EM725

Control Adaptivo de Sistemas

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8.0 Observadores Adaptivos

Introducción

- Teoría de Observadores adaptivos fines 60's primeros resultados principios 70's.

Caroll & Lindorff, U. de Connecticut

Lüders & Narendra, U. de Yale.

- Estimar el estado y los parámetros (A, B, C, D) del sistema a partir del conocimiento de la entrada u(t) y salida y(t).
- Teoría "clásica" y realizaciones mínimas y no mínimas de observadores adaptivos.
- Observadores adaptivos "mejorados" 4 tipos.
- Aplicación al control adaptivo.

Planteamiento del problema

$$Planta\begin{cases}\dot{x}_p(t) = Ax_p(t) + bu(t), \quad x_p(t_0) = x_{p0}\\ y_p(t) = c^T x(t)\end{cases}$$

A partir del conocimiento de

$$(u(t), y(t)) \quad \forall t \in [t_0, \infty)$$

Estimar simultáneamente A, b, c y x_p (t)

(c^T, A, b): n^2+2n parámetros

2n parámetros son suficientes

Realización mínima

$$\dot{x}_p(t) = \left[a \,\vdots\, \bar{A}\right] x_p(t) + bu(t)$$

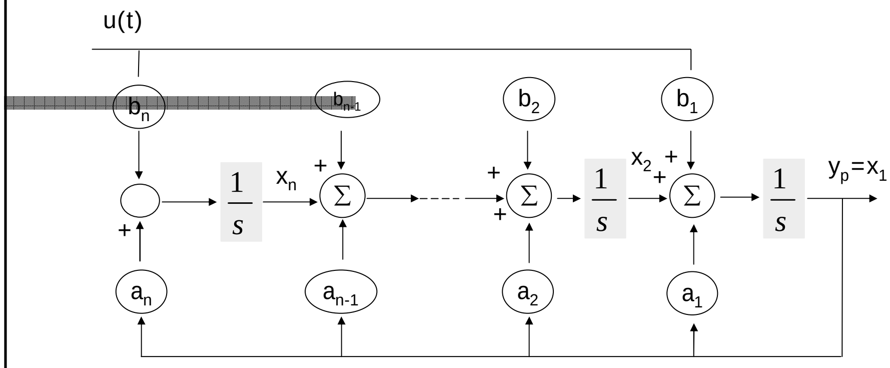
$$y_p(t) = c^T x_p(t) = x_1(t)$$

$$a = \left[a_1, a_2 ... a_n\right]^T, \quad b = \left[b_1, b_2, ... b_n\right]^T$$

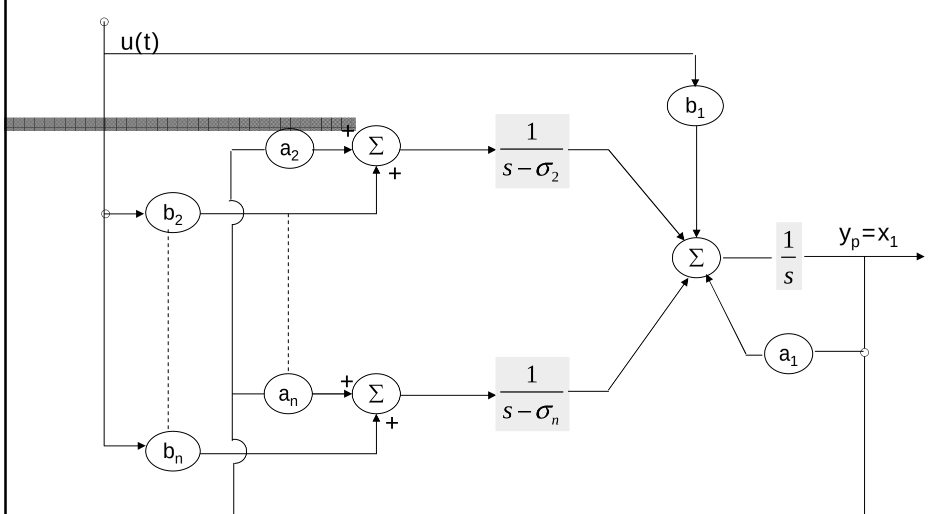
$$c = \left[1, 0 ... 0\right]^T, \quad \bar{A} \in \Re^{n\times(n-1)} \ conocida$$

$$\bar{A} = \begin{bmatrix} I_{n-1} \\ \cdots \\ 0 \end{bmatrix} \quad Caroll \ \& \ Lindorff \ 1973$$

$$\bar{A} = \begin{bmatrix} 1 & \cdots & 1 \\ \sigma_2 & & \\ & \ddots & \\ & & \sigma_n \end{bmatrix} \quad Lüders \ \& \ Narendra \ 1973$$



Caroll & Lindorff



Lüders & Narendra

Reexpresada como:

$$\dot{x}_p(t) = \left[-k \,\vdots\, \overline{A}\right] x_p(t) + g y_p(t) + b u(t)$$

$$y_p(t) = c^T x_p(t) = x_1(t)$$

$$k = \left[k_1 k_2 .....k_n\right]^T \qquad K = \left[-k \,\vdots\, \overline{A}\right] \quad \text{asintóticamente estable}$$

$$g = k + a$$

Parámetros desconocidos $g, b \in \Re^n$

Observador L & N

$$\dot{\hat{x}}_p(t) = K\hat{x}_p(t) + \hat{g}(t) y_p(t) + \hat{b}(t) u(t) + v_1(t) + v_2(t)$$

$$v_1, v_2 \quad \textit{señales auxiliares}$$

$$\hat{b}(t), \hat{g}(t) \ \textit{estimados de } b \ y \ g \ \textit{en ´} t \textit{`}.$$

$$\hat{y}_p(t) = c^T \hat{x}_p(t)$$

Ley de ajuste de parámetros

$$\dot{\hat{g}}(t) = -e_1(t) w^2(t) = -\Gamma_2(t) w^2(t) e_1(t)$$

$$\dot{\hat{b}}(t) = -e_1(t) w^1(t) = -\Gamma_1(t) w^1(t) e_1(t)$$

$$\Gamma_i(t) > 0, \ \text{ganancias adaptivas}$$

$$e_1(t) = \hat{y}_p(t) - y_p(t), \quad w^1(t) = G(s) u(t), \quad w^2(t) = G(s) y_p(t)$$

$$G(s) = \left[G_1(s), G_2(s), ...G_n(s)\right]^T$$

$$A_i = \begin{bmatrix} 0 & -d_i & -d_{i+1} & \cdots & -d_n & 0 & \cdots & 0 \\ 0 & 0 & -d_i & \cdots & -d_{n-1} & -d_n 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & & & & & \\ 0 & \cdots & 0 & -d_i & -d_{i+1} & \cdots & : & d_n \\ 0 & 1 & d_2 & d_3 & \cdots & d_{i-1} & 0\cdots & 0 \\ 0 & 0 & 1 & d_2 & \cdots & \cdots & d_{i-1}\cdots & 0 \\ \vdots & \ddots & \ddots & & & & & \\ 0 & & 0 & 1 & d_2 & d_3 & \cdots & d_{i-1} \end{bmatrix}$$

$$A_i \in \Re^{n\times n}, \quad i=1,2\ldots n$$

$$G_i(s) = \frac{s^{n-i}}{s^n + d_2 s^{n-2} + \cdots + d_n} \quad i=1,2\cdots n$$

$$d = [1, d_2, \ldots d_n]^T \quad t.q. \quad c^T(sI-k)^{-1}d \ \text{ sea } \ \text{e.r.p.}$$

$$v_1^T(t) = \dot{\hat{b}}^T(t)\left[0, A_2 w^1(t), \ldots., A_n w^1(t)\right]$$

$$v_2^T(t) = \dot{\hat{g}}^T(t)\left[0, A_2 w^2(t), \ldots, A_n w^2(t)\right]$$

$A_i, \quad i = 2, \ldots n$ se definen en términos de d

Se puede demostrar:

i) El sistema globalmente uniformemente estable (todas las señales uniformemente acotadas).

$$\lim_{t\to\infty} e_1(t) = 0$$

ii) Si u(t) es tal que $w(t) = \left[w^{1T}(t), w^{2T}(t)\right]^T \in \Re^{2n}$

es de excitación persistente en $\Re^{2n}$ entonces

$$\lim_{t\to\infty} e(t) = \hat{x}_p(t) - x_p(t) = 0$$
$$\lim_{t\to\infty} \hat{g}(t) = g$$
$$\lim_{t\to\infty} \hat{b}(t) = b$$

Realización No mínima

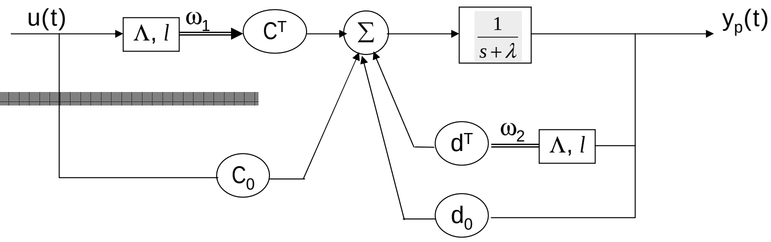
Representación 1

$$\dot{x}_p(t) = A\,x_p(t) + bu(t)$$
$$y_p(t) = h^T x_p(t)$$

$$\dot{x}_1(t) = -\lambda x_1(t) + \theta^T \omega(t)$$
$$\dot{\omega}_1(t) = \Lambda \omega_1(t) + \ell u(t)$$
$$\dot{\omega}_2(t) = \Lambda \omega_2(t) + \ell\, y_p(t)$$
$$y_p(t) = x_1(t)$$

$\theta \in \Re^{2n}$ Vector de parámetros

$$\omega(t) \in \Re^{2n}, \quad \omega(t) = \left[u(t), \omega_1^T(t), y_p(t), \omega_2^T(t)\right]$$
$$(\Lambda, \ell) \quad c.c., \quad \Lambda \in \Re^{(n-1)\times(n-1)}, \quad \lambda > 0, \quad \Lambda\ a.e.$$



$$c = \left[c_1 c_2 \ldots c_{n-1}\right]^T \in \Re^{n-1} \quad d = \left[d_1 d_2 \ldots d_{n-1}\right]^T \in \Re^{n-1}$$

$$\theta = \left[c_0, c^T, d_0, d^T\right]^T \in \Re^{2n}$$

2n-1 integradores para una planta de orden n

Observador serie-paralelo

$$\dot{\hat{\omega}}_1(t) = \Lambda \hat{\omega}_1(t) + \ell u(t)$$

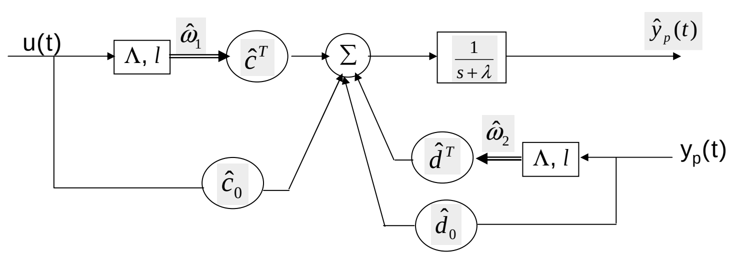
$$\dot{\hat{\omega}}_2(t) = \Lambda \hat{\omega}_2(t) + \ell y_p(t)$$

$$\dot{\hat{x}}_1(t) = -\lambda \hat{x}_1(t) + \hat{\theta}^T(t)\hat{\omega}(t)$$

$$\hat{y}_p(t) = \hat{x}_1(t)$$

$$\hat{\theta}(t) = \left[\hat{c}_0(t), \hat{c}^T(t), \hat{d}_0(t), \hat{d}^T(t)\right]^T \quad \in \Re^{2n}$$

$$\hat{\omega}(t) = \left[u(t), \hat{\omega}_1^T(t), y_p(t), \hat{\omega}_2^T(t)\right]^T \quad \in \Re^{2n}$$



$$e_1(t) = \hat{y}_p(t) - y_p(t)$$

$$\dot{\theta}_1(t) = -\lambda e_1(t) + \phi^T(t)\hat{\omega}(t) + \theta^T\tilde{\omega}(t) \quad \text{(Modelo de Error 2)}$$

$$\phi(t) = \hat{\theta}(t) - \theta$$

$$\tilde{\omega}(t) = \hat{\omega}(t) - \omega(t)$$

$\tilde{\omega}(t)$ decae exponencialmente

Ley de ajuste

$$\dot{\hat{\theta}}(t) = -e_1(t)\hat{\omega}(t)$$

$$V = \frac{1}{2}\left(e_1^2 + \phi^T\phi + \beta\ \tilde{\omega}^T P\ \tilde{\omega}\right)$$

$$i)\quad \dot{V} = -\lambda e_1^2 + e_1\theta^T\tilde{\omega} - \frac{\beta}{2}\tilde{\omega}^T Q\tilde{\omega}$$

$$\overline{\Lambda}^T P + P\overline{\Lambda} = -Q < 0, \quad \overline{\Lambda} = \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix}$$

β Se escoge mayor que $\dfrac{2\|\theta\|^2}{(\lambda\lambda_Q)}$ $\quad \lambda_Q = \min e.v.\ de\ Q$

$$\dot{V} \ \leq \ 0$$

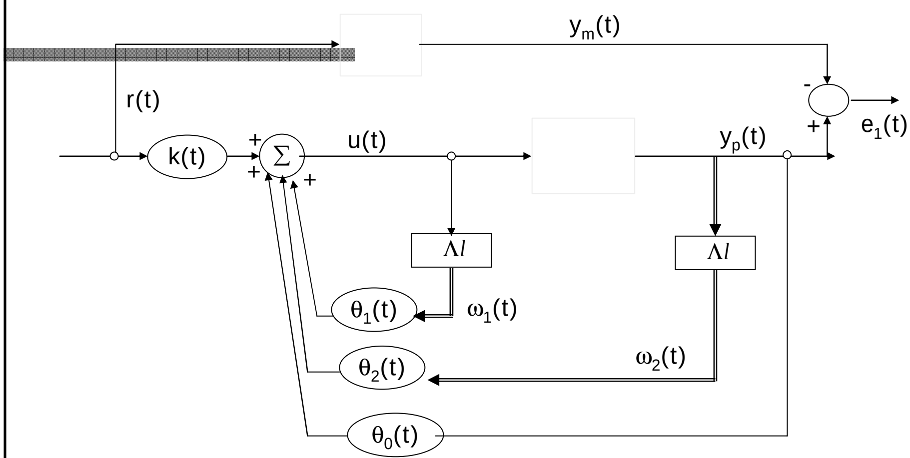
Todas las señales son acotadas

ii) $e_1 \in L^2, \quad \dot{e}_1 \in L^\infty \quad \therefore \quad \lim_{t\to\infty} e_1(t) = 0$

iii) $Si \quad \hat{\omega}(t) \in \Omega_{2n} \quad entonces \quad \lim_{t\to\infty} \hat{\theta}(t) = 0$

$$\lim_{t\to\infty} \hat{x}_p(t) = x_p(t)$$

Aplicaciones de Observadores Adaptivos



$$\dot{\theta}_1(t) = -e_1(t)\omega_1(t) \qquad n^* = 1$$

$$\dot{\theta}_2(t) = -e_1(t)\omega_2(t) \qquad n - m = 1$$

$$\dot{\theta}_0(t) = -e_1(t)y_p(t) \qquad \uparrow \quad \uparrow$$

$$\dot{k}(t) = -e_1(t)r(t) \qquad p \quad z$$

$$\lim_{t\to\infty} e_1(t) = 0$$

Todas las señales acotadas

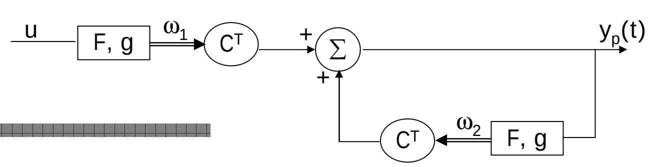
Si r(t) suficientemente rica t.q.

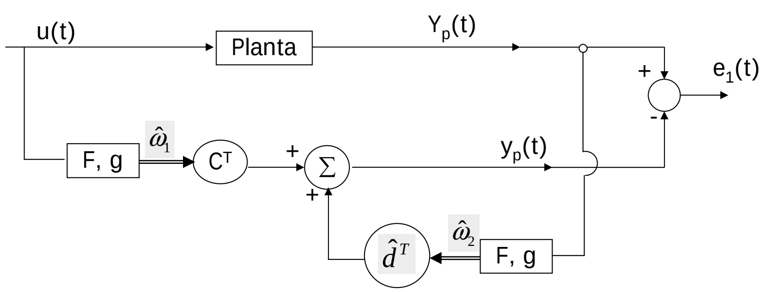
$$w(t) \in \Omega_{2n}, \quad \lim_{t\to\infty} \theta(t) = \theta^*$$

Representación 2 (No Mínima)

$$\dot{x}_p(t) = Ax_p(t) + bu(t)$$
$$y_p(t) = h^T x_p(t)$$

$$\dot{\omega}_1 = F\omega_1 + gu$$
$$\dot{\omega}_2 = F\omega_2 + gy_p$$
$$y_p = \theta^T \omega = c^T \omega_1 + d^T \omega_2 \qquad c, d \in \Re^n$$
$$F \in \Re^{n\times n}, \quad (F, g) \quad c.c.$$





$$\dot{\hat{c}}(t) = -e_1(t)\hat{\omega}_1(t)$$
$$\dot{\hat{d}}(t) = -e_1(t)\hat{\omega}_2(t)$$

$$\dot{\omega}_1(t) = F\,\omega_2(t) + gu(t) \qquad \omega_1 \in \Re^n$$

$$\dot{\omega}_2(t) = F\,\omega_2(t) + gy_p(t) \qquad \omega_2 \in \Re^n$$

$$y_p(t) = \theta^T \omega(t) \qquad \theta \in \Re^{2n}, \omega \in \Re^{2n}$$

$$\dot{\hat{\omega}}_1(t) = F\,\hat{\omega}_1(t) + gu(t) \qquad \hat{\omega}_1 \in \Re^n$$

$$\dot{\hat{\omega}}_2(t) = F\,\hat{\omega}_2(t) + gy_p(t) \qquad \hat{\omega}_2 \in \Re^n$$

$$\hat{y}_p(t) = \hat{\theta}^T(t)\hat{\omega}(t) \qquad \hat{\theta}, \hat{\omega} \in \Re^{2n}$$

$$e_1(t) = \hat{y}_p(t) - y_p(t) = \hat{\theta}^T(t)\hat{w}(t) - \theta^T w(t) \qquad \pm\,\theta^T \hat{w}(t)$$

$$e_1(t) = \left(\hat{\theta}(t) - \theta\right)^T \hat{w}(t) + \theta^T \underbrace{\left(\hat{w}(t) - w(t)\right)}_{\tilde{w}(t)}$$

$\tilde{w} \rightarrow$ exponencialmente

$$\boxed{e_1(t) = \phi^T(t)\hat{\omega}(t)}$$

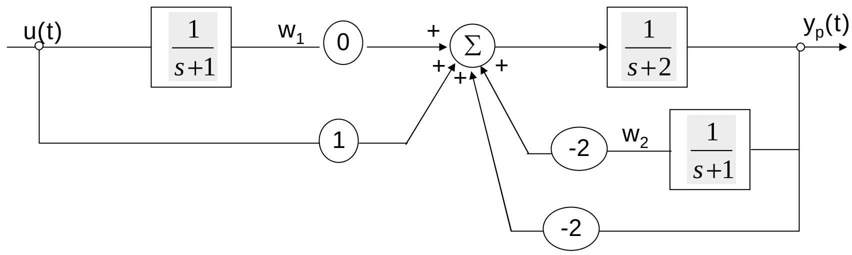
Modelo de Error 1

$$\phi(t) = \hat{\theta}(t) - \theta$$

$$\boxed{\dot{\phi}(t) = \dot{\hat{\theta}}(t) = -e_1(t)\hat{\omega}(t)}$$

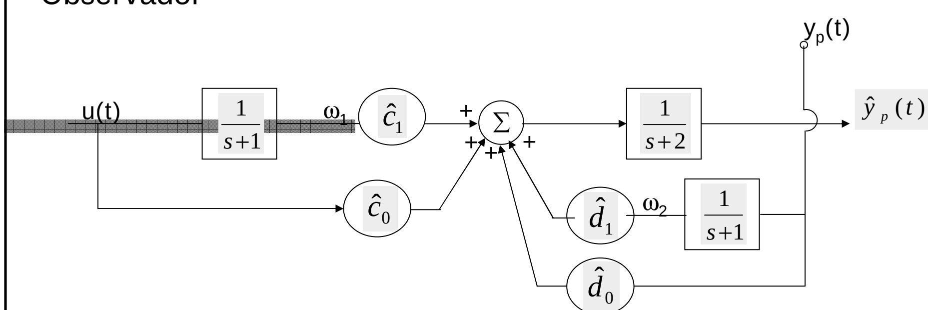
Ejemplo de Diseño

$$W_p(s) = \frac{s+1}{(s+2)(s+3)}$$



$$H(s) = \frac{\frac{1}{s+2}}{1 - \frac{1}{s+2}\left[\frac{-2}{s+1} - 2\right]} = \frac{s+1}{(s+2)(s+3)}$$

Observador



$$\dot{\hat{c}}_1 = -e(t)\omega_1(t)$$

$$\dot{\hat{c}}_0 = -e(t)u(t)$$

$$\dot{\hat{d}}_1 = -e(t)\omega_2(t)$$

$$\hat{d}_0 = -e(t)y_p(t)$$

Observadores Adaptivos Mejorados

a) Ganancias variables en el tiempo:

$$e(t) = \phi^T(t)\omega(t)$$

$$\dot{\phi} = \dot{\hat{\theta}}(t) = -\gamma\Gamma(t)e(t)\omega(t) \quad , \quad \gamma > 0$$

$$\dot{\Gamma}(t) = -\Gamma(t)\omega(t)\omega^T(t)\Gamma(t) \quad , \quad \Gamma(0) = \Gamma_0 > 0$$

$$V = \frac{1}{\gamma}\phi^T\Gamma^{-1}(t)\phi$$

$$\dot{V} = \frac{1}{\gamma}\dot{\phi}^T\Gamma^{-1}(t)\phi + \frac{1}{\gamma}\phi^T\Gamma^{-1}(t)\dot{\phi} - \frac{1}{\gamma}\phi^T\Gamma^{-1}\dot{\Gamma}\Gamma^{-1}\phi$$

$$\dot{V} = -e_1^2(t) \leq 0$$

$$\phi \in L^\infty$$

$$-\ si \quad \omega \in L^\infty \quad , \quad e \in L^\infty$$

$$e \in L^2 \quad , \quad si \quad \dot{\omega} \in L^\infty \quad , \quad \dot{e} \in L^\infty$$

$$\therefore \lim_{t \leftarrow \infty} e(t) = 0$$

$$-\ si \quad \omega \in \Omega_{2n} \Rightarrow \quad \lim_{t \to \infty} \phi(t) = 0$$

$$i.e. \quad \lim_{t \to \infty} \hat{\theta}(t) = 0$$

b) Algoritmo Integral:

$$\dot{\phi}(t) = -\gamma \int_{t_0}^{t} e_1(t,\delta)\omega(\delta)d\delta \qquad J = \int_{t_0}^{t} e_1^2(t,\delta)d\delta$$

$$e_1(t,\tau) = \phi^T(t)\omega(\tau) = \hat{\theta}^T(t)\omega(\tau) - y_p(\tau)$$

$$\dot{\phi}(t) = -\gamma\Gamma\phi(t) \qquad J = \int_{t_0}^{t} e^{-q(t-\tau)} e_1^2(t,\tau)d\tau$$

$$\dot{\Gamma}(t) = -q\Gamma(t) + \omega(t)\omega^T(t), \quad \Gamma(t_0) = 0$$

$$\dot{\phi}(t) = -\gamma\Gamma(t)\hat{\theta}(t) + \gamma\delta(t), \qquad \gamma > 0$$

$$\dot{\Gamma}(t) = -q\Gamma(t) + \omega(t)\omega^T(t), \qquad \Gamma(t_0) = 0$$

$$\dot{\delta}(t) = -q\delta(t) + \omega(t)y_p(t), \qquad \delta(t_0) = 0$$

Todas las señales acotadas

$$\lim_{t\to\infty} \quad e_1(t) \to 0$$

Si $w(t) \in \Omega_{2n}, \quad \theta(t) \to \theta$ con una tasa de convergencia no menor que min {q,H}

$$H = ke^{-qT}$$

$$\int_{t}^{t+T} \omega(\tau)\omega^T(\tau)d\tau \geq kI, \quad \forall t \geq t_0$$

c) Modelos Multiples

$$W_p(s)u(t) = y(t) \rightarrow e_1(t) = \phi^T \omega_1(t)$$

$$W_p(s)\frac{u(t)}{N(s)} = \frac{y(t)}{N(s)}$$

$$W_p(s)u_1(t) = y_1(t) \rightarrow e_2(t) = \phi^T \omega_2(t)$$

$$\vdots$$

$$e_{2n}(t) = \phi^T \omega_{2n}(t)$$

$$e(t) = \Omega(t)\phi(t)$$

$$e(t) = \left[e_1(t), e_2(t).....e_{2n}(t)\right]^T$$

$$\Omega(t) = \left[\omega_1(t) \vdots \omega_2(t) \vdots ...... \vdots \omega_{2n}(t)\right]^T$$

$$\omega_i(t) : \ell.i.$$

$$\dot{\phi}(t) = -\gamma \underbrace{\Omega^T(t)\Omega(t)}_{p.d.} \phi(t)$$

v.p. manejados por γ

$$Si \quad \gamma \rightarrow \infty, \quad \lim_{t\rightarrow\infty} \phi(t) = 0$$

d) Algoritmos Híbridos

$$e(t) = \phi^T(t)\omega(t)$$
$$\phi(t_{k+1}) = \phi(t_k) - \Gamma R(t_k) \quad , \quad \Gamma < 0$$
$$R(t_k) = \frac{1}{T_k}\int_{tk}^{tk+1} \frac{e(\tau)\omega(\tau)}{1+\omega^T(\tau)e(\tau)}\,d\tau$$

$$T_k = t_{k+1} - t_k \qquad \{t_k\} \quad k = 0,1,2...$$
$$sup_i\,[t_{i+1} - t_i] < \infty$$

- Todas las señales acotadas

$$\lim_{t\to\infty} e(t) = 0$$

$$Si \quad \omega(t) \in \Omega_{2n} \Rightarrow \lim_{t\to\infty} \phi(t) = 0$$
$$\lim_{t\to\infty} \hat{\theta}(t) = \theta$$

Robusta frente a perturbaciones externas

$$V(k) = \frac{1}{2}\phi_k^T\phi_k \qquad (\Gamma = I)$$
$$\Delta V(k) = V(k+1) - V(k) = \left[\phi_k^T + \frac{\Delta\phi_k^T}{2}\right]\Delta\phi_k$$
$$\Delta V(k) = -\frac{1}{2}\phi_k^T\,[2I - R_{k1k+1}]\,R_{k,k+1}\phi_k \le 0$$
$$R_{k,k+1} \ge 0$$

Conclusiones

- Principales aspectos de la teoría de observadores y observadores adaptivos.

- Importancia de la elección de la parametrización de la planta y del observador.

- Observadores	$\leftrightarrow$	Realimentación del estado
- Observadores Adaptivos	$\leftrightarrow$	Control adaptivo.
- Observadores:		Análisis de robusteza frente a variaciones en los parámetros de la planta.
- Observadores adaptivos:		Análisis en condiciones de incertidumbre; perturbaciones acotadas externas, parámetros variables en el tiempo y dinámica no modelada.