

EM725

Control Adaptivo de Sistemas

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4.0 Control Adaptivo por Referencia a Modelo (CARM) de Plantas de Grado Relativo Unitario($n^* = 1$)

Modelo de Referencia

$$W_m(s) = k_m \frac{Z_m(s)}{R_m(s)} \quad r(t) \rightarrow \boxed{W_m(s)} \rightarrow Y_m(t)$$

- $Z_m(s)$: mónico y Hurwitz grado $n-1$
 $R_m(s)$: mónico y Hurwitz grado n
(Fde T estrictamente real positiva)
 $r(\cdot)$: uniformemente acotada y continua por tramos
 k_m : ganancia de alta frecuencia

$$Z_m(s) = s^{n-1} + b_{n-1}^m s^{n-2} + \dots + b_1^m s + b_0^m$$
$$R_m(s) = s^n + a_{n-1}^m s^{n-1} + \dots + a_1^m s + a_0^m$$

Planta

$$W_p(s) = k_p \frac{Z_p(s)}{R_p(s)}$$

$Z_p(s)$: mónico y Hurwitz grado $n-1$

$R_p(s)$: mónico y Hurwitz grado n

k_p : ganancia de alta frecuencia

$$Z_p(s) = s^{n-1} + b_{n-2}s^{n-2} + \dots + b_1s + b_0$$

$$R_p(s) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0$$

Supuestos Clásicos

- i) Signo de k_p conocido
- ii) Cota superior de n conocida
- iii) grado relativo n^* conocido
- iv) ceros de la planta en \mathbb{C}^-

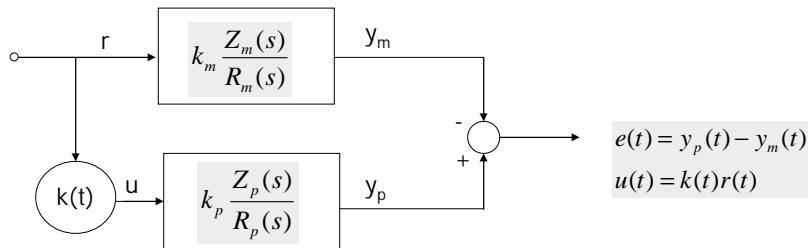
$$\dot{x}_p = A_p x_p + b_p u$$

$$y_p = h_p^T x_p$$

Objetivo de Control

$$\lim_{t \rightarrow \infty} (y_p(t) - y_m(t)) = 0$$

Caso 1: k_p desconocido (magnitud)



$$Z_m(s) = Z_p(s) \quad R_m(s) = R_p(s)$$

$$w(s) = \frac{Z_m(s)}{R_m(s)} = \frac{Z_p(s)}{R_p(s)}$$

Mónicos y Hurwitz grados $n-1$ y n respectivamente. Ambos conocidos

$$y_p(t) = k_p W(s) u(t) = k_p W(s) [k(t)r]$$

$$y_m(s) = k_m W(s) r(t)$$

$$e(t) = W(s) [k_p k - k_m] r = k_p W(s) \left[k - \frac{k_m}{k_p} \right] r$$

$$\psi(t) = k(t) - \frac{k_m}{k_p}, \quad k^* = \frac{k_m}{k_p}$$

$$e(t) = k_p W(s) [\psi(t) r(t)]$$

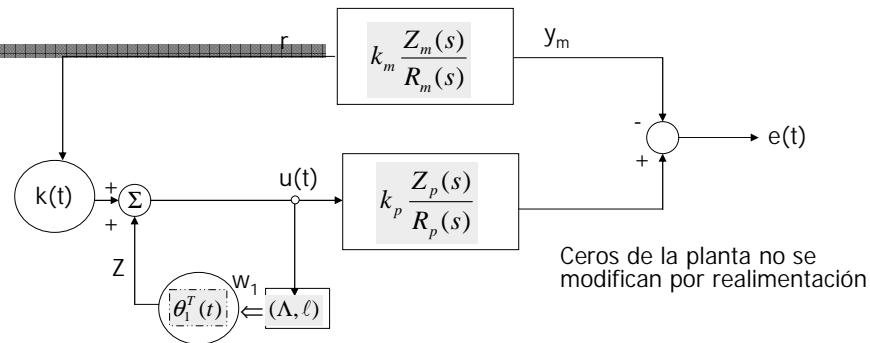
$$W_m(s) \text{ e.r.p.} \Rightarrow k_m > 0 \Rightarrow W(s) \text{ e.r.p.}$$

Ley de Ajuste:

$$\dot{k}(t) = \dot{\psi}(t) = -\operatorname{sgn}(k_p) e(t) r(t)$$

$$\lim_{t \rightarrow \infty} (y_p(t) - y_m(t)) = 0 \quad V = \frac{1}{2} \psi^T \psi$$

Caso 2: Ceros desconocidos (k_p también)



$R_m(s) = R_p(s)$: mónico y Hurwitz grado n (conocido)

$Z_p(s)$: mónico y Hurwitz grado n-1 (desconocido)

$Z_p(s)$: mónico y Hurwitz grado n-1 (conocido)

$$\dot{\omega}_l = \Lambda \omega_l + \ell u \quad \Lambda \in \Re^{(n-1) \times (n-1)}$$

$$u = \theta_l^T \omega_l + kr \quad \theta_l \in \Re^{n-1}$$

Existencia de k^*, θ_l^*

$$\theta_l^{*T} (SI - \Lambda)^{-1} \ell = \frac{\theta_l^*(s)}{\Lambda(s)}$$

$$k * \frac{1}{1 - \frac{\theta_l^*(s)}{\Lambda(s)}} \cdot k_p \frac{Z_p(s)}{R_p(s)} = k_m \frac{Z_m(s)}{R_p(s)}$$

$$k * k_p \frac{\Lambda(s) Z_p(s)}{\Lambda(s) - \theta_l^*(s)} = k_m Z_m(s), \quad \text{con} \quad k * k_p = k_m$$

Escoger

$$\begin{aligned} \Lambda(s) &= Z_m(s) \quad \therefore \\ \theta_l^*(s) &= -Z_p(s) + \Lambda(s) \\ \theta_l^*(s) &= Z_m(s) - Z_p(s) \end{aligned}$$

Notacion:

$$\begin{aligned} \psi(t) &= k(t) - k * \phi_l(t) = \theta_l(t) - \theta_l^* \\ \bar{\theta}_l(t) &= [k(t), \theta_l^T(t)]^T \quad \bar{\omega}_l(t) = [r(t), \omega_l^T(t)]^T \\ \bar{\phi}_l(t) &= [\psi(t), \phi_l^T(t)]^T \end{aligned}$$

Podemos escribir compactamente

$$\begin{aligned} \begin{bmatrix} \dot{x}_p \\ \dot{\omega}_l \end{bmatrix} &= \begin{bmatrix} A_p & b_p \theta_l^{*T} \\ 0 & \Lambda + \ell \theta_l^{*T} \end{bmatrix} \begin{bmatrix} x_p \\ \omega_l \end{bmatrix} + \begin{bmatrix} k^* & b_p \\ k^* & \ell \end{bmatrix} r + \begin{bmatrix} b_p \\ \ell \end{bmatrix} \bar{\phi}_l^T \bar{\omega}_l \\ y_p &= [h_p^T \quad 0^T]^T x \quad x = [x_p^T \quad \omega_l^T]^T \end{aligned}$$

$$\begin{bmatrix} \dot{\bar{x}}_p^* \\ \dot{\bar{\omega}}_1^* \end{bmatrix} = \begin{bmatrix} A_p & b_p \theta_1^{*T} \\ 0 & \Lambda \ell \theta_1^{*T} \end{bmatrix} \begin{bmatrix} \bar{x}_p^* \\ \bar{\omega}_1^* \end{bmatrix} + \begin{bmatrix} k^* b_p \\ k^* \ell \end{bmatrix} r$$

$$y_m = \begin{bmatrix} h_p^T & 0^T \end{bmatrix} x_{mn} \quad x_{mn} = \begin{bmatrix} \bar{x}_p^{*T} & \bar{\omega}_1^{*T} \end{bmatrix}^T$$

$$\dot{e}' = A_{mn} e' + b_{mn} \bar{\phi}_1^T \bar{\omega}_1 \quad , \quad e = h_{mn}^T e'$$

$$A_{mn} = \begin{bmatrix} A_p & b_p \theta_1^{*T} \\ 0 & \Lambda + \ell \theta_1^{*T} \end{bmatrix}, \quad b_{mn} = \begin{bmatrix} b_p \\ \ell \end{bmatrix}, \quad h_{mn} = \begin{bmatrix} b_p \\ 0 \end{bmatrix}$$

$$e' = x - x_{mn}, \quad e = y_p - y_m$$

$$h_{mn}^T (sI - A_{mn})^{-1} b_{mn} k^* = W_m(s)$$

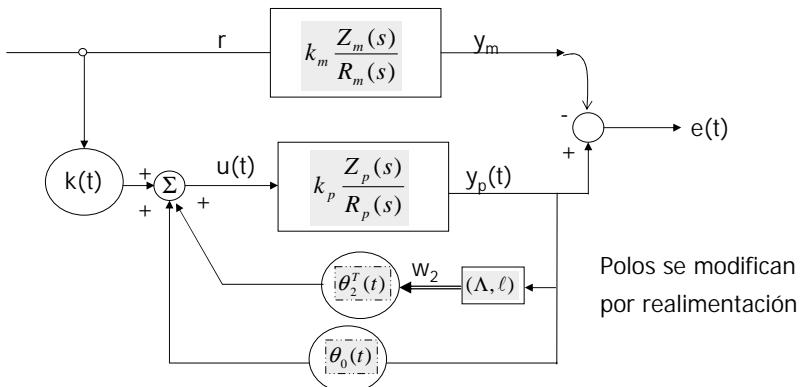
$$\therefore e = \frac{k_p}{k_m} W_m(s) \begin{bmatrix} \bar{\phi}_1^T \bar{\omega}_1 \end{bmatrix}$$

$$\bar{\phi}_1 = -\text{sgn}(k_p) e \bar{\omega}_1(t)$$

$$\lim_{t \rightarrow \infty} (y_p(t) - y_m(t)) = 0$$

$$V = e^{\tau T} P e' + \frac{1}{2} \bar{\phi}_1^T \bar{\phi}_1$$

Caso 3: polos desconocidos (k_p también)



$Z_p(s) = Z_p(s)$: momico y Hurwitz grado $n-1$ (conocido)

$R_p(s)$: momico y grado n (desconocido)

$R_m(s)$: momico y Hurwitz grado $n-1$ (conocido)

$$\dot{\omega}_2 = \Lambda \omega_2 + \ell y_p \quad \Lambda \in \Re^{(n-1) \times (n-1)}, \theta_2 \in \Re^{n-1}$$

$$u(t) = k(t)r + \theta_2^T(t)\omega_2(t) + \theta_0(t)y_p$$

Existencia de θ_0^*, k^* y θ_2^*

$$\theta_2^{*T} (sI - \Lambda^{-1}\ell) = \frac{\theta_2^*(s)}{\Lambda(s)}$$

$$\frac{\theta_2^*(s)}{\Lambda(s)} + \theta_0^* = \frac{\theta_2^*(s) + \theta_0^* \Lambda(s)}{\Lambda(s)}$$

$$k^* \frac{k_p \frac{Z_p}{R_p}}{1 - \frac{k_p Z_p}{R_p} \frac{(\theta_2^* + \theta_0^* \Lambda)}{\Lambda}} = ? k_m \frac{z_m}{R_m}, \Rightarrow k^* k_p = k_m$$

escoger $\Lambda(s) = Z_m(s) \therefore$

$$R_p(s) - k_p (\theta_2^*(s) + \theta_0^* Z_m(s)) = R_m(s)$$

Notacion

$$\psi(t) = k(t) - k^* \quad \phi_0(t) = \theta_0(t) - \theta_0^* \quad \phi_2(t) = \theta_2(t) - \theta_2^*$$

$$\bar{\theta}_2(t) \left[k(t), \theta_0(t) \theta_2^T(t) \right]^T, \bar{\theta}_2^* = \left[k^*, \theta_0^*, \theta_2^{*T} \right]^T$$

$$\bar{\phi}_2(t) = \bar{\theta}_2(t) - \bar{\theta}_2^*$$

$$\bar{\phi}_2(t) = \left[\psi(t), \phi_0(t), \phi_2^T(t) \right]^T, \quad \bar{\omega}_2(t) = \left[r \ y_p \ \omega_2^T \right]^T$$

Igual que antes

$$y_p(t) = \frac{k_p}{k_m} W_m(s) \left[k^* r(t) + \bar{\phi}_2^T(t) \bar{\omega}_2(t) \right]$$

$$y_m(t) = W_m(s) r(t)$$

$$e(t) = \frac{k_p}{k_m} W_m(s) \left[\bar{\phi}_2^T(t) \bar{\omega}_2(t) \right]$$

$$\dot{\bar{\phi}}_2(t) = -\text{sgn}(k_p) e(t) \bar{\omega}_2(t)$$

$$\dot{k}(t) = -\text{sgn}(k_p) e(t) r(t)$$

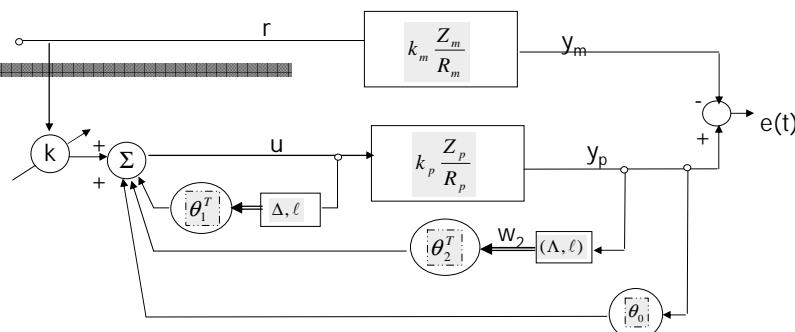
$$\dot{\theta}_0(t) = -\text{sgn}(k_p) e(t) y_p(t)$$

$$\dot{\theta}_2(t) = -\text{sgn}(k_p) e(t) \omega_2(t)$$

$$V = e^T P e + \frac{1}{2} \bar{\phi}_2^T \bar{\phi}_2$$

$$\lim_{t \rightarrow \infty} |y_p(t) - y_m(t)| = 0$$

Caso General $n^* = 1$



$Z_m(s), R_m$: mónico y Hurwitz grado $n-1$ y n (conocidos)

$W_m(s)$ e.r.p.

Z_p : mónico y grado $n-1$ (desconocido)

R_m : mónico y Hurwitz grado n (desconocido)

$$\begin{aligned}\dot{\omega}_1 &= \Lambda \omega_1 + \ell u & \Lambda \in \Re^{(n-1) \times (n-1)} \\ \dot{\omega}_2 &= \Lambda \omega_2 + \ell y_p \\ \omega &= \begin{bmatrix} r & \omega_1^T & y_p & \omega_2^T \end{bmatrix}^T & \theta = \begin{bmatrix} k & \theta_1^T & \theta_0 & \theta_2^T \end{bmatrix}^T \in \Re^{2n}\end{aligned}$$

Existencia de θ^*

$$\frac{k * k_p Z_p \Lambda}{(\Lambda - \theta_1^*) R_p - k_p Z_p (\theta_2^* + \theta_0^* \Lambda)} = k_m \frac{Z_m}{R_m}$$

$$\begin{aligned}k * k_p &= k_m \quad y \quad \text{escogemos} \quad \Lambda = Z_m \\ (Z_m - \theta_1^*) R_p - k_p Z_p (\theta_2^* + \theta_0^* Z_m) &= Z_p R_m\end{aligned}$$

$$\begin{aligned}\text{escogemos} \quad Z_m - \theta_1^* &= Z_p \\ R_p - k_p (\theta_2^* + \theta_0^* Z_m) &= R_m \\ \theta^* &= \begin{bmatrix} k^* & \theta_1^{*T} & \theta_0^* & \theta_2^{*T} \end{bmatrix}^T \in \Re^{2n}\end{aligned}$$

$$\begin{aligned}\dot{x}_p &= A_p x_p + b_p (\theta^T(t) \omega(t)) \\ \dot{\omega}_1 &= \Lambda \omega_1 + \ell (\theta^T(t) \omega(t)) \\ \dot{\omega}_2 &= \Lambda \omega_2 + \ell (h_p^T x_p)\end{aligned}$$

$$\begin{aligned}\phi(t) &= \theta(t) - \theta^* = \begin{bmatrix} k - k^* (\theta_1 - \theta_1^*)^T & \theta_0 - \theta_0^* (\theta_2 - \theta_2^*)^T \end{bmatrix}^T \\ &= \begin{bmatrix} \psi(t) & \phi_1^T(t) & \phi_0(t) & \phi_2^T(t) \end{bmatrix}^T\end{aligned}$$

$$\dot{x} = A_{mn}x + b_{mn} \left[\phi^T \omega + k^* r \right]$$

$$y_p = h_{mn}^T x$$

$$A_{mn} = \begin{bmatrix} A_p + b_p \theta_0^{*T} h_p^T & b_p \theta_1^{*T} & b_p \theta_2^{*T} \\ \ell \theta_0^{*T} h_p^T & \Lambda + \ell \theta_1^{*T} & \ell \theta_2^{*T} \\ \ell h & 0 & \Lambda \end{bmatrix}, \quad b_{mn} = \begin{bmatrix} b_p \\ \ell \\ 0 \end{bmatrix}$$

$$h_{mn} = \begin{bmatrix} h_p^T & 0 & 0 \end{bmatrix}^T \quad x = \begin{bmatrix} x_p^T & \omega_1^T & \omega_2^T \end{bmatrix}^T$$

$$\dot{x}_{mn} = A_{mn}x_{mn} + b_{mn}k^*r$$

$$y_m = h_{mn}^T x_{mn}$$

$$x_{mn} = \begin{bmatrix} x_p^{*T} & \omega_1^{*T} & \omega_2^{*T} \end{bmatrix}$$

$$h_{mn}^T [sI - A_{mn}]^{-1} b_{mn} = \frac{k_p}{k_m} W_m(s)$$

$$\dot{e}'(t) = A_{mn}e'(t) + b_{mn} \left[\phi^T \omega \right]$$

$$e(t) = h_{mn}^T e'(t)$$

$$e'(t) = x(t) - x_{mn}(t)$$

$$e(t) = y_p(t) - y_m(t)$$

Leyes de ajuste:

$$\dot{\psi}(t) = \dot{k}(t) = -\operatorname{sgn}(k_p)e(t)r(t)$$

$$\dot{\phi}_0(t) = \dot{\theta}_0(t) = -\operatorname{sgn}(k_p)e(t)y_p(t)$$

$$\dot{\phi}_1(t) = \dot{\theta}_1(t) = -\operatorname{sgn}(k_p)e(t)\omega_1(t)$$

$$\dot{\phi}_2(t) = \dot{\theta}_2(t) = -\operatorname{sgn}(k_p)e(t)\omega_2(t)$$

$$\dot{\phi}(t) = \dot{\theta}(t) = -\operatorname{sgn}(k_p)e(t)\omega(t)$$

$$\lim_{t \rightarrow \infty} |y_p(t) - y_m(t)| = 0$$

Estabilidad Lema (SPR)

Consecuencia directa Lemma:

$$\begin{aligned} e'(t), \phi(t) &\in L^\infty, \quad e' \in L^2 \\ e'(t) &\in L^\infty \Rightarrow x_p, \omega_1, \omega_2 \in L^\infty \\ \therefore \dot{e}'(t) &\in L^\infty \quad \therefore \lim_{t \rightarrow \infty} e'(t) \rightarrow 0 \end{aligned}$$

$$\therefore \lim_{t \rightarrow \infty} e(t) = 0$$

$$\begin{aligned} W_m(s) &\text{no es e.r.p.} \quad \downarrow n^* = 0, \text{ polos} \in \mathbb{C}^- \\ W_m(s) &= W(s) \bar{W}(s) \quad \underbrace{\text{e.r.p.}}_{\text{e.r.p.}} \end{aligned}$$

$$r'(t) = W(s)r(t) \quad \text{en lugar de } r(t)$$

Lema SPR

Sea

$$\dot{x} = Ax + b [\phi^T(t) \omega(t)]$$

$$y = h^T x$$

$$z_1 = k y(t)$$

(A,b) estabilizable

(controlable)

(h^T, A) detectable

(observable)

$$\text{con } h^T(sI - A)^{-1}b = H(s)$$

Entonces el punto de equilibrio ($x = 0, \phi = 0$) es uniformemente estable además de global

Demostración

$$\begin{aligned} H(s) &\text{ e.r.p. (KY Lema)} \quad \exists P \quad t.q. \\ A^T P + PA &= -Q \\ Pb &= h \end{aligned}$$

Sea

$$\begin{aligned} V &= x^T P x + \frac{1}{|k|} \phi^T \phi = \\ \dot{V} &= \dot{x}^T P x + x^T P \dot{x} + \frac{2}{|k|} \phi^T \dot{\phi} = \\ &= x^T (A^T P + PA)x + \phi^T \omega b^T P x + x^T P b \phi^T \omega - \frac{2}{k} \phi^T z_1 \omega \\ \dot{v} &= -x^T Q x \leq 0 \end{aligned}$$

Resumen

$$\begin{aligned} \dot{x}_p(t) &= -a_p x_p(t) + k_p u(t) && \text{Planta} \\ \dot{x}_m(t) &= -a_m x_m(t) + b_m r(t) && \text{Modelo} \\ &&& \text{de Referencia} \\ \dot{\hat{x}}_p(t) &= -a_i e_i(t) - \hat{a}_p(t) x_p(t) + \hat{k}_p(t) u(t) && \text{Modelo de} \\ &&& \text{Identificación} \\ u(t) &= \theta(t) x_p(t) + k(t) r(t) && \text{Ley de Control} \\ \dot{e}_c(t) &= -a_m e_c(t) + k_p \phi_\theta(t) x_p(t) + k_p \phi_k(t) r(t) && \text{Error de Control} \\ \dot{e}_i(t) &= -a_i e_i(t) - \phi_a(t) x_p(t) + \phi_{kp}(t) u(t) && \text{Error de Identification} \end{aligned}$$

Leyes de Ajuste

$$\begin{aligned}\dot{\phi}_a(t) &= \dot{\hat{a}}_p(t) = e_i(t)x_p(t) + \varepsilon_\theta(t) \\ \dot{\phi}_{kp}(t) &= \dot{\hat{k}}_p(t) = -e_i(t)u(t) - \theta(t)\varepsilon_\theta(t) - k(t)\varepsilon_k(t) \\ \dot{\phi}_\theta(t) &= \dot{\theta}(t) = -\text{sgn}(k_p)[e_c(t)x_p(t) + \varepsilon_\theta(t)] \\ \dot{\phi}_k(t) &= \dot{k}(t) = -\text{sgn}(k_p)[e_c(t)r(t) + \varepsilon_k(t)]\end{aligned}$$

Función de Lyapunov

$$\begin{aligned}V &= \frac{1}{2} \left(e_c^2 + |k_p| \phi_\theta^2 + |k_p| \phi_k^2 + e_i^2 + \phi_a^2 + \phi_{kp}^2 \right) \\ \dot{V} &= -a_m e_c^2 - a_i e_i^2 - \varepsilon_\theta^2 - \varepsilon_k^2\end{aligned}$$

A lo largo de cualquier trayectoria del sistema adaptivo.

Concluimos que todas las señales son acotadas.

$$\begin{aligned}e_c(t), e_i(t), \varepsilon_\theta(t) \text{ y } \varepsilon_k(t) &\in L^2 \\ \dot{e}_c(t), \dot{e}_i(t), \dot{\varepsilon}_\theta(t) \text{ y } \dot{\varepsilon}_k(t) &\in L^\infty \\ \lim_{t \rightarrow \infty} e_c(t), e_i(t), \varepsilon_\theta(t) \text{ y } \varepsilon_k(t) &= 0\end{aligned}$$

Ejemplo 1

$$W_p(s) = \frac{k_p}{(s + a_p)}, \quad W_m(s) = \frac{k_m}{(s + a_m)}$$

$$\begin{aligned}k_p &= 2, \quad a_p = -1 \\ k_m &= 1, \quad a_m = 1 \\ r(t) &\equiv 5 \text{ y } r(t) = 3 + 3\sin(t) \\ y_p(0) &= 0.5\end{aligned}$$

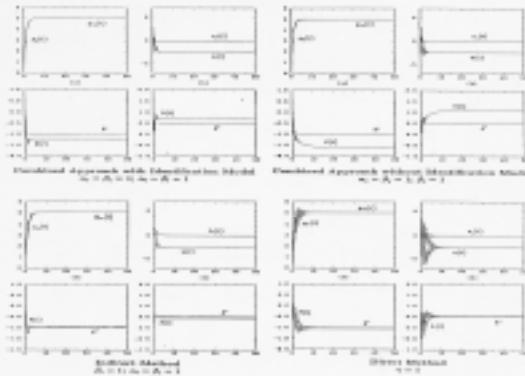


Figure 8.0: MRAC of a First Order Plant for $r(t) = 0$ and unity adaptive gains.

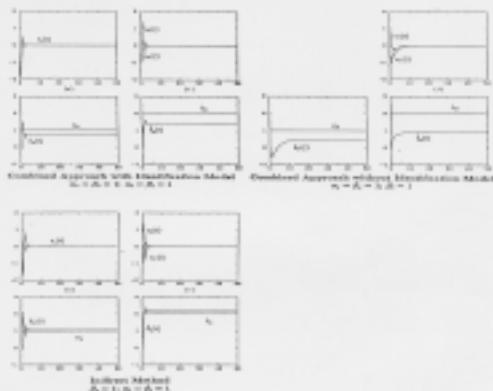


Figure 8.0: MRAC of a First Order Plant for $r(t) = 0$ and unity adaptive gains.
[Contd.]

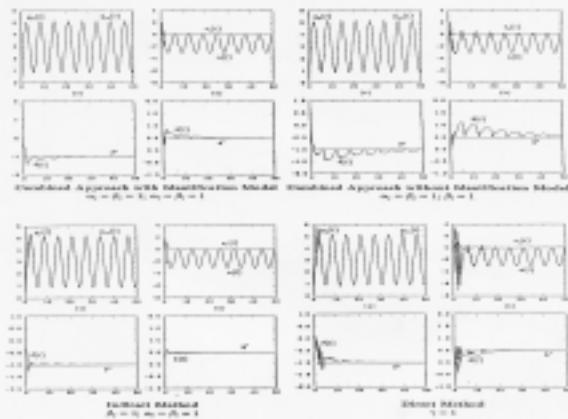


Figure 2.4: MIRAC of a First Order Plant for $r(t) = 3 + \sin(t)$ and unity adaptive gains.

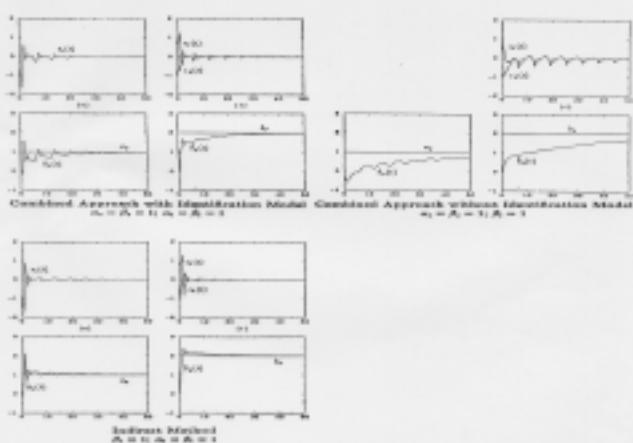


Figure 2.4: MIRAC of a First Order Plant for $r(t) = 3 + \sin(t)$ and unity adaptive gains. (Cont.)