrow \quad \tan\theta = \frac{v_z}{v_\perp}$$

$$\Rightarrow \quad \boxed{\tan\theta = - \frac{g\,(2n-1)\sqrt{R^2 - r^2}}{(wr)^2}}$$