CLASIFICACIÓN DE LOS PUNTOS DE EQUILIBRIO EN EL PLANO DE ESTADO DE UN SISTEMA DE SEGUNDO ORDEN

Sea el sistema de segundo orden: $\ddot{y} + a\dot{y} + by = o$

Tomando como variables de estado $x_1 = y \qquad x_2 = \dot{y}$ se tendrá:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \dot{x} = Ax \tag{1}$$

Si efectuamos la transformación lineal $x = Tu$ (T matriz de vectores propios) se tendrá:
$\dot{u} = Su \quad con \quad S = \; T^{-1}AT$

i) Valores propios distintos:

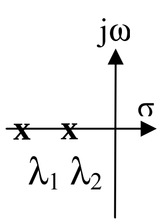
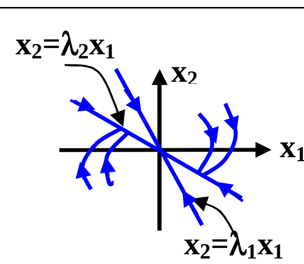
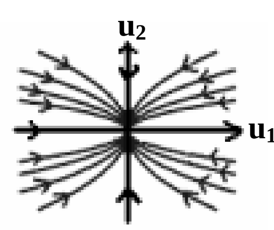
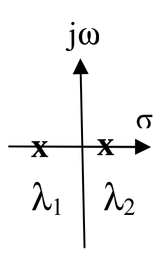
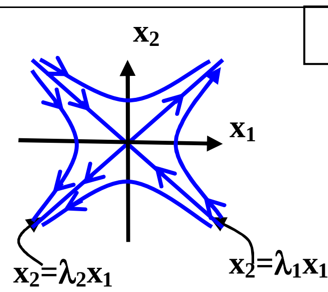
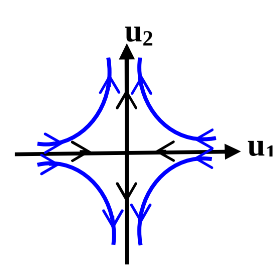
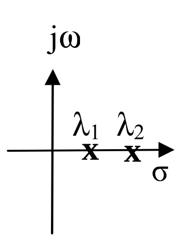
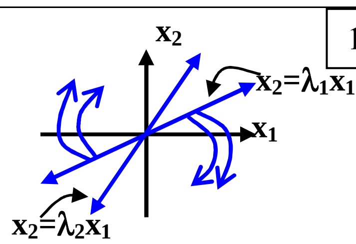
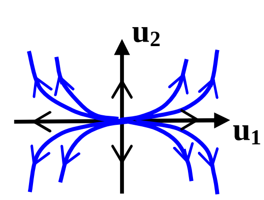
$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{2}$$

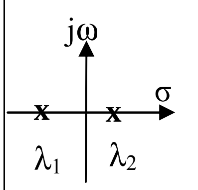
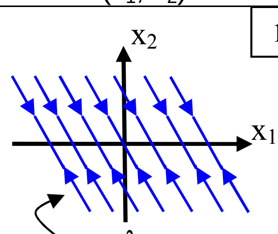
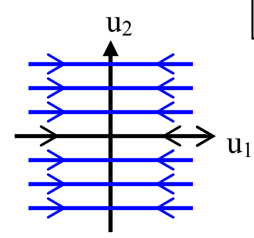
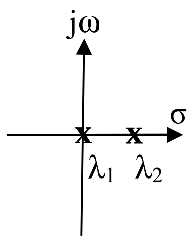
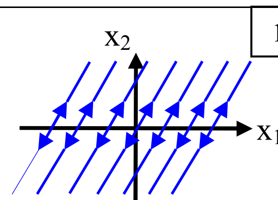
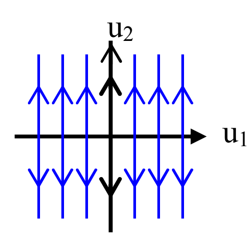
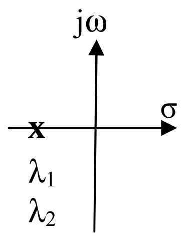
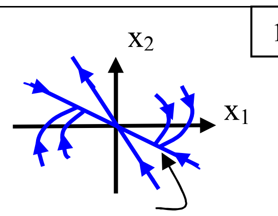
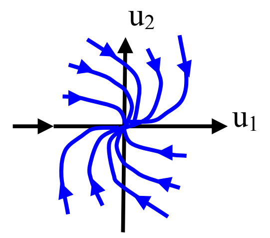
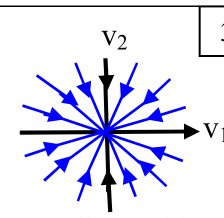
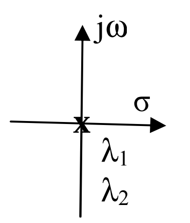
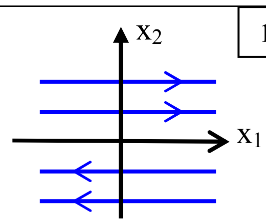
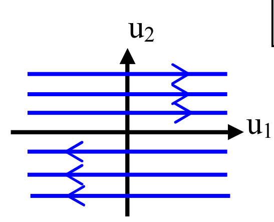
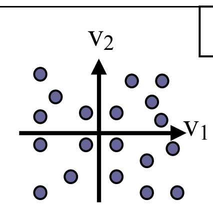
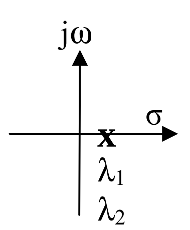
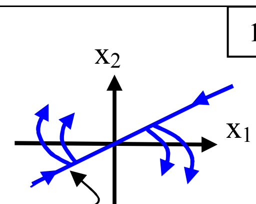
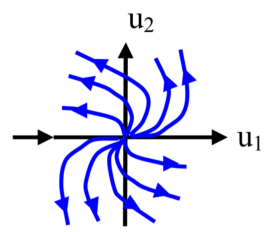
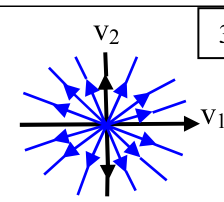
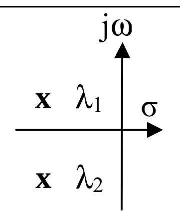
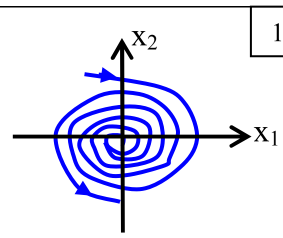
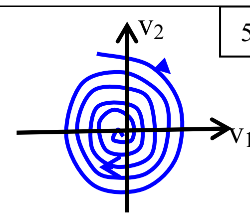
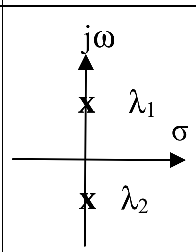
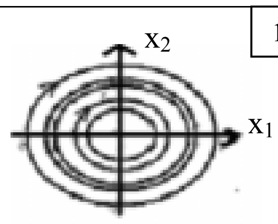
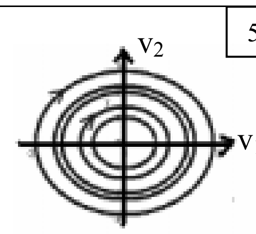
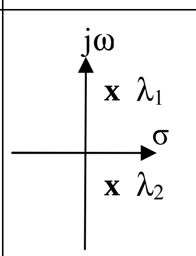
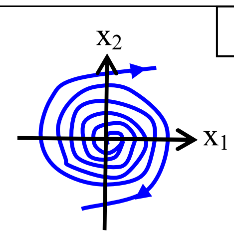
ii) En el caso que los valores propios sean iguales se tendrá alguna de las siguientes situaciones:

$$\begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (3) \qquad ó \qquad \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (3)$$

iii) En caso que los valores propios sean complejos conjugados:
$\lambda_1 = \sigma + j\omega \quad \lambda_2 = \sigma - j\omega$, se realiza una nueva transformación: $u = Rv$ tal que
$\dot{v} = Qv \quad con \quad R^{-1}SR$

$$\text{Se escoge} \quad R = \begin{bmatrix} \sigma - j\omega & -1 \\ \sigma + j\omega & -1 \end{bmatrix} \Rightarrow \quad Q = \begin{bmatrix} 0 & 1 \\ -\left(\sigma^2 + \omega^2\right) & 2\sigma \end{bmatrix} \qquad (Caso \quad \sigma \neq 0) \tag{4}$$

Punto de Equilibrio	Valores Propios	Plano de Fase (x_1, x_2)	Plano de Estado (u_1, u_2)	Plano de Estado (v_1, v_2)
Nodo Estable a > 0 a^2> 4b >0		1 	2 	———
Silla o Cuello a > 0 b < 0		1 	2 	———
Nodo Inestable a <0 a^2 > 4b >0		1 	2 	———

Punto de Equilibrio	Valores Propios	Plano de Fase (x_1, x_2)	Plano de Estado (u_1, u_2)	Plano de Estado (v_1, v_2)
b=0 a > 0		1  $x_2=\lambda_1 x_1+c$	2 	———
b = 0 a < 0		1  $x_2=\lambda_1\ x_1+c$	2 	———
α > 0 $a^2 = 4b$		1  $x_2=\lambda\ x_1$	4 	3  Estrella Estable
a = 0 b = 0		1 	4 	3 
a < 0 $a^2 = 4$ b		1  $x_2 = \lambda x_1$	4 	3  Estrella Inestable
Foco Estable a > 0 $0< a^2<4b$		1 	———	5 
Centro o Cima a = 0 b > 0		1 	———	5 
Foco Inestable a < 0 $0 < a^2< 4b$		1 	———	5