

Viscously Damped Free Vibration

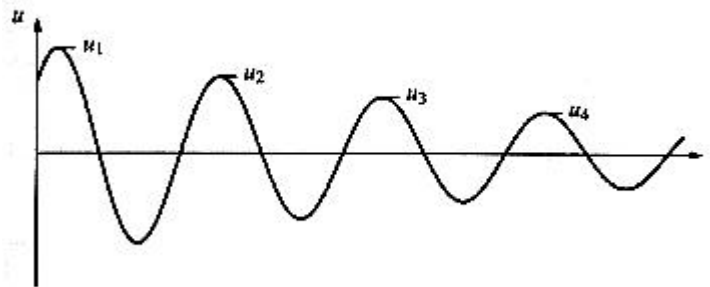


Figure 2.2.5

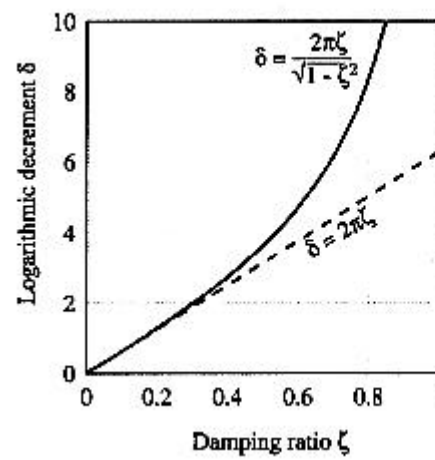


Figure 2.2.6 Exact and approximate relations between logarithmic decrement and damping ratio.

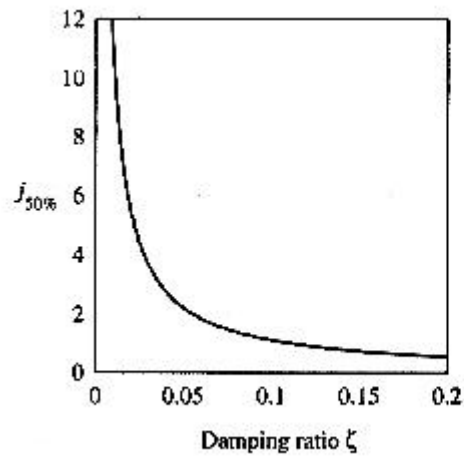


Figure 2.2.7 Number of cycles required to reduce the free vibration amplitude by 50%.

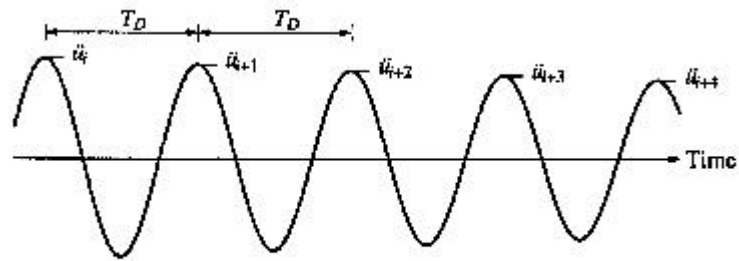


Figure 2.2.8 Acceleration record of a freely vibrating system.

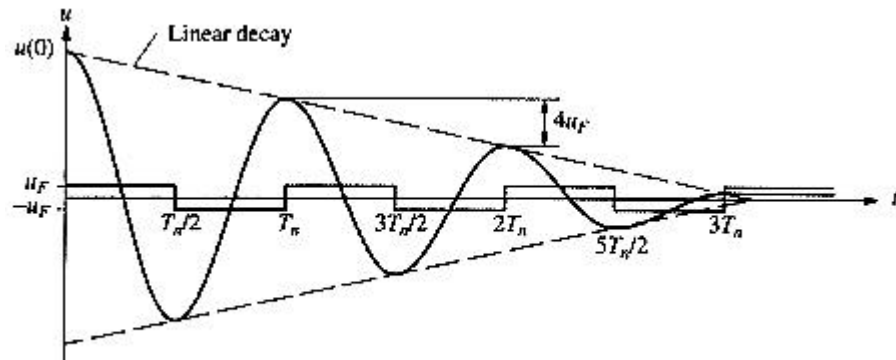


Figure 2.4.2 Free vibration of a system with Coulomb friction.

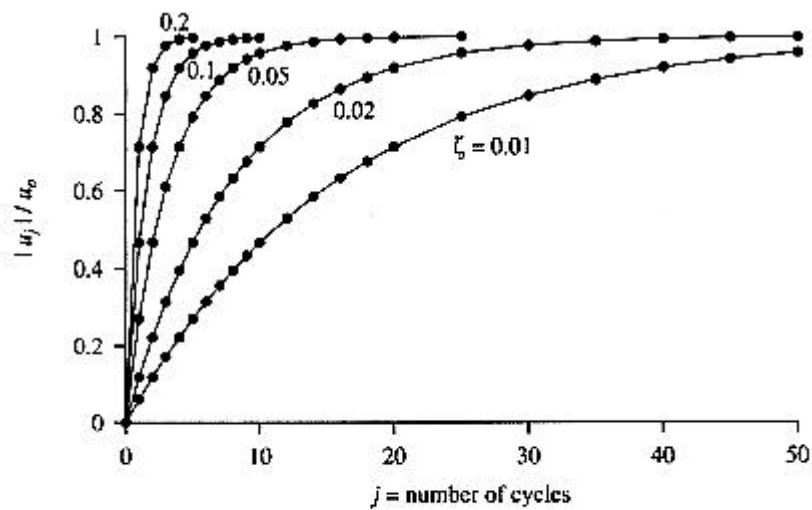


Figure 3.2.4 Variation of response amplitude with number of cycles of harmonic force with frequency $\omega = \omega_n$.

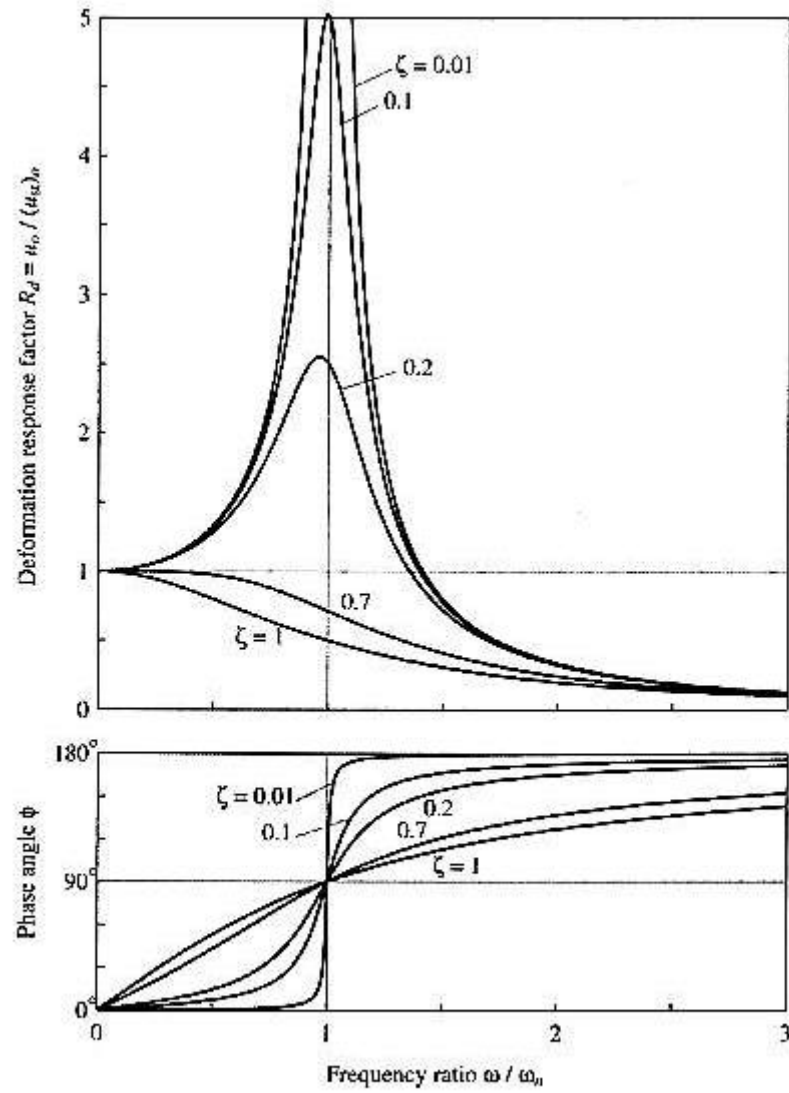


Figure 3.2.6 Deformation response factor and phase angle for a damped system excited by harmonic force.

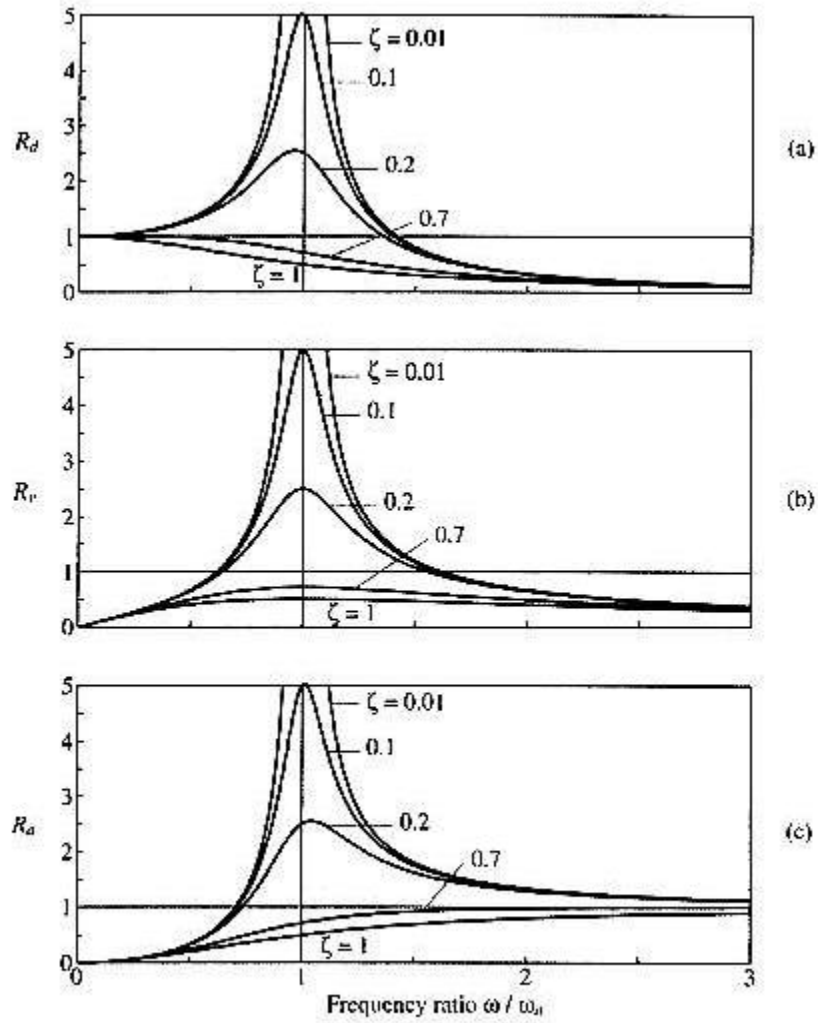


Figure 3.2.7 Deformation, velocity, and acceleration response factors for a damped system excited by harmonic force.

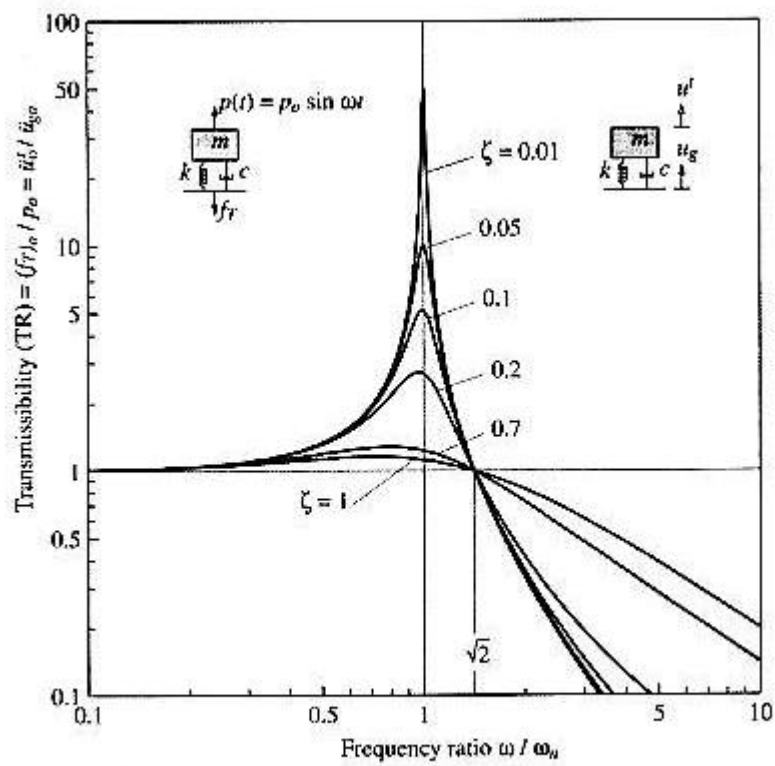


Figure 3.5.1 Transmissibility for harmonic excitation. Force transmissibility and ground motion transmissibility are identical.

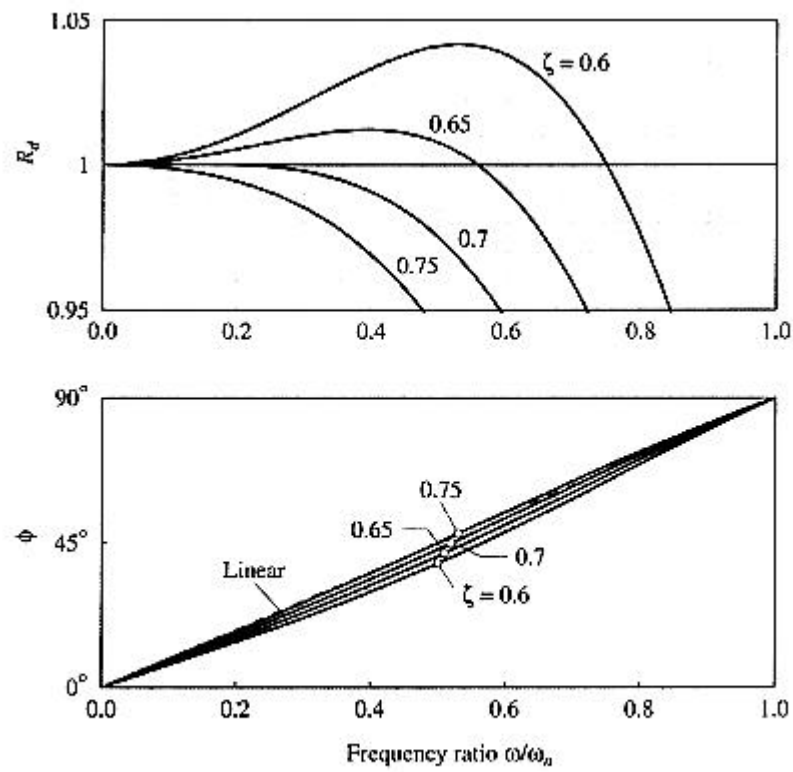


Figure 3.7.2 Variation of R_d and ϕ with frequency ratio ω/ω_n for $\zeta = 0.6, 0.65, 0.7$, and 0.75 .

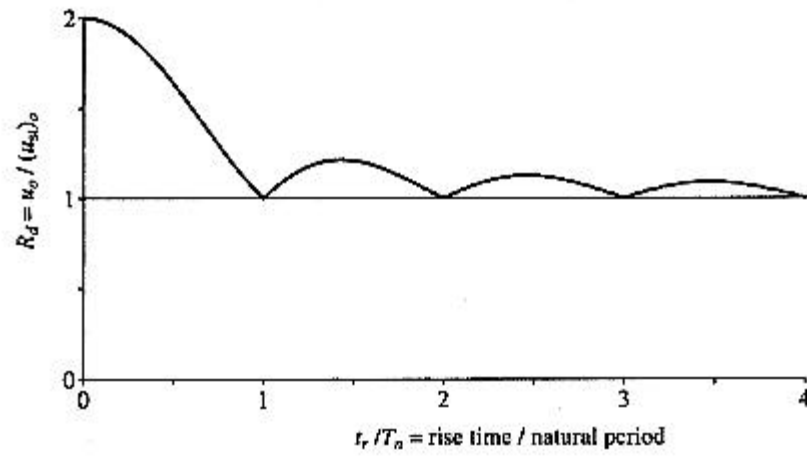


Figure 4.5.3 Response spectrum for step force with finite rise time.

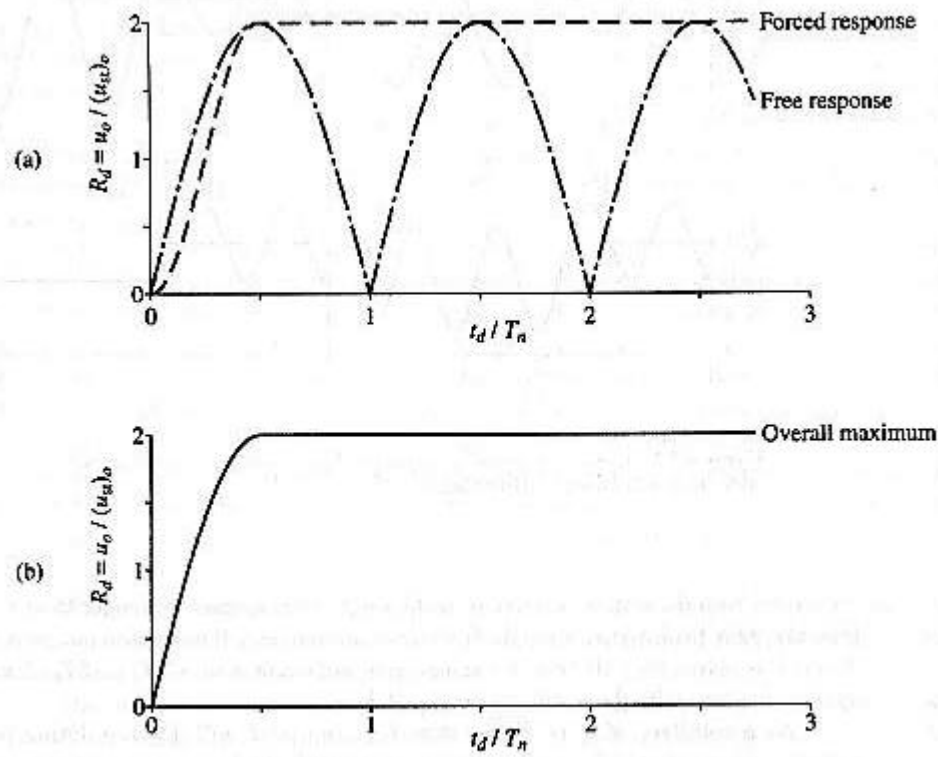


Figure 4.7.3 Response to rectangular pulse force: (a) maximum response during each of forced vibration and free vibration phases; (b) shock spectrum.

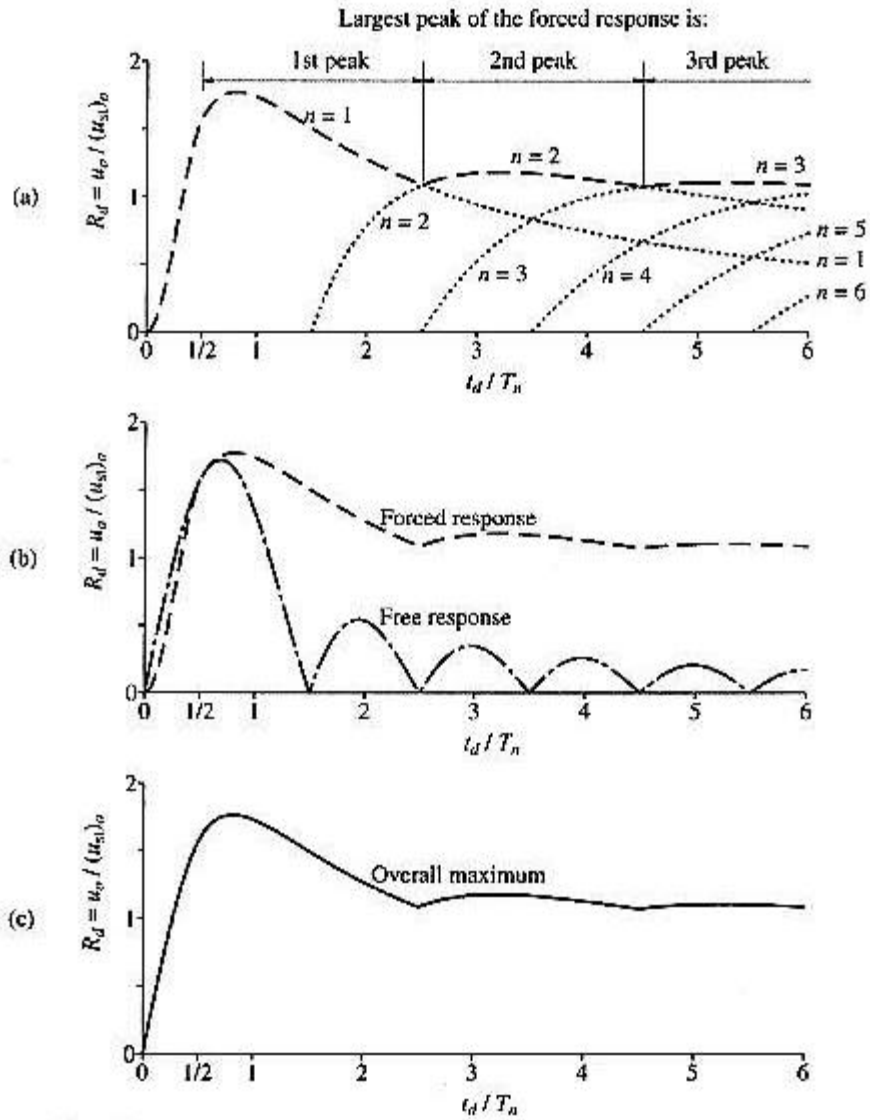


Figure 4.8.3 Response to half-cycle sine pulse force: (a) response maxima during forced vibration phase; (b) maximum responses during each of forced vibration and free vibration phases; (c) shock spectrum.

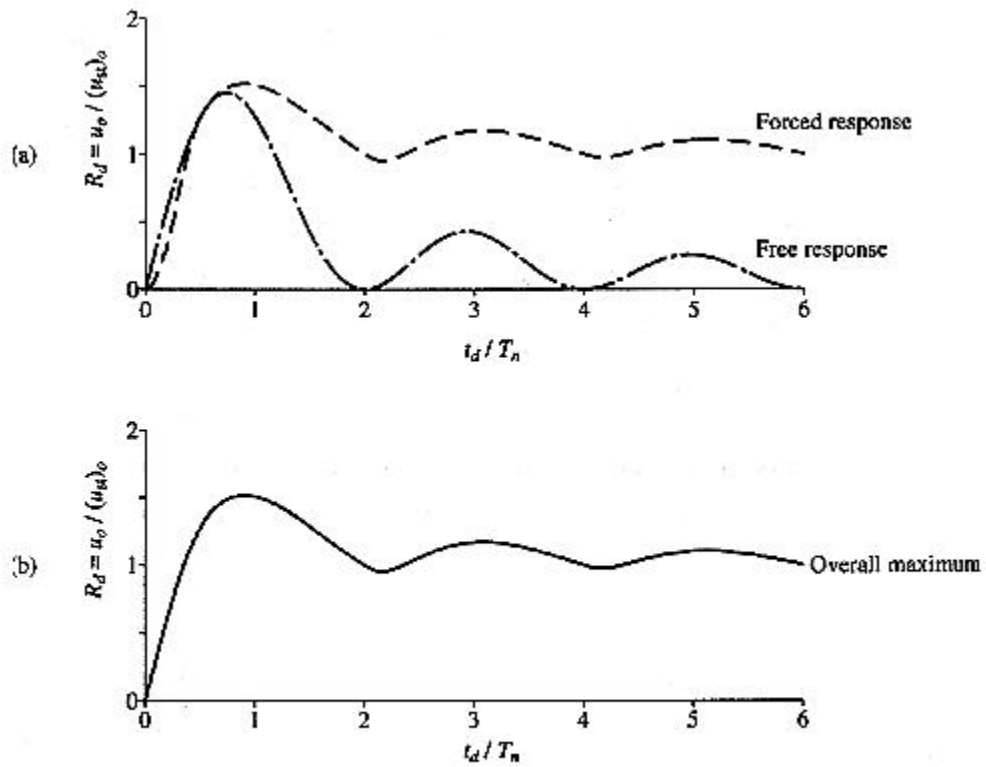


Figure 4.9.3 Response to triangular pulse force: (a) maximum response during each of forced vibration and free vibration phases; (b) shock spectrum.

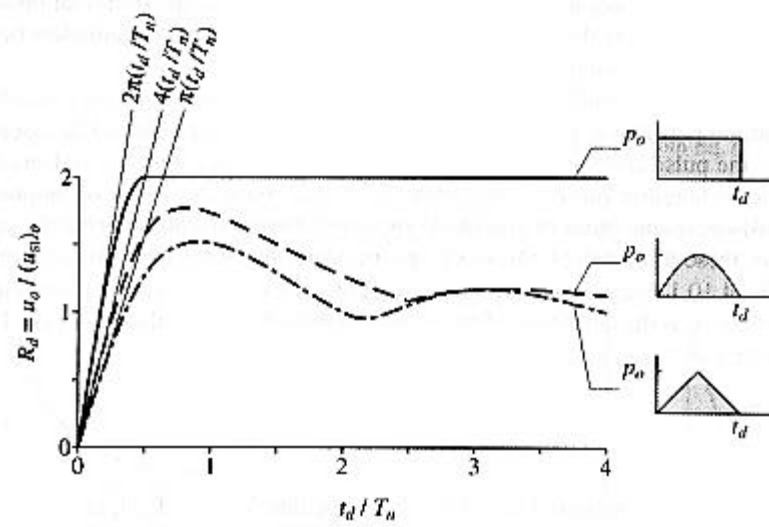


Figure 4.10.1 Shock spectra for three force pulses of equal amplitude.

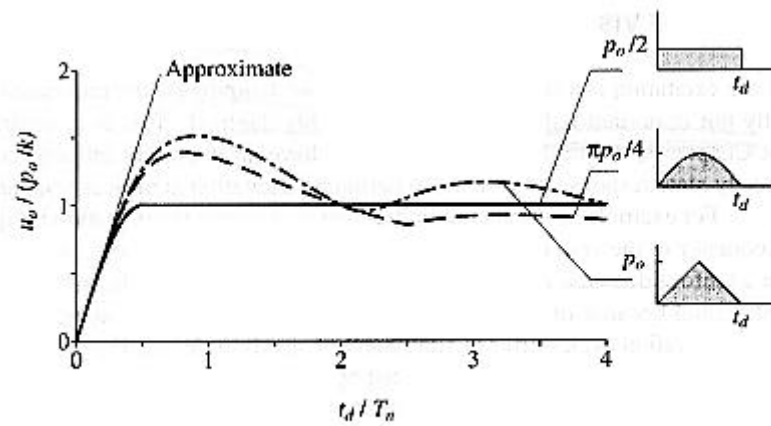


Figure 4.10.2 Shock spectra for three force pulses of equal area.

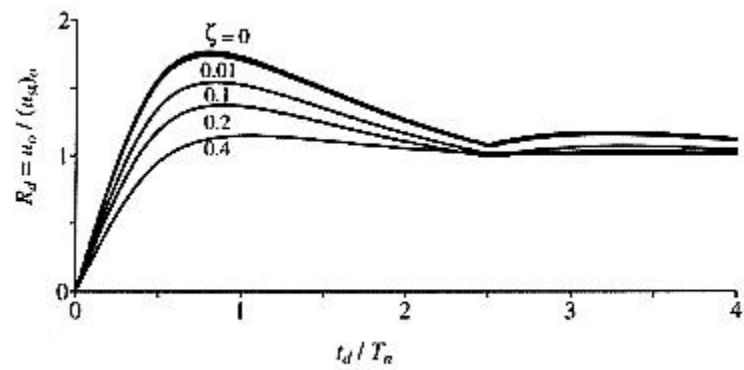


Figure 4.11.2 Shock spectra for a half-cycle sine pulse force for five damping values.

$$\Delta p_i = p_{i+1} - p_i \quad (5.2.1b)$$

and the time variable τ varies from 0 to Δt_i . For algebraic simplicity, we first consider systems without damping; later, the procedure will be extended to include damping. The equation to be solved is

$$m\ddot{u} + ku = p_i + \frac{\Delta p_i}{\Delta t_i} \tau \quad (5.2.2)$$

The response $u(\tau)$ over the time interval $0 \leq \tau \leq \Delta t_i$ is the sum of three parts: (1) free vibration due to initial displacement u_i and velocity \dot{u}_i at $\tau = 0$, (2) response to step force p_i with zero initial conditions, and (3) response to ramp force $(\Delta p_i/\Delta t_i)\tau$ with zero initial conditions. Adapting the available solutions for these three cases from Sections 2.1, 4.3, and 4.4, respectively, gives

$$u(\tau) = u_i \cos \omega_n \tau + \frac{\dot{u}_i}{\omega_n} \sin \omega_n \tau + \frac{p_i}{k} (1 - \cos \omega_n \tau) + \frac{\Delta p_i}{k} \left(\frac{\tau}{\Delta t_i} - \frac{\sin \omega_n \tau}{\omega_n \Delta t_i} \right) \quad (5.2.3a)$$

and

$$\frac{\dot{u}(\tau)}{\omega_n} = -u_i \sin \omega_n \tau + \frac{\dot{u}_i}{\omega_n} \cos \omega_n \tau + \frac{p_i}{k} \sin \omega_n \tau + \frac{\Delta p_i}{k} \frac{1}{\omega_n \Delta t_i} (1 - \cos \omega_n \tau) \quad (5.2.3b)$$

Evaluating these equations at $\tau = \Delta t_i$ gives the displacement u_{i+1} and velocity \dot{u}_{i+1} at time $i + 1$:

$$u_{i+1} = u_i \cos(\omega_n \Delta t_i) + \frac{\dot{u}_i}{\omega_n} \sin(\omega_n \Delta t_i) + \frac{p_i}{k} [1 - \cos(\omega_n \Delta t_i)] + \frac{\Delta p_i}{k} \frac{1}{\omega_n \Delta t_i} [\omega_n \Delta t_i - \sin(\omega_n \Delta t_i)] \quad (5.2.4a)$$

$$\frac{\dot{u}_{i+1}}{\omega_n} = -u_i \sin(\omega_n \Delta t_i) + \frac{\dot{u}_i}{\omega_n} \cos(\omega_n \Delta t_i) + \frac{p_i}{k} \sin(\omega_n \Delta t_i) + \frac{\Delta p_i}{k} \frac{1}{\omega_n \Delta t_i} [1 - \cos(\omega_n \Delta t_i)] \quad (5.2.4b)$$

These equations can be rewritten after substituting Eq. (5.2.1b) as recurrence formulas:

$$u_{i+1} = Au_i + B\dot{u}_i + Cp_i + Dp_{i+1} \quad (5.2.5a)$$

$$\dot{u}_{i+1} = A'\dot{u}_i + B'\dot{u}_i + C'p_i + D'p_{i+1} \quad (5.2.5b)$$

Repeating the derivation above for under-critically damped systems (i.e., $\zeta < 1$) shows that Eqs. (5.2.5) also apply to damped systems with the expressions for the coefficients A, B, \dots, D' given in Table 5.2.1. They depend on the system parameters ω_n, k , and ζ , and on the time interval $\Delta t \equiv \Delta t_i$.

Since the recurrence formulas are derived from exact solution of the equation of motion, the only restriction on the size of the time step Δt is that it permit a close approximation to the excitation function and that it provide response results at closely spaced time intervals so that the response peaks are not missed. This numerical procedure is especially

TABLE 5.2.1 COEFFICIENTS IN RECURRENCE FORMULAS ($\zeta < 1$)

$A = e^{-\zeta \omega_n \Delta t} \left(\frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_D \Delta t + \cos \omega_D \Delta t \right)$
$B = e^{-\zeta \omega_n \Delta t} \left(\frac{1}{\omega_D} \sin \omega_D \Delta t \right)$
$C = \frac{1}{k} \left\{ \frac{2\zeta}{\omega_n \Delta t} + e^{-\zeta \omega_n \Delta t} \left[\left(\frac{1-2\zeta^2}{\omega_D \Delta t} - \frac{\zeta}{\sqrt{1-\zeta^2}} \right) \sin \omega_D \Delta t - \left(1 + \frac{2\zeta}{\omega_n \Delta t} \right) \cos \omega_D \Delta t \right] \right\}$
$D = \frac{1}{k} \left[1 - \frac{2\zeta}{\omega_n \Delta t} + e^{-\zeta \omega_n \Delta t} \left(\frac{2\zeta^2-1}{\omega_D \Delta t} \sin \omega_D \Delta t + \frac{2\zeta}{\omega_n \Delta t} \cos \omega_D \Delta t \right) \right]$
$A' = -e^{-\zeta \omega_n \Delta t} \left(\frac{\omega_n}{\sqrt{1-\zeta^2}} \sin \omega_D \Delta t \right)$
$B' = e^{-\zeta \omega_n \Delta t} \left(\cos \omega_D \Delta t - \frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_D \Delta t \right)$
$C' = \frac{1}{k} \left\{ -\frac{1}{\Delta t} + e^{-\zeta \omega_n \Delta t} \left[\left(\frac{\omega_n}{\sqrt{1-\zeta^2}} + \frac{\zeta}{\Delta t \sqrt{1-\zeta^2}} \right) \sin \omega_D \Delta t + \frac{1}{\Delta t} \cos \omega_D \Delta t \right] \right\}$
$D' = \frac{1}{k \Delta t} \left[1 - e^{-\zeta \omega_n \Delta t} \left(\frac{\zeta}{\sqrt{1-\zeta^2}} \sin \omega_D \Delta t + \cos \omega_D \Delta t \right) \right]$

TABLE 5.4.1 AVERAGE ACCELERATION AND LINEAR ACCELERATION METHODS

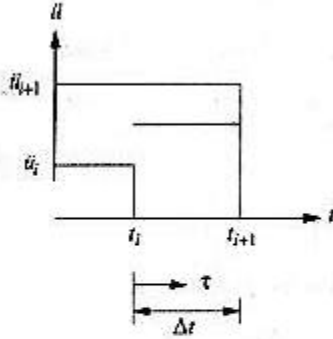
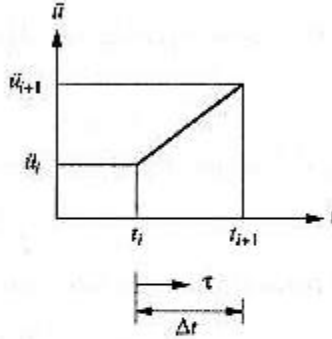
Average Acceleration	Linear Acceleration
	
$\bar{a}(\tau) = \frac{1}{2}(\ddot{u}_{i+1} + \ddot{u}_i)$	$\bar{a}(\tau) = \ddot{u}_i + \frac{\tau}{\Delta t}(\ddot{u}_{i+1} - \ddot{u}_i) \quad (5.4.2)$
$\dot{u}(\tau) = \dot{u}_i + \frac{\tau}{2}(\ddot{u}_{i+1} + \ddot{u}_i)$	$\dot{u}(\tau) = \dot{u}_i + \ddot{u}_i \tau + \frac{\tau^2}{2\Delta t}(\ddot{u}_{i+1} - \ddot{u}_i) \quad (5.4.3)$
$\dot{u}_{i+1} = \dot{u}_i + \frac{\Delta t}{2}(\ddot{u}_{i+1} + \ddot{u}_i)$	$\dot{u}_{i+1} = \dot{u}_i + \frac{\Delta t}{2}(\ddot{u}_{i+1} + \ddot{u}_i) \quad (5.4.4)$
$u(\tau) = u_i + \dot{u}_i \tau + \frac{\tau^2}{4}(\ddot{u}_{i+1} + \ddot{u}_i)$	$u(\tau) = u_i + \dot{u}_i \tau + \ddot{u}_i \frac{\tau^2}{2} + \frac{\tau^3}{6\Delta t}(\ddot{u}_{i+1} - \ddot{u}_i) \quad (5.4.5)$
$u_{i+1} = u_i + \dot{u}_i \Delta t + \frac{(\Delta t)^2}{4}(\ddot{u}_{i+1} + \ddot{u}_i)$	$u_{i+1} = u_i + \dot{u}_i \Delta t + (\Delta t)^2 \left(\frac{1}{8}\ddot{u}_{i+1} + \frac{1}{3}\ddot{u}_i \right) \quad (5.4.6)$

TABLE 5.4.2 NEWMARK'S METHOD: LINEAR SYSTEMS†

Special cases

(1) Average acceleration method ($\gamma = \frac{1}{2}, \beta = \frac{1}{4}$)

(2) Linear acceleration method ($\gamma = \frac{1}{2}, \beta = \frac{1}{6}$)

1.0 Initial calculations

$$1.1 \quad \ddot{u}_0 = \frac{p_0 - c\dot{u}_0 - ku_0}{m},$$

1.2 Select Δt .

$$1.3 \quad \hat{k} = k + \frac{\gamma}{\beta \Delta t} c + \frac{1}{\beta (\Delta t)^2} m.$$

$$1.4 \quad a = \frac{1}{\beta \Delta t} m + \frac{\gamma}{\beta} c; \text{ and } b = \frac{1}{2\beta} m + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) c.$$

2.0 Calculations for each time step, i

$$2.1 \quad \Delta \hat{p}_i = \Delta p_i + a\dot{u}_i + b\ddot{u}_i.$$

$$2.2 \quad \Delta u_i = \frac{\Delta \hat{p}_i}{\hat{k}}.$$

$$2.3 \quad \Delta \dot{u}_i = \frac{\gamma}{\beta \Delta t} \Delta u_i - \frac{\gamma}{\beta} \dot{u}_i + \Delta t \left(1 - \frac{\gamma}{2\beta} \right) \ddot{u}_i.$$

$$2.4 \quad \Delta \ddot{u}_i = \frac{1}{\beta (\Delta t)^2} \Delta u_i - \frac{1}{\beta \Delta t} \dot{u}_i - \frac{1}{2\beta} \ddot{u}_i.$$

$$2.5 \quad u_{i+1} = u_i + \Delta u_i, \dot{u}_{i+1} = \dot{u}_i + \Delta \dot{u}_i, \ddot{u}_{i+1} = \ddot{u}_i + \Delta \ddot{u}_i.$$

3.0 Repetition for the next time step. Replace i by $i + 1$ and implement steps 2.1 to 2.5 for the next time step.

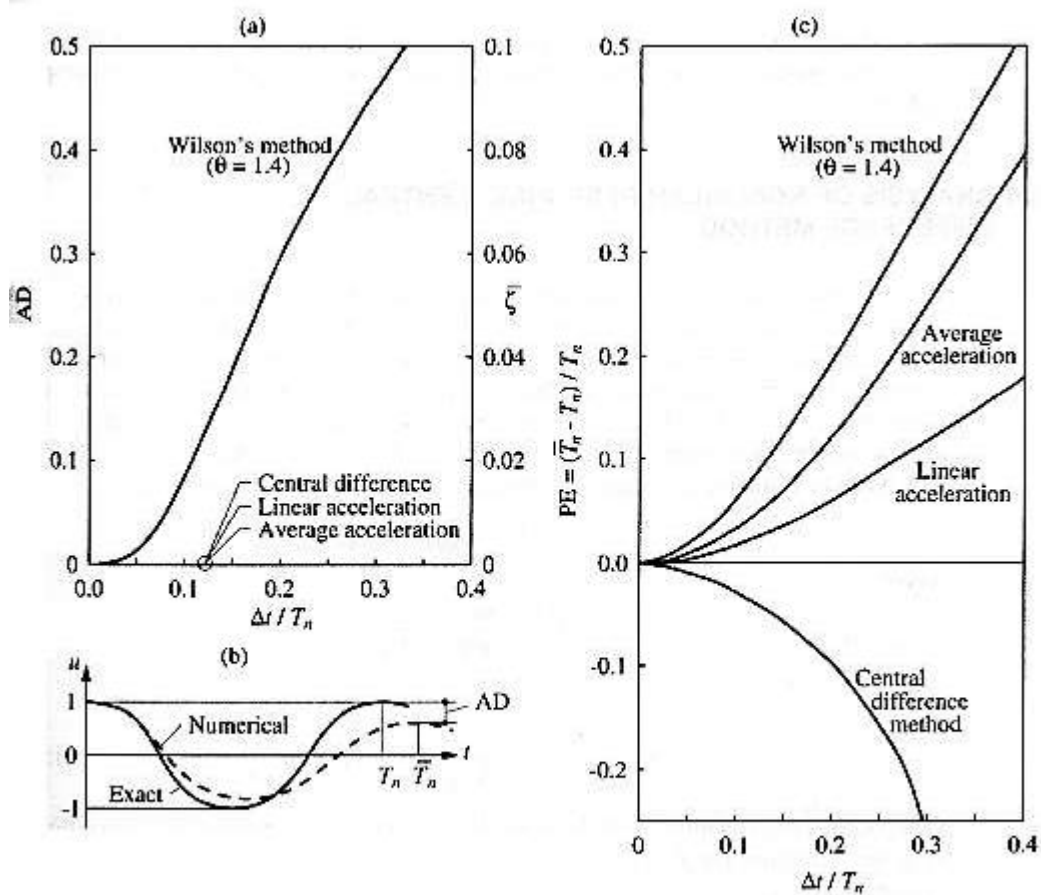


Figure 5.5.2 (a) Amplitude decay versus $\Delta t/T_n$; (b) definition of AD and PE; (c) period elongation versus $\Delta t/T_n$.

TABLE 5.7.1 MODIFIED NEWTON-RAPHSON ITERATION

1.0 *Initialize data.*

$$u_{i+1}^{(0)} = u_i \quad f_S^{(0)} = (f_S)_i \quad \Delta R^{(1)} = \Delta \hat{p}_i \quad \hat{k}_T = \hat{k}_i$$

2.0 *Calculations for each iteration, $j = 1, 2, 3, \dots$*

2.1 Solve: $\hat{k}_T \Delta u^{(j)} = \Delta R^{(j)} \Rightarrow \Delta u^{(j)}$.

2.2 $u_{i+1}^{(j)} = u_{i+1}^{(j-1)} + \Delta u^{(j)}$.

2.3 $\Delta f^{(j)} = f_S^{(j)} - f_S^{(j-1)} + (\hat{k}_T - k_T) \Delta u^{(j)}$.

2.4 $\Delta R^{(j+1)} = \Delta R^{(j)} - \Delta f^{(j)}$.

3.0 *Repetition for next iteration. Replace j by $j + 1$ and repeat calculation steps 2.1 to 2.4.*

TABLE 5.7.2 NEWMARK'S METHOD: NONLINEAR SYSTEMS

Special cases

(1) Average acceleration method ($\gamma = \frac{1}{2}, \beta = \frac{1}{4}$)

(2) Linear acceleration method ($\gamma = \frac{1}{2}, \beta = \frac{1}{6}$)

1.0 Initial calculations

$$1.1 \quad \ddot{u}_0 = \frac{p_0 - c\dot{u}_0 - (fs)_0}{m}$$

1.2 Select Δt .

$$1.3 \quad a = \frac{1}{\beta \Delta t} m + \frac{\gamma}{\beta} c; \text{ and } b = \frac{1}{2\beta} m + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) c.$$

2.0 Calculations for each time step, i

$$2.1 \quad \Delta \hat{p}_i = \Delta p_i + a\dot{u}_i + b\ddot{u}_i.$$

2.2 Determine the tangent stiffness k_i .

$$2.3 \quad \hat{k}_i = k_i + \frac{\gamma}{\beta \Delta t} c + \frac{1}{\beta (\Delta t)^2} m.$$

2.4 Solve for Δu_i from \hat{k}_i and $\Delta \hat{p}_i$ using the iterative procedure of Table 5.7.1.

$$2.5 \quad \Delta \dot{u}_i = \frac{\gamma}{\beta \Delta t} \Delta u_i - \frac{\gamma}{\beta} \dot{u}_i + \Delta t \left(1 - \frac{\gamma}{2\beta} \right) \ddot{u}_i.$$

$$2.6 \quad \Delta \ddot{u}_i = \frac{1}{\beta (\Delta t)^2} \Delta u_i - \frac{1}{\beta \Delta t} \dot{u}_i - \frac{1}{2\beta} \ddot{u}_i.$$

$$2.7 \quad u_{i+1} = u_i + \Delta u_i, \dot{u}_{i+1} = \dot{u}_i + \Delta \dot{u}_i, \ddot{u}_{i+1} = \ddot{u}_i + \Delta \ddot{u}_i.$$

3.0 *Repetition for the next time step.* Replace i by $i + 1$ and implement steps 2.1 to 2.7 for the next time step.

TABLE 14.4.1 GENERATION OF FORCE-DEPENDENT RITZ VECTORS

-
1. Determine the first vector, ψ_1 .
 - a. Determine y_1 by solving: $ky_1 = s$.
 - b. Normalize y_1 : $\psi_1 = y_1 \div (y_1^T m y_1)^{1/2}$.
 2. Determine additional vectors, $\psi_n, n = 2, 3, \dots, J$.
 - a. Determine y_n by solving: $ky_n = m\psi_{n-1}$.
 - b. Orthogonalize y_n with respect to previous $\psi_1, \psi_2, \dots, \psi_{n-1}$ by repeating the following steps for $i = 1, 2, \dots, n-1$:
 - $a_{in} = \psi_i^T m y_n$.
 - $\hat{\psi}_n = y_n - a_{in} \psi_i$.
 - $y_n = \hat{\psi}_n$.
 - c. Normalize $\hat{\psi}_n$: $\psi_n = \hat{\psi}_n \div (\hat{\psi}_n^T m \hat{\psi}_n)^{1/2}$.
-

TABLE 15.2.2 NEWMARK'S METHOD: LINEAR SYSTEMS

1.0 *Initial calculations*

$$1.1 \quad (q_n)_0 = \frac{\phi_n^T \mathbf{m} \mathbf{u}_0}{\phi_n^T \mathbf{m} \phi_n}; \quad (\dot{q}_n)_0 = \frac{\phi_n^T \mathbf{m} \dot{\mathbf{u}}_0}{\phi_n^T \mathbf{m} \phi_n}.$$

$$\mathbf{q}_0^T = ((q_1)_0, \dots, (q_J)_0) \quad \dot{\mathbf{q}}_0^T = ((\dot{q}_1)_0, \dots, (\dot{q}_J)_0)$$

$$1.2 \quad \mathbf{P}_0 = \Phi^T \mathbf{p}_0.$$

$$1.3 \quad \text{Solve: } \mathbf{M} \ddot{\mathbf{q}}_0 = \mathbf{P}_0 - \mathbf{C} \dot{\mathbf{q}}_0 - \mathbf{K} \mathbf{q}_0 \Rightarrow \ddot{\mathbf{q}}_0.$$

$$1.4 \quad \text{Select } \Delta t.$$

$$1.5 \quad \hat{\mathbf{K}} = \mathbf{K} + \frac{\gamma}{\beta \Delta t} \mathbf{C} + \frac{1}{\beta (\Delta t)^2} \mathbf{M}.$$

$$1.6 \quad \mathbf{a} = \frac{1}{\beta \Delta t} \mathbf{M} + \frac{\gamma}{\beta} \mathbf{C}; \quad \mathbf{b} = \frac{1}{2\beta} \mathbf{M} + \Delta t \left(\frac{\gamma}{2\beta} - 1 \right) \mathbf{C}.$$

2.0 *Calculations for each time step i*

$$2.1 \quad \mathbf{P}_i = \Phi^T \mathbf{p}_i.$$

$$2.2 \quad \Delta \hat{\mathbf{P}}_i = \Delta \mathbf{P}_i + \mathbf{a} \dot{\mathbf{q}}_i + \mathbf{b} \ddot{\mathbf{q}}_i.$$

$$2.3 \quad \text{Solve: } \hat{\mathbf{K}} \Delta \mathbf{q}_i = \Delta \hat{\mathbf{P}}_i \Rightarrow \Delta \mathbf{q}_i.$$

$$2.4 \quad \Delta \dot{\mathbf{q}}_i = \frac{\gamma}{\beta \Delta t} \Delta \mathbf{q}_i - \frac{\gamma}{\beta} \dot{\mathbf{q}}_i + \Delta t \left(1 - \frac{\gamma}{2\beta} \right) \ddot{\mathbf{q}}_i.$$

$$2.5 \quad \Delta \ddot{\mathbf{q}}_i = \frac{1}{\beta (\Delta t)^2} \Delta \mathbf{q}_i - \frac{1}{\beta \Delta t} \dot{\mathbf{q}}_i - \frac{1}{2\beta} \ddot{\mathbf{q}}_i.$$

$$2.6 \quad \mathbf{q}_{i+1} = \mathbf{q}_i + \Delta \mathbf{q}_i, \quad \dot{\mathbf{q}}_{i+1} = \dot{\mathbf{q}}_i + \Delta \dot{\mathbf{q}}_i, \quad \ddot{\mathbf{q}}_{i+1} = \ddot{\mathbf{q}}_i + \Delta \ddot{\mathbf{q}}_i.$$

$$2.7 \quad \mathbf{u}_{i+1} = \Phi \mathbf{q}_{i+1}.$$

3.0 *Repetition for the next time step.* Replace i by $i + 1$ and implement steps 2.1 to 2.7 for the next time step.

TABLE 15.3.1 AVERAGE ACCELERATION METHOD: NONLINEAR SYSTEMS

1.0 *Initial calculations*

1.1 Solve: $m\ddot{u}_0 = p_0 - c\dot{u}_0 - (f_S)_0 \Rightarrow \ddot{u}_0$.

1.2 Select Δt .

1.3 $a = \frac{4}{\Delta t}m + 2c$; and $b = 2m$.

2.0 *Calculations for each time step i*

2.1 $\Delta \hat{p}_i = \Delta p_i + a\dot{u}_i + b\ddot{u}_i$.

2.2 Determine the tangent stiffness matrix k_i .

2.3 $\hat{k}_i = k_i + \frac{2}{\Delta t}c + \frac{4}{(\Delta t)^2}m$.

2.4 Solve for Δu_i from \hat{k}_i and $\Delta \hat{p}_i$ using the iterative procedure of Table 15.3.2.

2.5 $\Delta \dot{u}_i = \frac{2}{\Delta t} \Delta u_i - 2\dot{u}_i$.

2.6 $\Delta \ddot{u}_i = \frac{4}{(\Delta t)^2} \Delta u_i - \frac{4}{\Delta t} \dot{u}_i - 2\ddot{u}_i$.

2.7 $u_{i+1} = u_i + \Delta u_i$, $\dot{u}_{i+1} = \dot{u}_i + \Delta \dot{u}_i$, and $\ddot{u}_{i+1} = \ddot{u}_i + \Delta \ddot{u}_i$.

3.0 *Repetition for the next time step.* Replace i by $i + 1$ and implement steps 2.1 to 2.6 for the next time step.

TABLE 15.3.2 MODIFIED NEWTON-RAPHSON ITERATION

1.0	Initialize data.
	$\mathbf{u}_{i+1}^{(0)} = \mathbf{u}_i \quad \mathbf{f}_S^{(0)} = (\mathbf{f}_S)_i \quad \Delta \mathbf{R}^{(1)} = \Delta \hat{\mathbf{p}}_i \quad \hat{\mathbf{k}}_T = \hat{\mathbf{k}}_i$
2.0	Calculations for each iteration, $j = 1, 2, 3, \dots$
2.1	Solve: $\hat{\mathbf{k}}_T \Delta \mathbf{u}^{(j)} = \Delta \mathbf{R}^{(j)} \implies \Delta \mathbf{u}^{(j)}$
2.2	$\mathbf{u}_{i+1}^{(j)} = \mathbf{u}_{i+1}^{(j-1)} + \Delta \mathbf{u}^{(j)}$
2.3	$\Delta \mathbf{f}^{(j)} = \mathbf{f}_S^{(j)} - \mathbf{f}_S^{(j-1)} + (\hat{\mathbf{k}}_T - \mathbf{k}_i) \Delta \mathbf{u}^{(j)}$
2.4	$\Delta \mathbf{R}^{(j+1)} = \Delta \mathbf{R}^{(j)} - \Delta \mathbf{f}^{(j)}$
3.0	Repetition for the next iteration. Replace j by $j + 1$ and repeat calculation steps 2.1 to 2.4.

TABLE 15.3.3 WILSON'S METHOD: NONLINEAR SYSTEMS**1.0 Initial calculations**

1.1 Solve $m\ddot{\mathbf{u}}_0 = \mathbf{p}_0 - \mathbf{c}\dot{\mathbf{u}}_0 - (\mathbf{f}_S)_0 \Rightarrow \ddot{\mathbf{u}}_0$.

1.2 Select Δt and θ .

1.3 $\mathbf{a} = \frac{6}{\theta \Delta t} \mathbf{m} + 3\mathbf{c}$; and $\mathbf{b} = 3\mathbf{m} + \frac{\theta \Delta t}{2} \mathbf{c}$.

2.0 Calculations for each time step, i

2.1 $\delta \hat{\mathbf{p}}_i = \theta(\Delta \mathbf{p}_i) + \mathbf{a}\dot{\mathbf{u}}_i + \mathbf{b}\ddot{\mathbf{u}}_i$.

2.2 Determine the tangent stiffness matrix \mathbf{k}_i .

2.3 $\hat{\mathbf{k}}_i = \mathbf{k}_i + \frac{3}{\theta \Delta t} \mathbf{c} + \frac{6}{(\theta \Delta t)^2} \mathbf{m}$.

2.4 Solve for $\delta \mathbf{u}_i$ from $\hat{\mathbf{k}}_i$ and $\delta \hat{\mathbf{p}}_i$ using the iterative procedure of Table 15.3.2.

2.5 $\delta \ddot{\mathbf{u}}_i = \frac{6}{(\theta \Delta t)^2} \delta \mathbf{u}_i - \frac{6}{\theta \Delta t} \dot{\mathbf{u}}_i - 3\ddot{\mathbf{u}}_i$; and $\Delta \ddot{\mathbf{u}}_i = \frac{1}{\theta} \delta \ddot{\mathbf{u}}_i$.

2.6 $\Delta \dot{\mathbf{u}}_i = (\Delta t) \ddot{\mathbf{u}}_i + \frac{\Delta t}{2} \Delta \ddot{\mathbf{u}}_i$; and $\Delta \mathbf{u}_i = (\Delta t) \dot{\mathbf{u}}_i + \frac{(\Delta t)^2}{2} \ddot{\mathbf{u}}_i + \frac{(\Delta t)^2}{6} \Delta \ddot{\mathbf{u}}_i$.

2.7 $\mathbf{u}_{i+1} = \mathbf{u}_i + \Delta \mathbf{u}_i$, $\dot{\mathbf{u}}_{i+1} = \dot{\mathbf{u}}_i + \Delta \dot{\mathbf{u}}_i$, and $\ddot{\mathbf{u}}_{i+1} = \ddot{\mathbf{u}}_i + \Delta \ddot{\mathbf{u}}_i$.

3.0 Repetition for the next time step. Replace i by $i + 1$ and implement steps 2.1 to 2.7 for the next time step.